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COHERENCE EFFECTS IN DEEP INELASTIC SCATTERING FROM NUCLEI

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Abstract. A complete theoretical picture of multiple scattering processes in QCD remains elusive. In deep inelastic scattering experiments (DIS), we hope to find out information about the internal structure of nuclei from inelastically scattering high-energy electrons off them. The electrons interact via virtual photon exchange with the target. In the target rest frame the virtual photon splits into a quark-antiquark pair which is then scattered off the target color field. At high energies, coherent multiple scattering within the nucleus takes place. We develop a model that uses a parameterization of scattering cross section of the quark-antiquark pair off the proton to predict the cross section suppression known as shadowing in larger nuclei. This model takes the possibility of multiple scattering into account using Glauber high-energy collision theory. In large nuclei we must also move beyond the eikonal approximation by correcting for the finite lifetime of the quark-antiquark pair inside the nucleus. Results and implications of this model in relation to available data will be discussed. Finally, application of this type of model to predicting gluon densities will be considered. Understanding this process can give us insights into the more complicated scattering taking place in heavy ion colliders such as RHIC and LHC.

DIS FROM PROTONS

Before turning to nuclear targets, it is useful to consider deep inelastic scattering (DIS) off protons first. DIS from protons can be understood using the parton model in which the proton is viewed as a noninteracting collection of quarks and gluons. A lepton interacts with the proton by exchange of a virtual photon γ^* . At the HERA collider at DESY, 27 GeV positrons are made to collide with 920 GeV protons. In an event where a photon scatters electromagnetically from a proton, it can be shown that the lepton-proton cross section is proportional to the structure function $F_2[1]$. This function is given in terms of Bjorken- x , the quark and antiquark densities, and z_f , the electric charge of the quark flavor as

$$F_2(x, Q^2) = x \sum_f z_f^2 [q_f(x, Q^2) + \bar{q}_f(x, Q^2)] \quad (1)$$

In a frame, where the proton is fast moving, Bjorken- x can be interpreted as the momentum fraction of the quark that is hit by the photon and $Q^2 = -q^2 > 0$ is the negative of the photons four-momentum squared. Thus, by observing the cross section of the scattered lepton, information about the internal structure of the proton can be deduced.

Typical values for Q^2 in DIS experiments reach from fractions of a GeV to several TeV. Experiment shows that scattering cross section is dependent on the value of Bjorken- x . The x dependence of the cross section is due to different contributions from valence quarks, which determine the quantum numbers of the nucleon, and sea quarks, virtual quark-antiquark pairs

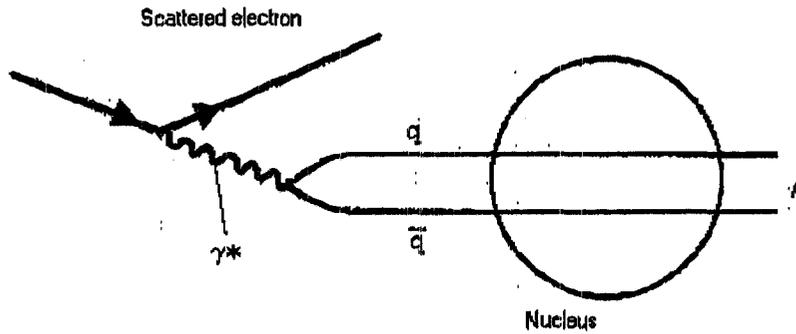


Figure 1: A lepton scatters off a nucleus via a virtual photon. The photon splits into a quark-antiquark pair which subsequently propagates through the target color field.

produced by gluons. Neutrino scattering shows that sea quarks contribute for very small x , while valence quarks dominate at larger x .

We are interested in small values of $x < 0.1$ where DIS is most conveniently described in the proton rest frame. In this frame we see the virtual photon split into a $q\bar{q}$ pair long before reaching the proton/nucleus as shown in figure 1. The probability distribution of the photon split depends on fractional momentum carried by the quark in the $q\bar{q}$ pair, α , and also on their transverse separation, ρ . The wave function of the virtual photon can be found in the literature [2].

Note that the dimensionless structure function F_2 can be written in terms of the scattering cross section of the virtual photon off the proton as

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \sigma^{\gamma^* p} \quad (2)$$

Using the photon wave function and the scattering cross section of the $q\bar{q}$ pairs, we can calculate the scattering cross section of the virtual photon on the proton,

$$\sigma^{\gamma^* p}(x, Q^2) = \int d\alpha \int d^2\rho |\Psi_{\gamma^*}(\alpha, \rho)|^2 \sigma_{q\bar{q}}(x, \rho) \quad (3)$$

The photon wavefunction represents the probability of a $q\bar{q}$ pair forming with separation ρ and momentum fraction α . The second term $\sigma_{q\bar{q}}$ represents the scattering cross section of this pair off the proton. We then integrate over these two quantities to calculate the structure function for a proton. Unfortunately, no theoretical calculation of the cross section of the $q\bar{q}$ pair has been derived. However, several good parameterizations have been devised.

A parameterization of the scattering cross section of the $q\bar{q}$ pair on a proton based on saturation effects has been given by Golec-Biernat and Wüsthoff [3]. At small separations, the pair can interact only via its dipole moment, so the cross section vanishes like ρ^2 (color transparency). However, at large ρ the cross section approaches a constant value. The functional form of their parameterization is given by

$$\sigma_{q\bar{q}}(x, \rho) = \sigma_0 \left(1 - e^{-\frac{\rho^2}{4R_0^2(x)}}\right) \quad (4)$$

where R_0^2 is the x -dependent saturation scale.

$$R_0^2(x) = \left(\frac{x}{x_0}\right)^\lambda \frac{1}{\text{GeV}^2} \quad (5)$$

This model is a simple formulation with only three parameters σ_0 , x , λ and a good fit for all available scattering data at low x , but is not entirely satisfactory. At high energies, this parameterization predicts a constant cross section. This is clearly not what is seen in experiments involving high energy mesons. Nevertheless, the Wüsthoff parameterization works well for all low- x DIS data.

SCATTERING FROM NUCLEI

Of course, an important question to ask is whether these theoretical calculations of proton structure can be extended to more complex nuclei. A first guess might be that for a nucleus with A nucleons $F_2^A = AF_2^p$. Experimental evidence shows this is not the case revealing a phenomenon known as "nuclear shadowing", i.e. a depletion of the nuclear cross section, $F_2^A < AF_2^p$, due to multiple scattering of the $q\bar{q}$ pair inside the nucleus. It is a special advantage of the target rest frame formulation that it allows the use of Glauber's high energy multiple scattering theory [4].

The γ^* -nucleus cross section is then simply calculated by replacing σ_{qq} with the cross section for scattering a $q\bar{q}$ pair off a nucleus. The latter reads in Glauber eikonal approximation:

$$\sigma_{q\bar{q}}^A(x, \rho) = 2 \int (1 - e^{-\frac{\sigma(x, \rho) T(b)}{2}}) d^2b \quad (6)$$

where $T(b)$ is the nuclear thickness, i.e. the integral over the nuclear density ρ_A ,

$$T(b) = \int_{-\infty}^{\infty} \rho_A(b, z) dz \quad (7)$$

and b is the impact parameter.

The eikonal approximation assumes that the quark-antiquark fluctuation lives an infinitely long time (compared to the nuclear radius) and that the transverse separation of the pairs remains frozen by Lorentz time dilation during propagation through the nucleus. These assumptions are fulfilled at infinitely high energies, however all data have to be measured at finite energy. Straightforward calculations with (6) grossly fail to describe experimental data and we have to account for the finite lifetime of the $q\bar{q}$ pair.

The lifetime l_c of the pair, the so-called coherence length, can be estimated from the uncertainty relation. When the virtual photon splits into the $q\bar{q}$ pair, the longitudinal momentum transfer can be calculated as

$$q_l = \frac{Q^2 \alpha (1 - \alpha) + m_f^2 + k_t^2}{2\nu \alpha (1 - \alpha)} = \frac{1}{l_c} \quad (8)$$

where ν is the photon energy. Note that the coherence length depends on the transverse momenta k_t of the particles in the $q\bar{q}$ pair and is therefore undefined in ρ representation. Although this problem was settled in the Green function technique developed in [5] by taking the transverse motion of the $q\bar{q}$ into account, it became necessary to introduce several simplifications, like a constant nuclear density, before one could perform numerical calculations. In this work we include finite coherence length effects only in an approximate way, so that the resulting formulae are simple enough to employ a realistic parameterization of the nuclear density and the dipole cross section.

How does a finite coherence length enter the multiple scattering formula? First it helps to rewrite our prior equation in another form. It can be decomposed into a multiple of the proton

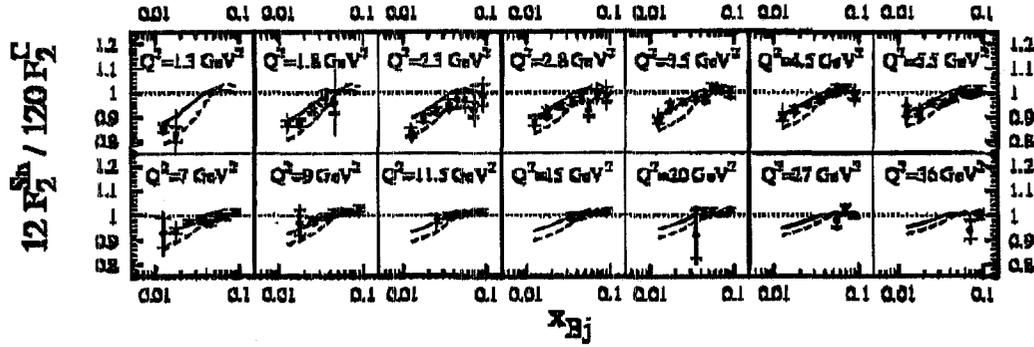


Figure 2: The x dependence of nuclear shadowing of tin relative to carbon. The solid line is calculated from equation 10. The dashed line shows a calculation in the double scattering approximation.

cross section minus a correction term for nuclear shadowing. It is this nuclear shadowing term which is of interest.

$$\sigma_A(x, \rho) = A\sigma_p(x, \rho) - \frac{1}{2} \int d^2b \int_{-\infty}^{\infty} dz_1 \int_1^{\infty} dz_2 \rho_A(\mathbf{b}, z_1) \rho_A(\mathbf{b}, z_2) \sigma_p(x, \rho)^2 e^{-\frac{\sigma(x, \rho)}{2} \int_{z_1}^{z_2} \rho(\mathbf{b}, z) dz} \quad (9)$$

A finite coherence length enters the shadowing term as an oscillating phase factor

$$\sigma_A(x, \rho) = A\sigma_p(x, \rho) - \frac{1}{2} \int d^2b \int_{-\infty}^{\infty} dz_1 \int_1^{\infty} dz_2 \rho_A(\mathbf{b}, z_1) \rho_A(\mathbf{b}, z_2) \sigma_p(x, \rho)^2 e^{-\frac{\sigma(x, \rho)}{2} \int_{z_1}^{z_2} \rho(\mathbf{b}, z) dz} e^{iq(z_1 - z_2)} \quad (10)$$

Note that this phase factor is not introduced ad hoc. In a more careful derivation of (10)[6], the overlap of the wavefunctions of photon, quark and antiquark, produces a factor $e^{iq_1 z_1}$. Taking the absolute square of the matrix element then yields the phase factor in (9). We still have to specify, what to take for the coherence length, i.e. for q_1 . Based on the analysis of [7], where an average coherence length was defined, we set

$$q_1 = \frac{5m_n x}{2} \quad (11)$$

where m_n is the nuclear mass.

We point out that (10) accounts for an arbitrary number of rescatterings as well as for a finite coherence length. This is an advantage over other approaches, which either have to assume an infinite coherence length (6) or can calculate only the double scattering contribution to shadowing.

Note that for small coherence length, i.e. for large q_L , the phase factor in (10) oscillates rapidly and suppresses shadowing. Indeed, the coherence length has to exceed the mean internucleon distance of about 2 fm in order to observe shadowing. One also sees, that since the dipole cross section enters the shadowing term squared, small dipoles do not contribute to shadowing. Only large dipoles, whose mean free path is shorter than the nuclear radius, are shadowed. For very long coherence length, (10) reduces to (9), which is identical to (6).

We can now compare to data without any adjustable parameter. Parameterization for nuclear densities are taken from [8]. The solid line in figure 2 shows calculations for shadowing of tin relative to carbon along with a comparison to a calculation not including only double scattering, i.e. the attenuation exponential in (10) is set to unity. Shadowing data are taken from [9]. Although both curves seem to describe the data well, in shadowing of lead ($A=207$) relative

to a proton, the multiple scattering term clearly has a significant effect at low x and has to be included. Note that the coherence length in actual data is typically of order of the nuclear radius.

As one can see, this calculation shows a slight excess over unity at higher x . This antishadowing effect is also present in the data. However, no such effect is incorporated in our approach. It occurs in our curves, because we divide two nuclear structure functions. Similar calculations for shadowing versus protons by construction do not yield antishadowing and therefore do not describe the data as accurately.

CONCLUSIONS

We calculated nuclear shadowing in DIS for various nuclei in an approach based on Glauber theory. Basic ingredient for our formulae is a phenomenological parameterization of the qq-proton cross section. Our formulation is a simplification of the Green function technique developed in [5,6]. An approximation for the coherence length allows us to employ a realistic parameterization of the dipole cross section and the nuclear density. The approach includes all higher order rescattering terms and parameter-free calculations are in good agreement with NMC data for shadowing of tin vs. carbon. We also compared to other shadowing data, not presented here. We find that for large nuclei the inclusion of all multiple scattering terms has a noticeable effect, so the double scattering approximation is insufficient.

This approach can be extended to calculate shadowing for gluons by postulating a particle that splits into a gluon-gluon pair analogous to our photon for quarks. It is crucial to have a good understanding of nuclear modifications of parton densities in order to get control over the initial conditions in heavy ion collisions at RHIC and LHC.

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