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Title: APPLICATION OF A FLIP-MPM-MFM METHOD FOR SIMULATING WEAPON-TARGET INTERACTION

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APPLICATION OF AN MPM-MFM METHOD FOR SIMULATING WEAPON-TARGET INTERACTION

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During the past two decades, Los Alamos National Laboratory (LANL) has developed computational algorithms and software for analysis of multiphase flow suitable for high-speed projectile penetration of metallic and nonmetallic materials, using a material point method (MPM)-multiphase flow method (MFM). Recently, ACTA has teamed with LANL to advance a computational algorithm for simulating complex weapon-target interaction for penetrating and exploding munitions, such as tank rounds and artillery shells, as well as non-exploding kinetic energy penetrators. This paper will outline the mathematical basis for the MPM-MFM method as implemented in LANL's *CartaBlanca* code. *CartaBlanca*, written entirely in Java using object-oriented design, is used to solve complex problems involving (a) failure and penetration of solids, (b) heat transfer, (c) phase change, (d) chemical reactions, and (e) multiphase flow. We will present its application to the penetration of a steel target by a tungsten cylinder and compare results with time-resolved experimental data published by Anderson, et. al., *Int. J. Impact Engng.*, Vol. 16, No. 1, pp. 1-18, 1995.

INTRODUCTION

ACTA Inc. and Los Alamos National Laboratory (LANL) have teamed to advance a robust, scalable, adaptive computational capability for the simulation of complex weapon-target interaction using LANL's FLIP (Fluid Implicit Particle) - MPM (Material Point Method) - MFM (Multiphase Flow Method) approach, developed as an alternative to the fully-Lagrangian approach, [4, 8, 9, 10, 14, 16]. The method is implemented in *CartaBlanca*, a component-based nonlinear physical system simulation prototyping package using object oriented design [15]. It is used to solve extremely complex coupled problems involving (a) failure and penetration of solids, (b) heat transfer, (c) phase change, (d) chemical reactions, and (e) multiphase flow. It features GUI capabilities and is designed to utilize multiple processors on a single computer or on computer clusters. Written entirely in the Java programming language, it can be easily ported to many computer platforms including MS Windows, Linux, Unix, etc.

NEED FOR A NEW ALGORITHM

The penetration of materials and structures by Army munitions, including Military Operations in Urban Terrain (MOUT) [7], require knowledge and understanding of weapon effects on these materials and structures. Air-dropped weapons tend to be larger and are designed to damage a target by detonation in, on or near the structure. Army munitions, such as tank shells and artillery shells, tend to be smaller and are designed to explode on impact or after partial penetration. Traditional Lagrangian-based nonlinear finite element programs such as DYNA3D are used to simulate weapon-target interaction problems when weapons are exploded at a distance from the targeted structure or structural component. However, many Army munitions explode after partially penetrating the structure. Severe deformations near the explosion and resulting secondary debris are not easily simulated using traditional Lagrangian-based codes due to severe mesh tangling. *CartaBlanca* is also capable of modeling secondary explosions due to impact of kinetic energy projectiles. There is therefore a need for a validated computational capability that can simulate these complex weapon-target interaction problems. The proposed FLIP-MPM-MFM approach in *CartaBlanca* is ideally suited for these types of complex problems.

THE FLIP-MPM-MFM APPROACH

The survivability of systems due to blast and high velocity impact by K.E. penetrators can be addressed by a variety of computational methods. Two popular techniques involve the use of either a purely Eulerian approach or a purely Lagrangian approach. With the Eulerian approach, a fixed computational mesh is used and material is advected from mesh cell to mesh cell to follow the flow. With the Lagrangian approach, the computational mesh itself is distorted to follow the material motion. Each has its advantages and disadvantages.

The Eulerian approach has the advantage that set up is often very straightforward because simple Cartesian meshes can be used. Additionally, mesh-tangling issues are altogether avoided. The disadvantage of the Eulerian approach is that all information must be transported via advection, which is subject to numerical diffusion errors that can be quite important when dealing with non-fluid materials. Moreover, there is no clear conservation principle for the advection of non-conserved quantities such as stress history.

The purely Lagrangian approach, on the other hand, avoids the problems associated with advection by letting the mesh points follow the motion of the flow. Thus no advection is required. Often, the generation of the initial mesh for the Lagrangian approach can be more challenging than that for the Eulerian approach if body fitting is required. Another difficulty with the purely Lagrangian approach, is that for complex flows with material distortion, the mesh can become tangled to the point where the computation cannot be continued. One solution to this is the generation of a new mesh (rezoning) and the remapping of the data from the nearly tangled mesh to the new mesh. The remapping operation is, however, a form of advection, and thus brings with it the numerical diffusion problems of the Eulerian method discussed above. Furthermore, remapping items with no clear conservation principle for the advection of quantities like stress history, is questionable.

For penetration problems, an additional concern associated with Lagrangian treatments is the complexity and difficulty associated with slide lines and so-called eroding slide lines. These are implemented to allow slip between two separate material bodies with strength. These representations in software can be quite complex and difficult to maintain.

A new alternative to the purely Lagrangian and Eulerian techniques discussed above is the use of the so-called FLIP (Fluid Implicit Particle) and MPM (Material Point Method) methods developed by the LANL T-3 group and collaborators at Univ. of New Mexico, [4, 8, 9, 10, 14]. The essence of these techniques is the use of meshless Lagrangian particles for the transport of conserved quantities for materials with strength and communication between these particles and the computational mesh through the use of interpolation functions called shape functions. This technique eliminates the advection diffusion problems of the Eulerian technique and the mesh tangling and slide line problems of the Lagrangian techniques. The LANL T-3 group has also married the FLIP/MPM method with their multiphase flow capabilities to produce a powerful general capability that has been used for a wide variety of fluid structure interaction problems including those with material penetration and material failure. Addressio et al. [2] provide a summary of this technique including examples of the hydrodynamic ram problem in which a bullet penetrates a composite airplane wing tank filled with fluid.

Currently, the FLIP-MPM-MFM approach is being used at LANL to address issues of high explosives safety and high explosives microstructure, metals loaded with high explosives and granular materials. It is also beginning to be used to investigate shock induced plastic deformation in complex metal alloys. The FLIP-MPM-MFM approach is currently implemented in LANL's CFDLIB code as well as its CartaBlanca code, both developed by the LANL T-3 group.

CARTABLANCA

CartaBlanca was developed recently at Los Alamos National Laboratory to exploit both modern object oriented programming techniques and recent developments in numerical methods for non-linear systems of algebraic equations. CartaBlanca is written in the Java programming language and employs a component-like design using Java's object oriented programming facilities. As a result, CartaBlanca is a developer friendly environment; adding new physical models, conservation equations, or numerical methods is easy compared to traditional multipurpose solvers. More on the object oriented design of CartaBlanca can be found in [15].

MULTIPHASE FLOW EQUATIONS

One of the main physics modules in CartaBlanca is for the simulation of multiphase flow. The derivation of the governing equations for multiphase flow is provided in [5, 6, 1]. In the following sections, we present the final forms of the governing equations. We shall make use of the following general form of the conservation equation. For an arbitrary control volume V with bounding surface A the generic conservation statement is of the form

$$\frac{d}{dt} \int_V q dV + \oint_A [q(\vec{u} - \vec{u}_A) + \vec{f}] \cdot \vec{n} dS + \int_V s dV = 0, \quad (1)$$

where q is the density of some conserved quantity such as mass, momentum or energy, \vec{u} is the material velocity field, \vec{u}_A is the velocity of the boundary of the control volume, \vec{f} is the local flux of this conserved quantity due to a variety of mechanisms such as diffusion, \vec{n} is an outward normal vector defined on the surface of the control volume, and s is a generalized source density. The first and third integrals in Equation (1) are over the entire space of the control volume; the second integral is over the surface of the control volume. The derivative on the first integral quantity in Equation (1) is with respect to time.

MASS CONSERVATION

The mass conservation equations for each phase in a multiphase flow are obtained using Equation (1) with $q = \rho_k$, the average mass density for phase k and with $s = \Gamma_k$, the net rate of appearance of phase k due to mass exchange from or to other phases.

MOMENTUM CONSERVATION

The momentum conservation equation for phase k is obtained from Equation (1) where $q = \rho_k \vec{u}_k$ in which \vec{u}_k is the local mass average velocity for phase k . In addition, the source and flux terms are given by

$$s = -\theta_k \nabla p + \Gamma_k \vec{u}^* + \rho_k \vec{g} + \theta_k \sum_{l \neq k} \theta_l K_{kl} (\vec{u}_l - \vec{u}_k) + \theta_k \nabla \cdot \tau_m + K_{D-k} \vec{u}_k \quad (2)$$

and

$$\vec{f} = \left[\theta_k (p_k^o - p) \right] I + \left[\theta_k (\tau_k^o - \tau_m) \right] \quad (3)$$

where θ_k is the volume fraction, p is the pressure field common to all phases, p_k^o is the phase k pressure, \vec{u}^* is the velocity of the transferring material, \vec{g} is the body force, τ_m is the deviatoric stress of the mixture, τ_k^o is the deviatoric stress in phase k and K_{kl} is the momentum exchange coefficient for interactions such as drag between phase k and phase l and I is the identity tensor. The last term in Equation (2) is a Darcy drag term with coefficient, K_{D-k} .

For the example considered in this proposal, we assume inviscid flow and that $p_k^o = p$. We also will neglect the effect of gravity. We furthermore neglect as often unimportant the mass transfer term in Equation (2). Thus, Equations (2) and (3) become

$$s = -\theta_k \nabla p + \theta_k \sum_{l \neq k} \theta_l K_{kl} (\vec{u}_l - \vec{u}_k) + K_{D-k} \vec{u}_k \quad (4)$$

and

$$\vec{f} = 0 \quad (5)$$

ENERGY CONSERVATION

For energy conservation, we choose the enthalpy wherein $q = \rho_k h_k$ where h_k is the average enthalpy per unit mass of phase k . The source and flux terms are given by

$$s = \theta_k \left\{ \sum_{l \neq k} \theta_l R_{kl} (T_l - T_k) + \beta \dot{p} + \tau_m : \nabla \tilde{u}_m \right\} + \Gamma_k h_k^* + \theta_k \nabla \cdot k_m \nabla T_m \quad (6)$$

and

$$\vec{f} = \theta_k \left[k_m \nabla T_m - k_k^{eff} \nabla T_k \right] \quad (7)$$

where R_{kl} is a heat transfer coefficient for exchange between phase k and phase l , T_k is the temperature of phase k , β_k is the thermal expansivity for phase k , \dot{p} is the material time derivative of pressure, h_k^* is the enthalpy of the exchanged material and k_m is the mixture thermal conductivity. The subscript m denotes a mixture quantity.

For the example considered in this proposal, we neglect the work term, the viscous dissipation term and the deviatoric heat flux. Thus, Equations (6) and (7) simplify to

$$s = \theta_k \sum_{l \neq k} \theta_l R_{kl} (T_l - T_k) + \Gamma_k h_k^* + \theta_k \nabla \cdot k_m \nabla T_m \quad (8)$$

and

$$\vec{f} = 0 \quad (9)$$

We will identify the mass transfer term later.

CONTINUITY EQUATION

The continuity equation or condition for multiphase flows is that the sum of the volume fractions must equal one.

$$\sum_k \theta_k = 1. \quad (10)$$

This equation essentially governs the pressure variable. The volume fraction is computed as

$$\theta_k = \frac{\rho_k}{\rho_k^o(p_k^o, T_k)} \quad (11)$$

where ρ_k^o is the material k density, which is a function of pressure and temperature via an equation of state.

NUMERICAL ALGORITHM

In CartaBlanca, the multiphase flow equations are solved using the finite-volume discretization and the Lagrangian-Eulerian time stepping technique. Many of the details of the algorithm take advantage of the procedure of Kashiwa, et al. [11]. CartaBlanca adopts the node-based version of this scheme with edge-based connectivity, [12,13]. We provide here a very simplified outline of the method. For numerical computations, Equation (1) is discretized in time and in space on a computational grid. On such a grid, conservation nodes are connected by edges as shown in Figure 1.

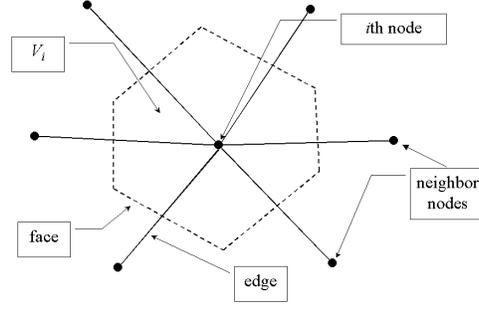


Figure 1. Control volume for the i th node.

Each node is associated with a polyhedral control volume, V_i , as depicted in Figure 1. For each node, the averaged value of the conserved density is defined as

$$q_i \equiv \frac{1}{V_i} \int_{V_i} q dV. \quad (12)$$

The quantities q_i are, typically, the state variables for the numerical simulation. Similarly, the average source over each control volume is

$$s_i \equiv \frac{1}{V_i} \int_{V_i} s dV. \quad (13)$$

Let the subscript e denote the average value of quantities on the control volume face associated with edge e . Then, if we integrate the Equation (1) over a time step, Δt , using, for example, a first-order difference approximation for the time derivative, we obtain the discretized form of the conservation equation

$$q_i^{n+1} V_i^{n+1} - q_i^n V_i^n + \Delta t \left\{ \sum_{edges} \langle q \rangle_e (\bar{u} - \bar{u}_A)_e \cdot \bar{n}_e A_e + \sum_{edges} \bar{f}_e \cdot \bar{n}_e A_e + s_i V_i \right\} = 0, \quad (14)$$

where the superscripts n and $n+1$ denote the present and future time levels, respectively. The angle brackets, $\langle \rangle_e$, denote the average over the time interval of the conserved quantity passing through the face e . Of course, the fluxes and source terms are generally functions of space, time and the state variables, q_i . Thus, the set of discretized conservation equations for all nodes and all types of conservation quantities forms a nonlinear algebraic system. The physics for a given application lies in the definition of the fluxes and sources in Equation (14). The aim of CartaBlanca is to provide scientists and engineers a friendly environment using object-oriented Java for the implementation of component-like physics and solver objects for the solution of the corresponding coupled nonlinear conservation equations.

GRID-PARTICLE INTERPOLATION

To integrate the equations of motion for phases with material strength interpolation of data is required from the grid to the particle and vice versa. This is accomplished using finite-element shape functions (usually 1st order). The interpolation formulas are of the general form

$$q_{particle} = \sum_n q_n S_{n-p} \quad (15)$$

$$q_{node} = \frac{\sum_p m_p q_p S_{n-p}}{\sum_p m_p S_{n-p}} \quad (16)$$

$$(f_i)_{node} = \frac{-\sum_p V_p \sigma_p \frac{\partial S_{n-p}}{\partial x_i}}{\sum_p V_p S_{n-p}} \quad (17)$$

The first formula interpolates a quantity q (*mass density, velocity, etc.*) from the nodes in an element to the particle by summing over the nodes in the element and weighting the node quantity with the node shape function S . The second formula scatters information from the particles to the nodes by summing over all the particles in the elements corresponding to the node in a mass weighted fashion. In the third equation, the force density at a node is obtained by interpolating the divergence of the stress, σ , using the spatial derivative of the shape function and particle volume weighting. These interpolation formulas enable information to be carried on the particles and then brought down to the grid for the final integration of the momentum equation to advance velocity, position, etc. Items such as stress can be evolved on the particle so that history effects can be accounted for in a straightforward manner and effects of advection can be avoided.

EXPERIMENTS OF ANDERSON, ET. AL.

Anderson, et. al., [3], performed a series of experiments to examine the penetration behavior of a tungsten alloy long rod penetrator into high-hard steel. Two impact velocities were investigated, 1.25 km/s and 1.70 km/s. The tungsten rod was 0.4 cm in diameter and 5.0 cm in length. Two steel blocks were used as targets. In the direction of the impact, the two block thicknesses were 2.90cm and 4.95cm. In the lateral dimensions, both blocks were 4.0 cm by 7.0 cm. The positions of the nose and tail of the projectile were measured by means of a 600 kV flash X-ray system at different times during the penetration.

DEMONSTRATION OF FLIP-MPM-MFM APPROACH

Simulation of the experiments of Anderson, et al. [3] on the penetration of a tungsten rod through a steel target were performed using the CartaBlanca code that employs the FLIP-MPM-MFM algorithm developed by LANL's fluid dynamics group for fluid-structure interaction problems. Figure 2 provides a schematic diagram of the experimental and computational setup.

Set Up Information

$S_d = 2.9\text{cm}$, Steel depth.

$h_0 = 5.0\text{ cm}$, Initial height of tungsten bar

P_d Penetration depth

$h(t)$ height of tungsten bar at time t

Cylindrical Coordinates:

Mesh 250x135 (radius 5.4, height 10)

Number of particles in Tungsten 2490

Number of particles in Steel 35712

Tungsten radius $r=0.2\text{cm}$

Steel radius $r=5\text{cm}$

Three phases in simulation:

air, tungsten, high-hard steel.

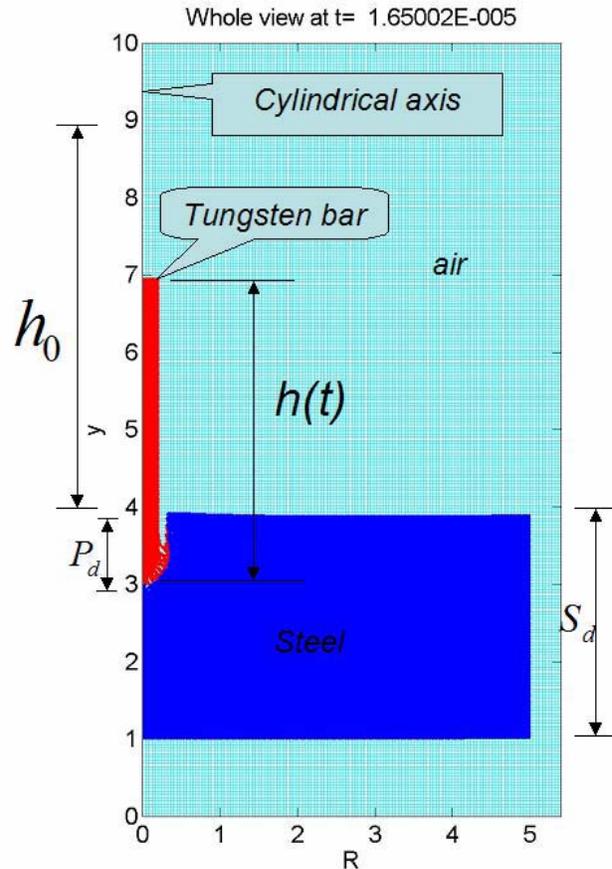


Figure 2. Schematic diagram of the simulation of the experiments of Anderson, et. al.

In order to compare with the CTH calculation reported on by Anderson, et. al. and to save computer time, a cylindrical coordinate system was used for the CartaBlanca simulations. In addition, the Johnson-Cook material model was used to model the behavior of both the tungsten and the high-hard steel. The material parameters from Anderson, et. al. were used in the CartaBlanca implementation of that model. In the simulations, we tried both a two-phase approach and a three-phase approach. In the two-phase calculation, we treated the tungsten and the steel as components within in the same “phase.” By phase, we mean that the materials share the same velocity and temperature field. The surrounding air was the second phase with a separate velocity and temperature field. The two phases were coupled through the use of momentum and heat exchange terms. In the three-phase case, we treated the tungsten and the steel as separate phases. This required additional modelling of momentum and heat exchange between not only the metals and the air but also between the two metals. The results from these calculations are shown in the figures below.

Penetration depth P_d and Tungsten bar's length h varies with time

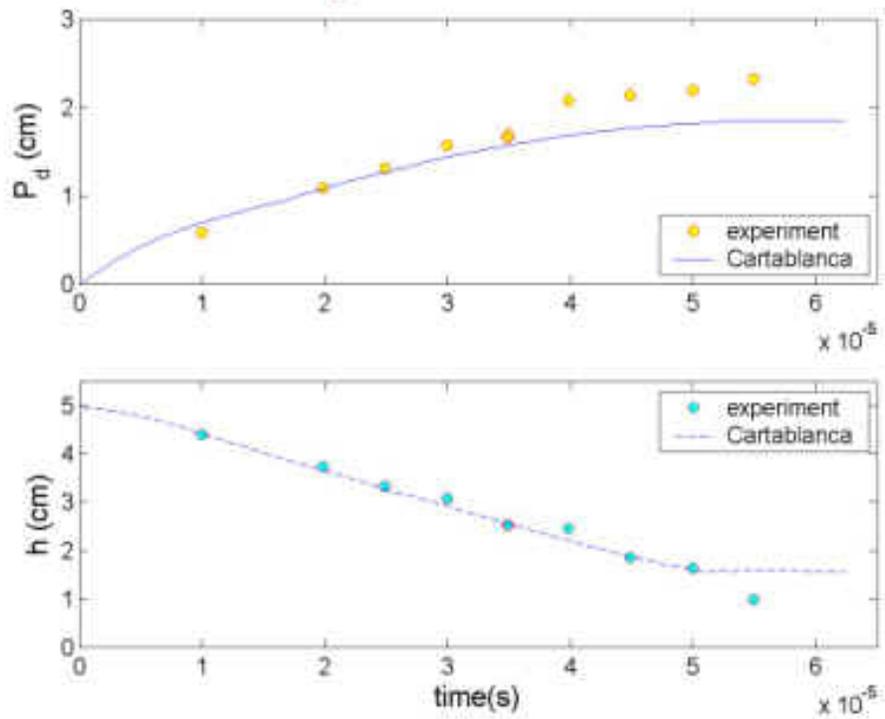


Figure 3. Penetration depth and bar height as a function of time for 1.25km/s impact velocity case (three phases)

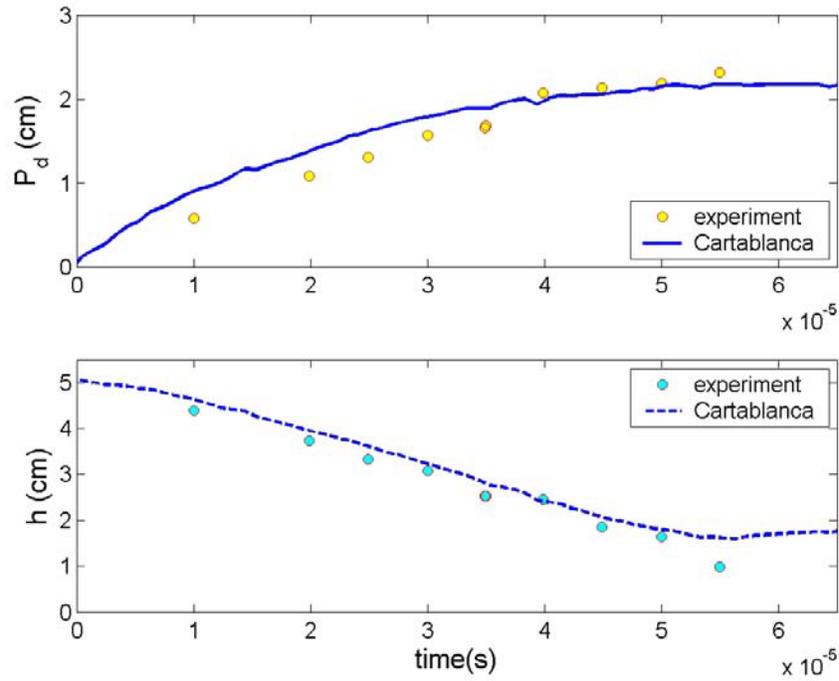


Figure 4. Penetration depth and bar height as a function of time for 1.25km/s impact velocity case (two phases)

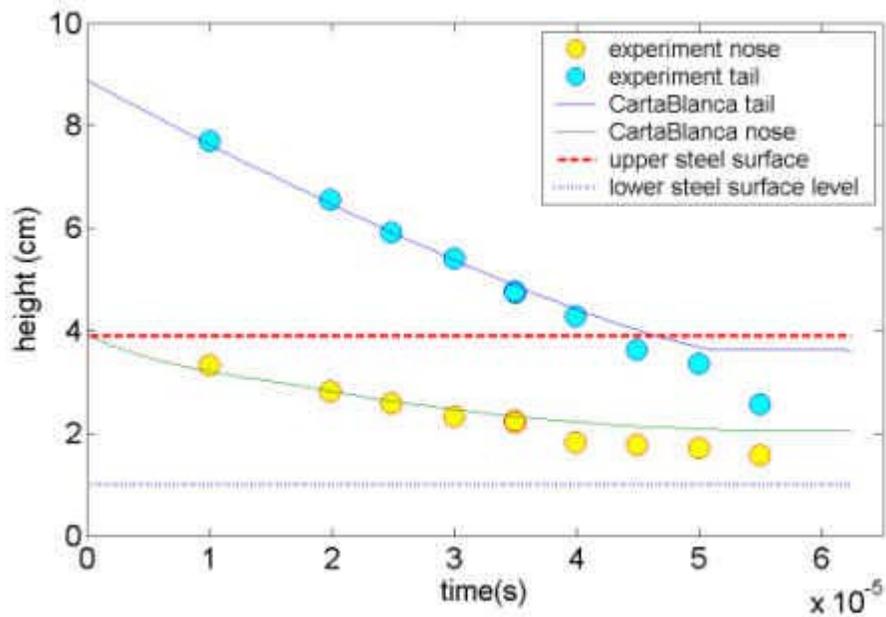


Figure 5. Nose and tail position as a function of height for 1.25km/s impact velocity case (three phases).

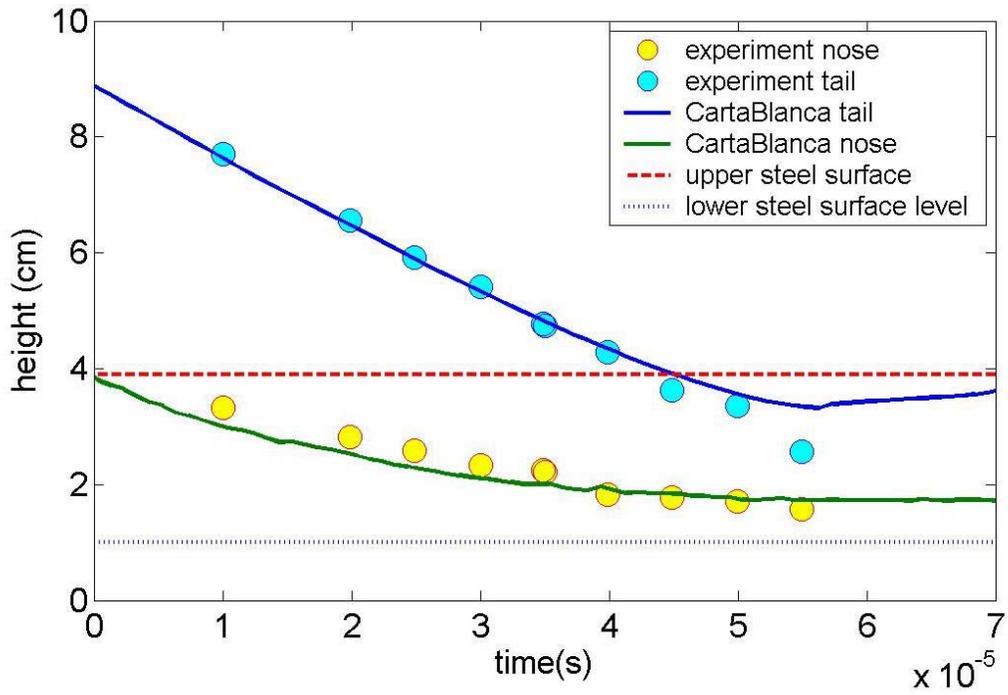


Figure 6. Nose and tail position as a function of height for 1.25km/s impact velocity case (two phases)

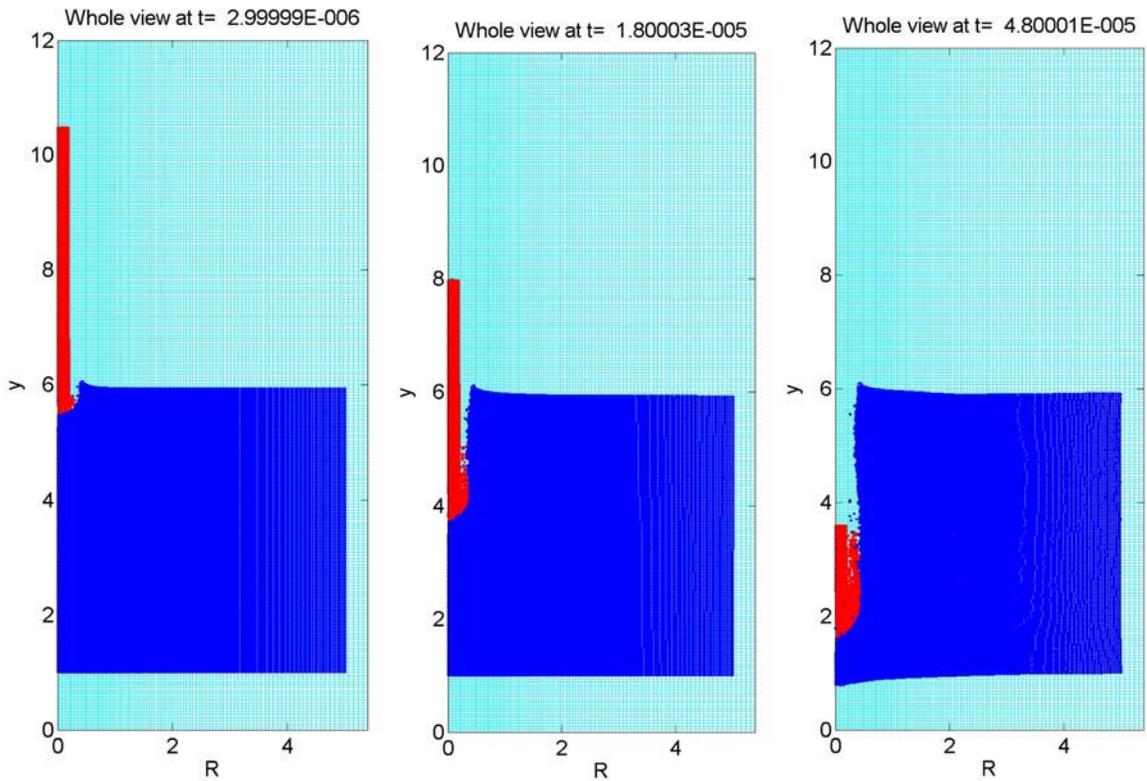


Figure 7. Snapshots showing the time progression of penetration of impact velocity 1.70km/s.

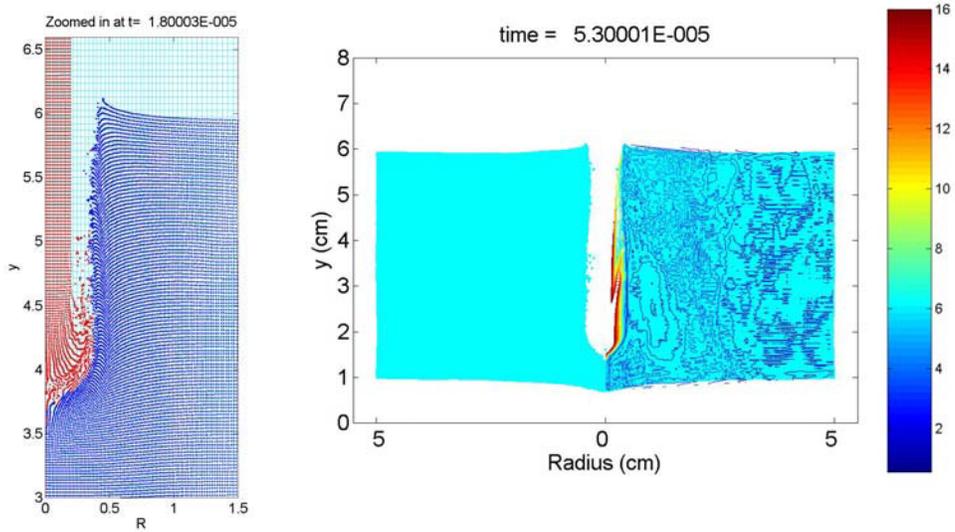


Figure 8. Crater shape and plastic strain of impact velocity 1.70km/s.

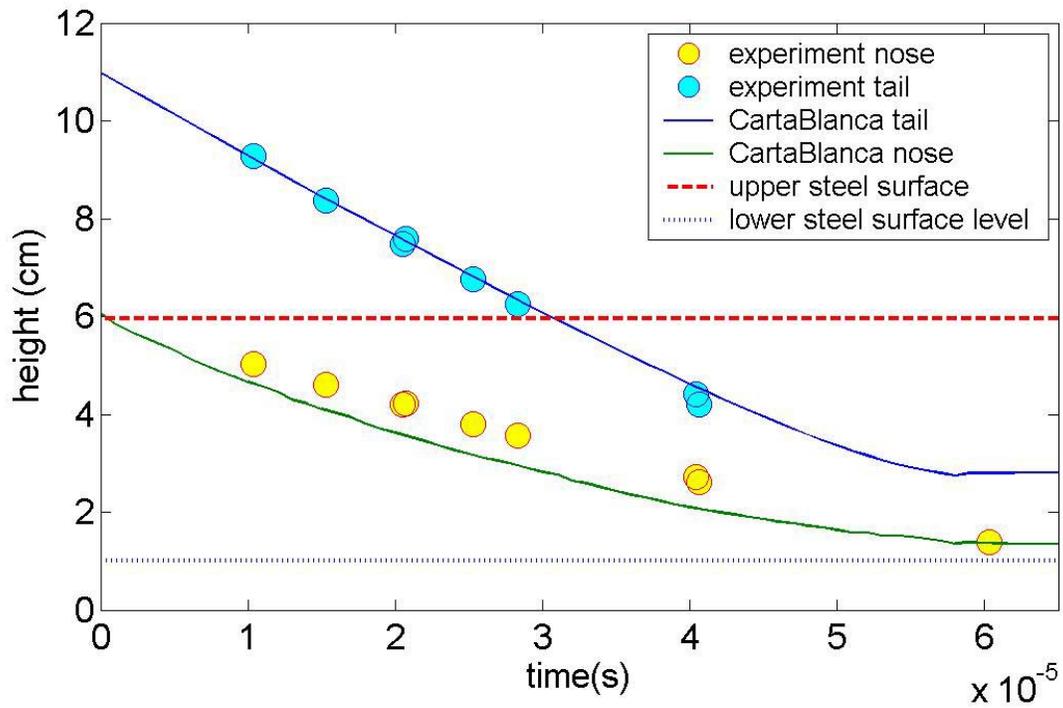


Figure 9. Nose and tail position as a function of height for 1.25km/s impact velocity case (two phases).

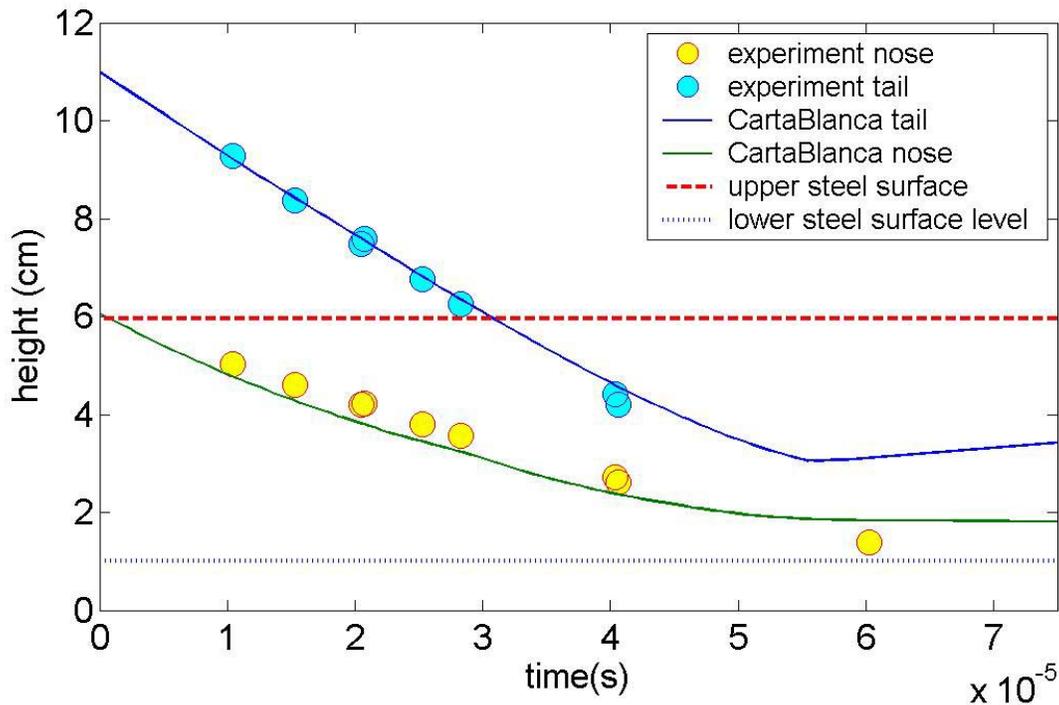


Figure 10. Nose and tail position as a function of height for 1.75km/s impact velocity case(three phases).

CONCLUSIONS

The above results show that CartaBlanca and the FLIP-MPM-MFM method can successfully simulate and model the material deformation and impact phenomena associated with ductile-ductile weapon target interactions of interest to the defense community.

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REFERENCES

1. Addressio, F.L., J. R. Baumgardner, J. K. Dukowicz, N. L. Johnson, B. A. Kashiwa, R. M. Rauenzahn, C. Zemach, CAVEAT: A Computer Code for Fluid Dynamics Problems with Large Distortion and Internal Slip, Los Alamos National Laboratory Report LA-10613-MS, Rev. 1, May, 1992.
2. Addressio, F.L., B.A. Kashiwa, M.A. Kenamond, M.W. Lewis, R.M. Rauenzahn, E.A. Rodribuez, M.W. Schraad, and T.L. Wilson, Physics Based Damage Prediction for Simulating Testing & Evaluation (T&E) Experiments, Los Alamos National Laboratory Report LA-UR-97-4877.

3. Anderson, C. E. V. Hohler, J. D. Walker and A. J. Stilp, "Time-Resolved Penetration of Long Rods into Steel Targets", *Int. J. Impact Engng.*, Vol . 16, No. 1, pp. 1-18, 1995.
4. Brackbill, J. U., D. B. Kothe, H. M. Ruppel, "FLIP: a low dissipation, particle-in-cell method for fluid flow, *Computer Physics Communications*, Jan. 1988, vol. 48(10), 25-38.
5. Drew, D.A. and S. L. Passman, *Theory of Multicomponent Fluids*, Springer, New York, 1999.
6. Ferziger, J.H., and M. Peric, *Computational Methods for Fluid Dynamics*, Springer, New York, 1999.
7. Fordyce, David F., White Paper on Modeling and Estimating Building Damage from Army Munitions – The Need for Testing (Second Draft), March 11, 2003.
8. Guilkey, J. E., T. Harman, A. Xia, B. Kashiwa, P. McMurtry, "An Eulerian-Lagrangian approach for large deformation fluid-structure interaction problems, part 1: Algorithm development," in *Adv. Fluid Mechanics*, Second International Conference on Fluid Structure Interactions, 2003, Jun 26-28, Cadiz, Spain.
9. Harman, T., J. E. Guilkey, B. Kashiwa, J. Schmidt, P. McMurtry, "An Eulerian-Lagrangian approach for large deformation fluid-structure interaction problems, part 2: Multi-physics simulations within a modern computational framework," in *Adv. Fluid Mechanics*, Second International Conference on Fluid Structure Interactions, 2003, Jun 26-28, Cadiz, Spain.
10. Kashiwa, B. A., M. L. Lewis, T. Wilson, Fluid structure interaction modeling, Los Alamos Tech Report, LA-13111-PR, Los Alamos National Laboratory (1996).
11. Kashiwa, B., N. T. Padial, R. M. Rauenzahn and W. B. VanderHeyden, A Cell-Centered ICE Method for Multiphase Flow Simulations, *FED-Vol. 185, Numerical Methods in Multiphase Flows*, ASME, 185:159-176, 1994.
12. O'Rourke, P.J., and M. S. Sahota, *CHAD: A Parallel, 3-D. Implicit, Unstructured-Grid, Multimaterial, Hydrodynamics Code for All Flow Speeds*, Los Alamos National Laboratory Report LA-UR-98-5663, October, 1998.
13. Selmin, V., The Node-Centered Finite Volume Approach: Bridge between Finite Differences and Finite Elements, *Comput. Methods in Appl. Mech. Engrg.*, 102(1):107-138, January, 1993.
14. Sulsky, D., Z. Chen, H. L. Chen, "A particle method for history dependent materials," *Comp. Methods Appl. Methods Engrg.*, 118, 1994, 179-196.
15. Vanderheyden WB, ED Dendy, NT Padial-Collins, "CartaBlanca– A Pure-Java, Component-based Systems Simulation Tool for Coupled Nonlinear Physics on Unstructured Grids—An Update," *Concurrency and Computation: Practice and Experience*, 2003, 15, 431-458.
16. Zou, Q., D. Z. Zhang, W. B. VanderHeyden, W. Wathugala and T. K. Hasselman, "Application of a FLIP-MPM-MFM Method for Simulating Weapon-Target Interaction," 75th Shock & Vibration Symposium, Virginia Beach, VA, Oct 18-22, 2004.