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FAILURE CURVE FOR ANISOTROPIC POLYCRYSTALS

*Author(s):* THOMAS A. MASON, MST-8  
PAUL J. MAUDLIN, T-3

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# CONSIDERATION OF THE HANCOCK-MACKENZIE FAILURE CURVE FOR ANISOTROPIC POLYCRYSTALS

T.A. Mason<sup>†</sup> and P.J. Maudlin<sup>+</sup>

<sup>†</sup>Materials Science and Technology Division, Mail Stop G755, tmason@lanl.gov

<sup>+</sup>Theoretical Division, Mail Stop B216, pjm@lanl.gov

Los Alamos National Laboratory

Los Alamos, New Mexico, 87545

**ABSTRACT-** Previously, a modified Hancock-Mackenzie failure curve has been used as the strain-to-failure criterion in a Gurson-based continuum damage model. The analysis employed in obtaining the parameters for this surface has been inherently isotropic plasticity and damage. Recent tests of polycrystalline materials with significant crystallographic texture have shown that the classic data reduction methods for the Hancock-Mackenzie curve are problematic. A number of the inherent assumptions employed in obtaining the failure surface will be reexamined in light of significantly anisotropic materials.

**INTRODUCTION:** A variety of failure criteria have been employed in recent continuum damage modeling efforts. The criterion used in the Gurson-based, tensile plasticity theory (TEPLA) developed at Los Alamos has the form shown in Eqn. (1) (Johnson and Addessio, [1988]). This equation describes a failure surface in a space comprised of equivalent strain, porosity, and triaxiality parameters. This surface is coupled to a temperature and strain-rate dependent constitutive model through the yield strength in the triaxiality ( $P/Y$ ). The failure porosity,  $\phi_f$ , is the porosity at which a material fails under spall conditions. The material parameters,  $D_1$ ,  $D_2$  and  $D_3$ , are determined from experimental testing of smooth and notched geometry bars. These parameters have been reported for a number of materials (Johnson and Cook, [1985]). If the porosity,  $\phi$ , is assumed to be very small during these tests then Eqn. (1) describes a curve in the equivalent strain-triaxiality plane.

$$\varepsilon_f = \sqrt{1 - \frac{\phi}{\phi_f}} \left( D_1 + D_2 \exp D_3 \frac{P}{Y} \right) \quad (1)$$

Increasingly, the role of anisotropic materials is more recognized and required in engineering practice. Previously, these materials have not been completely characterized and many assumptions inherent in the standard analyses must be revisited. In this case, these assumptions include constant triaxiality throughout the test, a scalar triaxiality and at least transverse-isotropic material properties with respect to the tensile axis. These assumptions and other points required to successfully address anisotropic failure will be discussed below.

**PROCEDURES, RESULTS AND DISCUSSION:** In this paper, preliminary results of quasistatic failure testing of textured zirconium will be presented. A number of smooth and notched bars were machined from clock-rolled, high-purity zirconium plates. This material exhibited a strong c-axis texture resulting in transverse-isotropic material properties with respect to the plate normals. The test specimens were machined with their axes in in-plane directions. The bar geometries included smooth, E, A, and D notches with initial Bridgman triaxialities of 0.33, 0.45, 0.74, and 1.03, respectively. The yield strength in the through-thickness (TT) direction of the plates was approximately four times greater than that in an in-plane (IP) direction. The cross-sections of all the tested bars went oval or elliptical.

During initial tension tests, the extension and the diametral strains were monitored using extensometers. The weaker, in-plane diameter was monitored with the through-thickness strain assumed to be negligible. During subsequent testing the extension and the circumferential strain were monitored. The circumference was measured using a length of high-strength monofilament line looped around the specimen and attached to an extensometer. Neither of these approaches, taking the extension and the circumference or taking the extension and a single in-plane diametral measure, was completely satisfactory as neither set of strains yielded a unique equivalent strain measure of the resulting oval cross-sections in the notch roots.

The plastic anisotropy of the zirconium violates the first basic assumption in the Bridgman analysis that the specimens exhibit isotropy with respect to the tensile axis. The classic Bridgman triaxiality is given in Eqn. (2). Here A and R denote the radius of the cross-section and of the notch, respectively.

$$\frac{P}{Y} = \frac{1}{3} + \ln \left( 1 + \frac{A}{2R} \right) \quad (2)$$

The second flaw in applying the classic approach to anisotropic materials is the assumption that the triaxiality is constant during the test. Preliminary examination of arrested E and D notch specimens clearly shows that the triaxiality is changing. Figure 1 shows a comparison of the initial profile of a D-notch specimen with the profile of a similar specimen pulled to a nominal extension of 46%. It appears that the notch radius has actually increased in the hard, TT direction while it is about the same as the original notch radius in the soft, IP direction. The anisotropic reduction of the cross-sectional area makes use of Eqn. (2) clearly of questionable value.

These two violated assumptions make it nearly impossible to construct the Hancock-Mackenzie failure curve for anisotropic materials without resorting to numerical simulations to obtain reasonable values for the triaxiality in the deforming specimens. These simulations will, in turn, require that the material's constitutive response and anisotropy are well characterized beforehand.

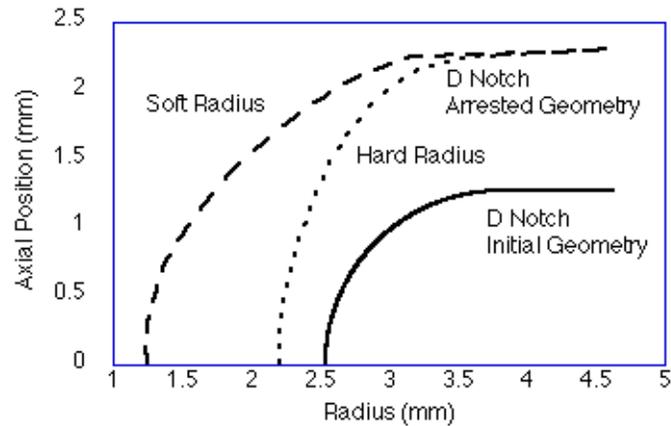


Figure 1. Comparison of the initial and final geometry of a D-notch specimen pulled to a nominal elongation of 46%.

The circumferential strain was used in order to capture a measure of the strain in both the TT and IP directions since it is not possible to put two orthogonal, diametral extensometers in the same notch. Extraction of the minor and major axes for a cross section with a given convex, elliptic circumference requires the knowledge of the aspect ratio of the notch as a function of strain. An examination of available tensile data on this zirconium, both smooth and arrested notch bars, reveals that the tensile aspect ratio follows a monotonically increasing, linear relationship with the equivalent plastic strain. This knowledge allows the true stress-true strain to be calculated for the notched bars which can then be employed with the boundary conditions and final geometry to calculate triaxialities for each of the anisotropic tests.

**CONCLUSIONS:** A coupled experimental and numerical research effort is required to produce a useful Hancock-Mackenzie curve for anisotropic materials. Ultimately, a suite of separate curves or a new tensoral form must be determined to capture the contribution of each principle direction in the material.

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**REFERENCES:**

Johnson, J.N. and Addessio, F.L. 1988, "Tensile plasticity and ductile fracture", *J. Appl. Phys.*, **64**, 6699.  
 Johnson, G.R. and Cook, W.H. 1985, "Fracture characteristics of three metals subjected to large strains, high strain rates, and high temperatures", *J. Eng. Fracture*, **21**, 31.