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# ELEMENTARY THEORY OF THE PILE

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# ELEMENTARY THEORY OF THE PILE

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I shall attempt in what follows to give an outline of the theory of a chain reacting pile working with natural uranium and graphite.

The results and the methods that I am going to discuss have been obtained partly independently and partly in collaboration with many people who participated in the early development work on the chain reaction. Very important contributions to the theoretical ideas were given by Szilard and Wigner. Many physicists contributed experimental results that helped to lead the way. Among them H. L. Anderson and W. H. Zinn first at Columbia University and later at the Metallurgical Laboratory of the University of Chicago; R. R. Wilson and E. Creutz at Princeton; Allison, Whitaker and V. C. Wilson at the University of Chicago. The production of the chain reaction was finally achieved in the Metallurgical Laboratory directed by A. H. Compton.

## Absorption and Production of Neutrons in a Pile

We consider a mass, "the pile", containing uranium spread in some suitable arrangement throughout a block of graphite. Whenever a fission takes place in this system an average number $\nu$ of neutrons is emitted with a continuous distribution of energy of the order of magnitude of a million of electron volts. After a neutron is emitted, its energy decreases by elastic collisions with the atoms of carbon and to some extent also by inelastic collisions with the uranium atoms. In the majority of cases the neutrons will be slowed down to thermal energies.

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This process requires about 100 collisions with carbon atoms. After the energy of the neutron is reduced to thermal value, the neutron keeps on diffusing until it is finally absorbed. In several cases, however, it will happen that the neutron is absorbed before the slowing down process is completed.

The neutron may be absorbed by either the carbon or the uranium. The absorption cross-section of carbon for neutrons of thermal energy is quite small; its value being approximately $.005 \times 10^{-24} cm^2$. For graphite of density 1.6 this corresponds to a mean free path for absorption of about 25 m. It is believed that the absorption cross-section follows the $1/v$ law and consequently the absorption cross-section which is already quite small at thermal energies becomes practically negligible for neutrons of higher energy. It is, therefore, a sufficiently good approximation to assume that absorption by carbon during the slowing down process can be neglected.

The absorption of a neutron by uranium may lead either to fission or to absorption by an $(n, \gamma)$ process. We shall refer to this last possibility as the process of resonance absorption. The relative importance of fission and resonance absorption in the different energy intervals is not the same. In this respect we can consider roughly three intervals.

a. Neutrons with energy above the fission threshold of $U^{238}$. We can call these conventionally "fast neutrons". For fast neutrons the most important absorption process is fission which normally takes



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place in the abundant isotope $U^{238}$. Resonance absorption is smaller but not negligible.

b. Neutrons of energy below the fission threshold of $U^{238}$ and above thermal energy, we shall refer to as "epi-thermal neutrons". For epi-thermal neutrons the most important absorption process is the resonance capture. The cross-section for this process as a function of energy is quite irregular and presents a large number of resonance maxima that can be fairly well represented by the Breit-Wigner theory.

c. Neutrons having thermal agitation energy or "thermal neutrons". For thermal neutrons both the resonance and fission absorption processes are important. In this energy range both cross-sections follow approximately the 1/v law and therefore their relative importance becomes practically independent of the energy. Let $\sigma_f$ and $\sigma_r$ be the cross-sections for fission and resonance absorption for neutrons of energy kT and $\eta$ be the average number of neutrons emitted when a thermal neutron is absorbed by uranium. $\eta$ differs from $\nu$ since only the fraction $\sigma_f/(\sigma_r + \sigma_f)$ of all the thermal neutrons absorbed by uranium produces a fission. It is, therefore,

(1) $$\eta = \nu\, \sigma_f/(\sigma_f + \sigma_r)$$

The preceeding discussion leads one to conclude that only a fraction of the original fast neutrons produced will end up by producing a fission process. For systems of finite size further losses of neutrons will be expected by leakage outside the pile.

Limiting ourselves for the present to systems of practically infinite dimensions we shall call P the probability that a fast neutron ultimately is absorbed by the fission process. The average number of

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neutrons produced in the "second generation" by the first neutron will then be

(2) $$k = P\nu$$

k is usually called the "reproduction factor" of the system. A self-sustaining chain reaction evidently is possible only when $k > 1$. If this is the case the reaction actually will take place provided the leakage loss of neutrons is sufficiently small. This of course can always be achieved if the size of the pile is large enough.

Life History of a Neutron

When a fast neutron is first emitted in our pile, the following events may take place.

a. The neutron has a small probability to be absorbed by uranium before its energy has been appreciably decreased. If this is the case, the absorption leads often to fission of $U^{238}$. The probability of such fast fissions however is usually only a few percent. Indeed, if the system contains little uranium and a large amount of carbon, the elastic collisions with carbon tend to reduce very rapidly the energy to a value below the fission threshold of $U^{238}$. If the system on the other hand is very rich in uranium, the inelastic collision processes become very probable and rapidly reduce the energy of the original fast neutron to a fairly low value before it has a chance to produce a fission in $U^{238}$.

b. In the large majority of the cases, therefore, the neutron is not absorbed as a fast neutron and it looses rapidly its energy mostly due to collision against the carbon atoms. One can prove in an elementary way that it takes about 6.3 collisions against the carbon

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atoms to reduce the energy by an average factor of e . Consequently, it will take about 14.6 collisions in order to reduce the energy by a factor of 10; and about 110 collisions to reduce the energy from one million of electron volts to the thermal energy value of 1/40 of a volt. While this slowing down process is in progress the neutron may be absorbed by the resonance process in uranium. We shall call p the probability that a neutron is not absorbed before reaching thermal energy. One of the most important factors in designing a pile consists in trying to minimize the probability that neutrons are removed from the system by resonance absorption during the slowing down.

c. If the neutron is not absorbed during the slowing down process it eventually reaches thermal energy and ultimately will be absorbed by either uranium or carbon. If uranium and carbon were mixed uniformly, the probability for these two events would be in the ratio of absorption cross-sections of uranium and carbon for thermal neutrons multiplied by the atomic concentrations of the two elements. Since actually the mixture is not uniform this is only approximately true. We shall call f the probability that a thermal neutron is absorbed by uranium. In designing a chain reacting pile one will normally try to adjust things so as to have both f and p as large as possible. Unfortunately, the two requirements are contradictory because in order to make f large one shall try to build a system very rich in uranium in order to reduce the probability of absorption of thermal neutrons by carbon. On the other hand in a system containing a relatively small amount of carbon the slowing down process will be relatively

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slow and consequently the probability of resonance absorption during the slowing down will be large.

It is clear therefore that one shall have to conciliate two opposite requirements by finding an optimum value for the ratio of uranium to carbon.

In a homogeneous mixture of uranium and carbon the values of f and p depend only on the relative concentrations of the two elements. If we do not restrict ourselves, however, to homogeneous mixtures only, one can try to obtain a more favorable situation by proper arrangement of the geometrical distribution of the two components. This actually is possible to a considerable extent due to the following circumstances. The resonance absorption which is responsible for the loss of neutrons during the slowing down has very sharp cross-section maxima of the Breit-Wigner type. If the uranium, therefore, instead of being spread through the graphite mass is concentrated in rather sizeable lumps, we will expect that the uranium in the interior of a lump will be shielded by a thin surface layer from the action of neutrons with energy close to a resonance maximum. Therefore, the resonance absorption of a uranium atom inside the lump will be much less than it would be for an isolated atom. Of course, self-absorption in a lump reduces not only the resonance absorption but also the thermal absorption of uranium. One can expect theoretically, however, and experiment has confirmed, that at least up to a certain size of lumps the gain obtained by reducing the resonance loss of neutrons over-balances by a considerable amount the loss due to a lesser absorption of thermal neutrons.

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The typical structure of a pile is a lattice of uranium lumps embedded in a matrix of graphite. The lattice may be for instance a cubic lattice of lumps or a lattice of rods of uranium. This latter arrangement is slightly less efficient from the point of view of the neutron absorption balance but often presents some practical advantages since it makes the removal of the heat produced by the pile more easy. In the present discussion we shall consider only lattices of lumps.

In order to evaluate the reproduction factor one must be able to calculate the probabilities for the various processes mentioned. We shall briefly indicate some points of view which may be used in the practical calculation.

Probability of Fission Before Slowing Down

The value of this quantity is very easily calculable for a very small lump of uranium. In this case it is obviously given by

(3) $$P_p = \sigma_p \, n d$$

where $\sigma_p$ is the average value of the fission cross-section for fission neutrons; n is the concentration of uranium atoms in the lump, and d is the average value of the distance that the neutron produced in the lump must travel before reaching the surface of the lump. The case of a lump of larger size is more complicated since then multiple collision processes become important and both elastic and inelastic scattering play a considerable role. In particular the last process for a lump of large size effectively slows down the neutrons before



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the fission threshold of $U^{238}$ and brings them down to an energy level in which they are readily absorbed by the resonance process.

Resonance Absorption

If we had a single atom of uranium in a graphite medium where fast neutrons are produced and slowed down to thermal energy, the probability per unit time of a resonance absorption process of neutrons with energy larger than thermal energy would be given by the following expression:

$$(4) \qquad \frac{q\lambda}{.158} \int \sigma(E) \frac{dE}{E}$$

where q is the number of fast neutrons entering the system per unit time and unit volume; $\lambda$ is the mean free path and $\sigma(E)$ is the resonance absorption cross-section at energy E. The integral must be taken between a low limit just above thermal energy and an upper limit equal to the average energy of the fission neutrons. One will expect that the largest contribution to the integral will be due to the Breit-Wigner peaks of $\sigma(E)$.

The above formula would be very much in error in the case of a lattice of lumps. As already indicated, this is due to the fact that inside a lump there is an important self-screening effect that reduces very considerably the density of neutrons having energy close to a resonance maximum.

The best approach to a practical solution to the problem is therefore a direct measurement of the number of neutrons absorbed by resonance in lumps of uranium of various sizes.



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Measurements of this type have been performed first at Princeton University and the results have been summarized in practical formulae that are used in the calculations.

Probability of Absorption at Thermal Energies

If uranium and carbon were uniformly mixed a thermal neutron would have a probability

$$(5) \qquad \frac{N_U \sigma_U}{N_C \sigma_C + N_U \sigma_U}$$

to be absorbed by uranium. In this formula $N_C$ and $N_U$ represent the numbers of atoms of carbon and of uranium per unit volume; and $\sigma_C$ and $\sigma_U$ represent the cross-sections of carbon and uranium for thermal neutrons.

More complicated is the case of a lattice distribution of lumps of uranium in graphite, since the density of thermal neutrons throughout the system is not uniform but is large at the places far from the uranium lumps and smaller near and inside the uranium lumps, due to the fact that the absorption of thermal neutrons is much greater in uranium than in graphite. Let $\overline{n_C}$ and $\overline{n_U}$ be the average densities of thermal neutrons in graphite and in the uranium lumps. The number of thermal neutrons absorbed by uranium and by carbon will be proportional to $N_U \sigma_U \overline{n_U}$ and $N_C \sigma_C \overline{n_C}$ and we will have therefore, instead of Eq. (5) the corrected formula

$$(6) \qquad f = \frac{N_U \sigma_U \overline{n_U}}{N_U \sigma_U \overline{n_U} + N_C \sigma_C \overline{n_C}}$$

For practical purposes it is usually sufficiently accurate to calculate $\overline{n_C}$ and $\overline{n_U}$ using the diffusion theory. The approximation is made to substitute the lattice cell by a spherical cell having volume equal to that of the actual cell with the boundary condition that the radial



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derivative of the density of neutrons vanishes at the surface of the sphere. It is also assumed that the number of neutrons that are slowed down to thermal energies per unit time and unit volume is constant throughout in the graphite part of the cell. This approximation is fairly correct provided the dimensions of the cell are not too large. With these assumptions one finds the following formula for the probability f that thermal neutrons be absorbed by uranium:

$$(7) \quad f = \frac{3\alpha^2}{\alpha^3 - \beta^3} \, \frac{(1-\alpha)(1+\beta)e^{-(\beta-\alpha)} - (1+\alpha)(1-\beta)e^{\beta-\alpha}}{(\alpha+s-s\alpha)(1+\beta)e^{-\beta+\alpha} - (\alpha+s+s\alpha)e^{\beta-\alpha}}$$

where $\alpha$ and $\beta$ represent the radius of the lump and the radius of the cell expressed taking the diffusion length in graphite $l = \sqrt{\lambda\Lambda/3}$ as unit of length. It is further

$$(8) \quad s = \frac{\lambda}{\sqrt{3}\,l} \, \frac{1+\gamma}{1-\gamma}$$

where $\gamma$ is the reflection coefficient of the lump for thermal neutrons.

## Lattice Containing a Large Number of Cells

The density of neutrons of any given energy in a lattice containing a large number of cells is a function of the position in the lattice. One can arrive at a simple mathematical description of the behavior of such a system by neglecting in first approximation the local variation of such functions due to the periodic structure of the lattice and substituting for the actually inhomogeneous system an equivalent homogeneous system. In this section we shall accordingly simplify the problem by substituting for all densities of neutrons, values obtained by averaging the actual values over the volume of the cell. The densities will then be represented by smooth functions such as one would expect in



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homogeneous uranium-graphite mixture.

Let $Q(x, y, z)$ be the number of fast neutrons produced per unit time and unit volume at each position in the lattice. These neutrons diffuse through the mass and are slowed down. During this process some of the n are absorbed at resonance. Let $q(x, y, z)$ be the number of neutrons per unit time and unit volume which become thermal at the position $x, y, z$. $q$ is called the "density of the nascent thermal neutrons."

We shall assume that if an original fast neutron is generated at a point O the probability that it becomes thermal at a given place has a Gaussian distribution around O. This assumption may be justified by considering that the diffusion process of slowing down consists of very many free paths. Experimentally one finds that the distribution curve of the nascent thermal neutrons around a point source of fast neutrons is represented only approximately by a Gaussian distribution and formulas have been used in which the actual distribution is described as a superposition of two or three Gaussian curves with different ranges. For the purpose of the present discussion, however, we shall take only one. For each fast neutron produced only p neutrons reach thermal energy. The distribution of nascent thermal neutrons produced by a source of strength 1 placed at the origin of the coordinate shall then be represented by

$$(9) \qquad q_1 = \frac{p}{\pi^{3/2} r_0^3} e^{-\frac{r^2}{r_0^2}}$$

For graphite of density 1.6 the range $r_0$ is of the order of 35 cm. The density of nascent thermal neutrons at the point P can be expressed in terms of Q by adding up the contribution of all the infinitesimal sources $Q(P')d\tau'$ ($d\tau'$ represents the volume element around the point P').

We obtain in this way



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$$(10) \qquad q(P) = \frac{P}{\pi^{3/2} r_0^2} \int Q(P') e^{-\frac{(P'-P)^2}{r_0^2}} d\tau'$$

The density $n(x, y, z)$ of the thermal neutrons is connected to q by the differential equation

$$(11) \qquad \frac{\lambda v}{3} \Delta n - \frac{v}{\Lambda} n + q = 0$$

where $\lambda$ is the collision mean free path of thermal neutrons; v is their velocity and $\Lambda$ is the mean free path for absorption of a thermal neutron. Equation (11) is obtained by expressing a local balancing of all processes whereby the number of thermal neutrons at each place tends to increase or decrease. The first term represents the increase in number of neutrons due to diffusion ( $\lambda v/3$ is the diffusion coefficient of thermal neutrons). The second term represents the loss of neutrons due to absorption and the third term represents the effect of the nascent thermal neutrons.

It should be noted that the absorption mean free path $\Lambda$ in Eq. (11) is much shorter than the corresponding quantity $\Lambda_0$ in pure graphite. Indeed the absorption in a lattice is due mostly to the uranium. In first approximation $\Lambda$ is given by

$$(12) \qquad \Lambda = (1 - f) \Lambda_0$$

In practical cases $\Lambda$ may be of the order of magnitude of 300 cm, whereas $\Lambda_0$ in graphite without uranium is about 2500 cm.

When a thermal neutron is absorbed by uranium $\eta$ new neutrons are produced by fission. This number should be increased by a few percent in order to take into account the effect of the small probability of fast fission. Let $\varepsilon\eta$ be the total number of fast neutrons so corrected.



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The number of thermal neutrons absorbed per unit volume and unit time is $\frac{vn}{\Lambda}$. Of these the fraction f is absorbed by uranium. We have, therefore,

$$(13) \qquad Q = f\eta\varepsilon \frac{v}{\Lambda} n + Q_0$$

where $f\eta\varepsilon\frac{v}{\Lambda}$ n represents the number of fast neutrons produced in the chain reaction process and $Q_0$ represents the number of fast neutrons produced by an outside source if one is present. In most cases, of course, $Q_0$ shall be equal to 0. From Eqs. (10), (11) and (13) we can eliminate all unknowns except n and we find

$$(14) \qquad \frac{1}{\lambda\Lambda} n - \Delta n = \frac{3p\varepsilon\eta f}{\Lambda \pi^{3/2} r_0^3 \Lambda\lambda} \int n(P') e^{-\frac{(P'-P)^2}{r_0^2}} d\tau'$$

$$+ \frac{3p}{\pi^{3/2} r_0^3 \lambda v} \int Q_0(P') e^{-\frac{(P'-P)^2}{r_0^2}} d\tau$$

A solution of this equation is obtained readily by developing both $Q_0$ and n in a Fourier series. The general term of this development corresponding to $Q_0$ of the form $Q_0 \sin \omega_1 x \sin \omega_2 y \sin \omega_3 z$, is:

$$(15) \qquad n = \frac{(\Lambda p Q_0/v) \sin \omega_1 x \sin \omega_2 y \sin \omega_3 z}{\left(1 + \frac{\lambda\Lambda}{3}\omega^2\right) e^{\frac{r_0^2}{4}\omega^2} - \xi p f \eta}$$

where $\omega^2 = \omega_1^2 + \omega_2^2 + \omega_3^2$.

When the dimensions of the pile are finite but very large compared with the mean free path, the boundary condition is that all densities must vanish at the surface. If the pile, for example, is a cube of side a and the origin of the coordinates is taken in one of the corners it is:



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(16) $$\omega_1 = \frac{\pi n_1}{a}, \quad \omega_2 = \frac{\pi n_2}{a}, \quad \omega_3 = \frac{\pi n_3}{a}$$

where $n_1$, $n_2$, $n_3$ are positive integral numbers that define the various Fourier components. The critical dimensions of the system are such that the denominator of the Eq. (15) vanishes for the 1, 1, 1 harmonic, since in this case the density of the neutrons becomes infinitely large. The critical condition can be expressed therefore by the equation

(17) $$\left(1 + \frac{3\pi^2}{a^2} \cdot \frac{\lambda\Lambda}{3}\right) e^{\frac{3\pi^2}{a^2} \frac{r_0^2}{4}} = \varepsilon p f \eta$$

The right hand side in this formula is the reproduction factor k for a system of infinite size. We can therefore write the critical condition as follows:

(18) $$k = \left(1 + \frac{3\pi^2}{a^2} \frac{\lambda\Lambda}{3}\right) e^{\frac{3\pi^2}{a} \frac{r_0^2}{4}}$$

In most cases both the exponent of e and the term added to 1 in the parenthesis are small compared with 1 and so the previous expression can be simplified to

(19) $$k = 1 + \frac{3\pi^2}{a^2} \left(\frac{\lambda\Lambda}{3} + \frac{r_0^2}{4}\right)$$

Description of A Graphite Pile at Argonne Laboratory

The first pile was erected under the West Stands on the campus of the University of Chicago at the end of 1942. After having been operated there for a few months it was moved to the Argonne Laboratory near Chicago where it has been used until now for various research purposes. A brief description of the main features of that pile follows.



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The lattice of the pile is not the same throughout the structure. Since only a small amount of uranium metal was available at that time, metal has been used in the central portion of the pile and uranium oxide in the outer portion.

The intensity of operation of the pile is recorded by a number of $BF_3$ ionization chambers connected to amplifiers or to galvanometers.

Since this pile has no cooling devices built into it, the power produced is limited by the necessity of avoiding an excessive temperature rise. The pile could be operated indefinitely at a power of 2 kw and is often operated for periods of the order of 1 or 2 hours up to about 100 kw.

One feature that is often used for neutron research work is the thermal column. This is a column of graphite having side of about 5' x 5' which is built on the center of the top of the pile and goes through the top shield. The neutrons that diffuse from the pile into this column are rapidly reduced to thermal energy so that the neutrons inside the column a few feet above the top of the pile are practically pure thermal neutrons.

The pile is also equipped with a number of holes in the shield and removable stringers of graphite that make it possible to explore phenomena inside the pile or to introduce samples for neutron irradiation.



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