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Title:

EFFECTS OF LONG AND SHORT RELAXATION TIMES OF PARTICLE INTERACTIONS IN DENSE AND SLOW GRANULAR FLOWS

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Effects of Long and Short Relaxation Times of Particle Interactions in Dense and Slow Granular Flows

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In this work, dense granular flows are numerically simulated using a discrete element method. The interaction of a pair of colliding particles is modeled as a parallel connection of a linear spring and a linear dashpot. Although the force model for particle interactions is simplistic for many practical problems, a significant amount of meaningful new physics can be extracted from the numerical simulations by studying the behavior of particle interaction time and its probability distribution. For instance, it is found that the probability distribution of particle contact ages is exponential for long-term contacts. The time scale of the exponential decay of the contact age probability is related to the rheological properties of the dense granular medium.

It is observed that rheological properties and the duration of particle interactions in a dense granular media are closely related to the formation of particle interaction networks. The formation and behavior of particle interaction networks depend not only on the particle volume fractions but also on friction between particles. For a large friction coefficient between particles, particle interaction networks can form at a relatively low particle volume fraction (< 0.57). For frictionless particles, a particle interaction network may not form at particle volume fraction greater than 0.62, the random dense packing volume fraction for monodisperse spheres. Formation of particle interaction networks greatly changes the behavior of the granular stress in the system. Before network formation, particle interactions are short in time and mostly binary. In most cases, the binary collision time is short compared to the macroscopic hydrodynamic time of the problem and can be neglected. Under this condition, the granular medium can be modeled as a viscous fluid with variable viscosity as in kinetic theory.

Formation of particle interaction networks dramatically increases particle interaction time and results in a phase transition in the constitutive relations of the granular medium. After the phase transition, the relaxation time is inversely proportional to the macroscopic shear rate in simple shear flows. This relaxation time is comparable to the hydrodynamics time scale of the macroscopic motion. In this case, the granular medium can be modeled as a viscoelastic material with the stress relaxation time as a function of the macroscopic shear rate. This particular relation between the stress relaxation time and the macroscopic strain rate results in many interesting and unique behaviors of the granular flows. For instance, for small shear rates, the stresses in the granular medium are independent of macroscopic shear rates in simple shear flows. As the shear rate approaches zero, the relaxation time approaches infinity, and the shear stress approaches a finite value instead of zero. Under this condition, the medium behaves as a solid with a finite yield stress.

We also studied the relaxation behavior of the stress tensor under time dependent shear rates. The dynamics of the particle interaction network leads to nonlinear behaviors of stress relaxation that are not present in ordinary viscoelastic materials, such as polymeric fluids.

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1. Introduction

If the stress relaxation time in a material is small compared to the time scale of a physical process, the material behaves like a fluid. If the stress relaxation time is large compared to the time scale of the physical process, the material behaves like a solid. Many polymers have stress relaxation times comparable to the time scale of the problem, and therefore behave as viscoelastic materials. In this paper, we shall show that in dense granular flows, the stress relaxation time is not only comparable to the time scale of the flow, but also often depends on the time scale of the flow. In other words, in contrast to many polymeric materials, the relaxation time for granular systems is not a material property but rather a flow property. This explains why a dense granular system can behave both like a fluid or a solid depending on the nature of externally applied loading conditions.

In this paper, we shall examine these behavior changes and explain physical mechanisms behind them. To understand the behavior of a dense granular system, we perform numerical simulations using a discrete element method. The forces arising from the interaction of a pair of particles is modeled as a parallel connection of a linear spring and a linear dashpot. Although this force model for particle interactions is simplistic for many practical problems, a significant amount of meaningful physics can be extracted from the numerical simulations by studying the behavior of particle interaction time and its probability distribution.

In our recent papers (Zhang and Rauenzahn, 1997, 2000) we elucidated the viscoplasticity of a dense granular system as it undergoes a slow shear motion. The main physical reason for viscoplasticity in the system is the behavior of the stress relaxation time. We found that the stress relaxation time corresponds to the time constant in the probability distribution of contact ages. Because of the random particle motions in a granular system, contacts between particles lose memory of their origins in a short time, and the motion of a contacting pair quickly becomes history-independent. This history independence has two immediate consequences. First, the interaction forces between particles become independent of their contact ages. Second, the probability distribution of contact ages decays exponentially as a function of contact time. This decay of the age distribution results from loss of particle contacts, which causes stress relaxation. These physical explanations are actually verified in our numerical simulations that are described in detail in our papers (Zhang and Rauenzahn, 1997, 2000). The rate of decay of long-term contacts is inversely proportional to rate of shear in the system. As the shear rate approaches zero, the stress relaxation time approaches infinity, and the granular system behaves as a solid material.

All of the phenomena mentioned above are results of the long contact times of particles. However, even in dense granular systems, such as spheres at a volume fraction of 0.6, it is still possible for particles to experience collisions that occur over a short time scale, such as binary collisions. In this paper, we shall show coexistence of these long and short time scales in stress relaxation and in the relaxation of the particle contact age probability. The short relaxation time is responsible for the fluid-like behavior of the dense granular flow, and the long relaxation time is responsible for the solid-like behavior.

2. Rheological Behavior

To study rheological behavior of a granular material, we perform three typical experiments, here done numerically, used to study rheological properties of polymers.

We shall see that the stress in a dense granular system behaves quite differently from the corresponding experiments performed with a typical polymeric material.

Details of the numerical simulation scheme are described in our recent papers mentioned before, which are only briefly summarized here. In the initial condition, 864 particles are placed into a cubic cell according to face center cubic (FCC) array and then subjected to several thousand random motions. Our previous experience indicates that when more than 100 particles are used in such simulations, the results do not depend on the number of particles. Initial velocities of particles are specified according to a prescribed shear motion. For simplicity, all particles are identical and spherical, and the force model is similar to the one used by Jenkins and Strack (1993). The normal force between particles is modeled as a parallel connection of linear spring with coefficient K_n and a dashpot with resistance R. The tangential force is proportional to the total tangential deformation incurred after initial contact between particles. The magnitude of the tangential force is limited by Coulomb friction force determined by the product of the friction coefficient μ and the normal force. Time is non-dimensionalized by $200\sqrt{m/K_n}$, where m is the particle mass, and stress is non-dimensionalized by $K_n/(200a)$, where a is the particle radius. The value of the resistance R is chosen such that the restitution coefficient for a binary collision between particles is 0.8. To eliminate the effects of boundaries, periodic boundary conditions are used in the simulation. All examples present in this paper are performed with the particle volume fraction 0.6, and the friction coefficient μ equals 0.3.

As the first numerical experiment, we simulated a granular system subjected to an external shear that varied sinusoidally in time. The imposed shear rate is illustrated in blue line in Figure 1, and the resulting shear stress is plotted in red and is in phase with the shear rate. The system therefore behaves like a fluid, suggesting that the stress relaxation time in this system is small.

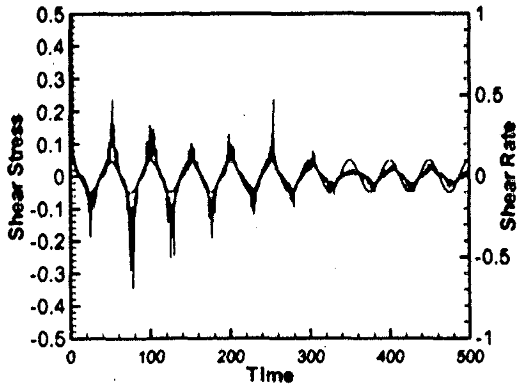

Figure 1. Relation between shear stress and shear rate with sinusoidal variation in the shear rate, which precludes the formation of particle interaction networks.

In the second example, the system is sheared with initial shear rate 0.1. At a time equal to 100, the direction of shear is suddenly reversed. As shown in figure 2, the stress decreases almost immediately to zero following the reversal of the shear rate, then rebuilds with the opposite sign. In this case, stress shows some delay in following the strain rate. This phenomenon can neither be categorized as solid-like behavior nor as fluid-like behavior.

The third example is sudden cessation of shear. The granular system is sheared at shear rate 0.1. After 100 time units, the shear is suddenly stopped. As shown in figure 3, the stress decreases immediately following the cessation of shear, but maintains a constant non-zero value for rest of the simulation.

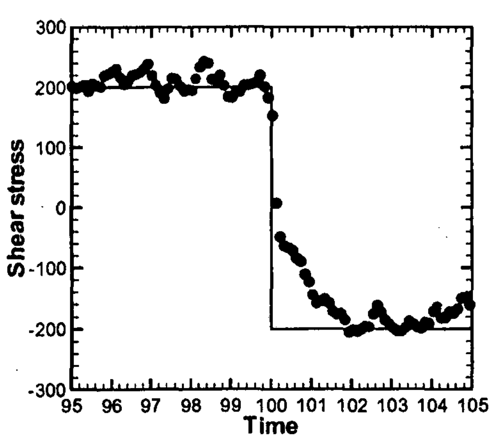

Figure 2. Shear stress change during the reversal of the shear direction.

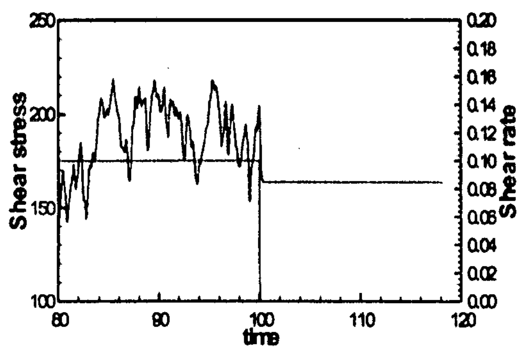

Figure 3. Stress does not relax completely after cessation of shear, showing the persistence of long time contacts.

These three examples show that the stress relaxation in a dense granular flow is substantially different from a typical polymeric material. To explain these phenomena, we need to study stress relaxation mechanisms in granular systems. Although many analogies can be drawn from polymer theory, direct application of those analogies is not possible. In the following section, we shall use the behavior of particle contact age and its probability distribution to explain the observed phenomena.

3. Effects of particle contact age and its probability distribution

At any time, particle contacts have different ages, defined as the differences between current time and the time that the contacts were formed. Contacts of various ages are in different stages of development. To study the behavior of the stress in a granular system, we need to understand the evolution of forces during the life span of a contact. As shown in figure 4 and in our earlier paper, the probability distribution function for long contacts is exponential. For short contacts, the probability distribution of contact age is not exponential. As shown in figure 5, the pdf drops off much more steeply than exponentially for small times, but the age distribution for long-term contacts does follow an exponential trend. The fastest decay occurs at approximately the binary contact time, which is 0.011 in this simulation.

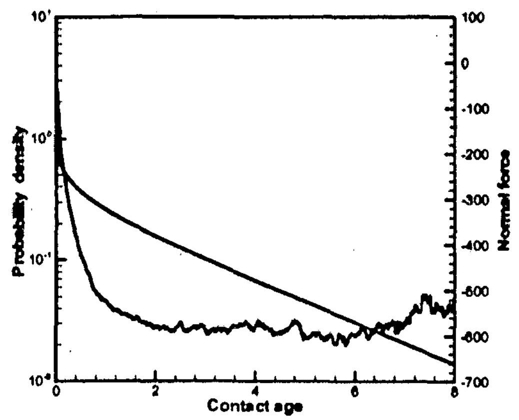

Figure 4. The probability distribution function for long contacts.

The decline in particle contact age probability distribution function is a result of contact destruction. Therefore, the relaxation in the age distribution is directly related to the stress relaxation. Figures 4 and 5 show that there are multiple relaxation time scales in the granular system. A relatively young contact that has lived longer than the binary collision time has a smaller relaxation time than old contacts. These different relaxation time scales and their relationships to the externally imposed shear motion determine the various behaviors observed in the numerical experiments described in the previous section.

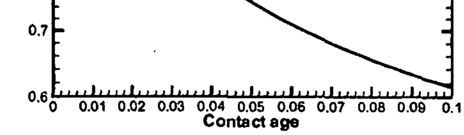

Figure 5. Probability distribution function for short contact ages. A close-up look of figure 4 at small contact ages.

In the first example, the granular system is sheared back and forth, and particle clusters do not have sufficient time to form. Most of the particle contacts have small ages, and their relaxation time is approximately equal to the binary collision time, as shown in figure 5. This relaxation time is small compared to the period of the sinusoidal motion; as a result, the system behaves as fluid with no phase delay between the shear stress and applied shear rate.

In the second case, the system has been sheared long enough for interacting clusters to form. After the sudden reversal of the shear direction, all contacts terminate fairly quickly, and then reform in response to the reversed shear rate. The active destruction of all particle contacts results in disappearance of the stress within a short time. The time required for the stress to recover corresponds to the period needed to rebuild the particle interaction clusters.

In the last example the granular system is again continuously sheared until clusters form. According to the probability distribution of contact ages shown in figures 4 and 5, there are both long and short time contacts. After cessation of shear, only contacts with small ages terminate, doing so within the time scale of a binary collision. This results in rapid stress relaxation, as shown in figure 3. In contrast, the relaxation of long contacts is inversely proportional to the shear rate (Zhang and Rauenzahn, 2000). Because the shear rate vanishes after a time of 100, the relaxation time is infinite. The stress arising from long-term contacts persists in this case, as demonstrated in the numerical experiment.

4. Conclusions

There are interesting spatial correlations in a dense granular system, such as particle clusters and force chains. The temporal correlations associated with these spatial structures cannot be visualized easily and hence are not well studied. We show in this paper that these temporal correlations control the rheological behavior of dense granular systems. In our study of particle interaction history, we found that the probability distribution of contact ages has both short and long relaxation time scales. The short time scale is related to binary collision time between particles, and the long time scale is related to particle interactions within clusters and is inversely proportional to the imposed macroscopic shear rate. The long time scale controls the long exponential tail in the probability distribution of contact age. Depending on loading conditions, the short or long time scales in the age probability distribution function can dictate the flow behavior, thereby determining whether a granular system behaves like a fluid or a solid.

Acknowledgements

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