

# Los Alamos

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## memorandum

TO: Distribution  
FROM: Bruce Robinson *BAR*  
SYMBOL: ESS-4-82-431  
SUBJECT: DETERMINING THE TRUE RESIDENCE TIME  
DISTRIBUTION CURVE OF PHASE I SYSTEM

DATE: August 24, 1982

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### Summary

Previous engineering analyses of the Br<sup>82</sup> tracer experiments failed to account for the fact that the fluid was being recirculated during these tests. Thus, the concentration vs. volume curves shown in the Run Segments 4 and 5 reports and elsewhere are not really the response of the system to a pulse of tracer. These data are complicated by the fact that at later times most of the tracer being measured was not the original pulse, but the tracer on its second or third pass through the reservoir. When this recirculation effect is subtracted out of the original concentration vs. volume curves, the true residence time distribution (RTD) for the Phase I system indicates that the "long tail" on these curves is not caused by dispersion but results almost entirely from recirculation. The RTD curve for this system cannot be modeled precisely using a one parameter model, but can probably be described by a combination of hydrodynamic and turbulent dispersion in a single fracture. Alternatively, flow through multiple fractures could easily result in the RTD curves determined during Run Segments 4 and 5.

## Mathematical Analysis

The response of a continuous flow-through vessel (or, in our case, a reservoir) to a pulse of inert tracer material is often used to characterize the degree of backmixing in the system. This so-called E curve may also be thought of as the exit age distribution:  $E(t)dt$  is the fraction of fluid in the exit stream which has been in the system for a length of time between  $t$  and  $t + dt$ . The two extreme cases of mixing are a perfectly mixed system, which results in an exponentially-decaying E curve, and plug flow with no backmixing (the E curve is the dirac delta function in this case).

Since the E curve is actually the exit age distribution, it is easy to show (Levenspiel, 1972) that the response of a system to an arbitrary inlet concentration  $C_{in}(t)$  is

$$C_{out}(t) = \int_0^t C_{in}(t-t')E(t')dt' \quad (1)$$

Equation (1) is known as the convolution integral.

The normal situation in chemical reaction engineering is to use the convolution integral to compute  $C_{out}(t)$  for a system for which  $E(t)$  has been determined experimentally. Our problem is just the opposite: we seek to compute  $E(t)$  from a tracer experiment in which we have measured  $C_{in}$  and  $C_{out}$ . Note that if  $C_{in}$  were simply a pulse of tracer at time  $t = 0$ , then Equation (1) predicts (correctly) that the response of the system ( $C_{out}$ ) would be the E curve itself. However, since we recirculated the fluid in these experiments,  $C_{in}(t)$  is actually a pulse at time  $t = 0$ , followed by

$C_{out}(t)$ . Since  $C_{out}(t)$  is measured in these experiments, we can use the convolution integral to calculate  $E(t)$ .

The computer code TRACE calculates  $E(t)$  for a tracer experiment where the input is a pulse of tracer followed by the recirculated fluid of concentration  $C_{out}(t)$ . It is important to note that according to Equation (1),  $E(t)$  may be calculated up to time  $t$  without knowing the  $E$  curve or the concentrations at times greater than  $t$ . Thus, the computer code marches forward in time, calculating the convolution integral at each time step, using the input and output concentrations and the value of the  $E$  curve found at the preceding times. The code also corrects for dilution with make-up fluid at the surface and pore fluid in the reservoir.

To check the computer code TRACE, two analytical solutions were used: a single continuous stirred tank reactor (CSTR) and 2 CSTR's in series. The  $E$  curves for these idealized systems are known, and Equation (1) may be solved analytically for  $C_{out}(t)$  for the case of completion recirculation. The code takes the values of  $C_{out}(t)$  as input data and calculates the  $E$  curve. Details of this analysis are given in the Appendix.

### Analysis of the Phase I Tracer Experiments

The computer code can be used to analyze the Phase I  $Br^{82}$  tracer data to obtain  $E(t)$  for the reservoir. In the analysis of this real system, there are the added complications of net water loss and dilution with pore fluid. In order for the computer simulation to be accurate, the concentrations must be normalized with respect to the total amount of tracer recovered, which in turn depends on the water loss. For Experiment 217-A2 (5/9/80), even after correcting for 10% dilution with make-up fluid

and 5% with pore fluid, the total recovery of tracer was still only 89%. However, despite the fact that about 10% of the tracer loss cannot be accounted for in this analysis, it is still felt that the E curve generated by the computer code TRACE is a reasonably accurate representation of the RTD for the Phase I system.

Figure 1 compares the experimental data obtained in the May 9, 1980 Br<sup>82</sup> tracer test (Experiment 217-A2) to the E curve generated from this data in the present analysis. The E curve matches the concentration data almost exactly up to a volume of about 125,000 gallons. The slight discrepancy is due to the fact that the fluid is being diluted with ~5% pore fluid before being measured at the surface. At volumes greater than 125,000 gallons, the outlet concentration is a combination of tracer in its first pass through (the E curve) and tracer which has been recirculated two or more times. At very large volumes, the measured output is almost entirely made up of recirculated fluid.

#### Reservoir Modeling Using the New RTD Curve

Important conclusions with respect to reservoir modeling of the Phase I system can be inferred from the RTD curve calculated in the preceding section. First, the long tail on the concentration vs. volume tracer curves need not be considered as it has been shown to be due to recirculation of the fluid. As a result, the overall dispersion of the system is less than has been suggested in the Run Segments 4 and 5 reports and elsewhere (i.e. Tester et al., 1982). Modeling of the Phase 1 reservoir with a one-fracture, one-parameter model now seems more justified, given the shape of the E curve in Figure 1.

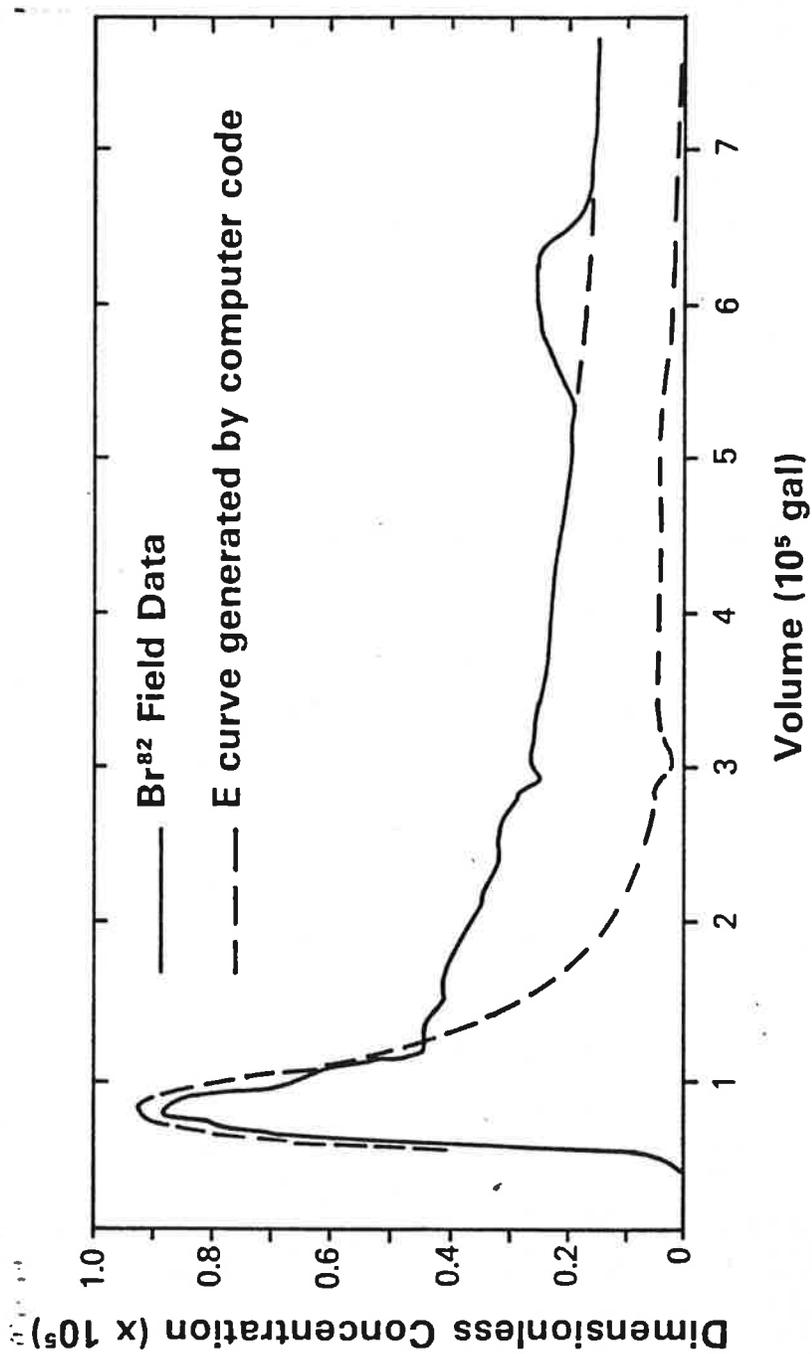


Figure 1. Comparison of the Br<sup>82</sup> field data for Experiment 217-A2 with the E curve generated by the computer code TRACE.

Figure 2 shows two attempts to model the Phase I system using one parameter models. The axial dispersion model was calculated using an analytical expression developed by Levenspiel and Smith (1957):

$$E_{\theta} = \frac{1}{2\sqrt{\theta Pe}} \exp\left[-\frac{(1-\theta)^2 Pe}{4\theta}\right], \quad (2)$$

where  $\theta = V/\bar{V}$  = volume divided by the integral mean volume. The optimum value of Pe for the Phase I reservoir is 6.96, determined by using the following relationship for the variance  $\sigma_{\theta}^2$  governed by Equation (2).

$$\sigma_{\theta}^2 = \frac{2}{Pe} + \frac{8}{Pe^2} \quad (3)$$

where

$$\sigma_{\theta}^2 = \frac{\sigma^2}{\bar{V}^2} = \frac{\sum V_i^2 C_i}{\bar{V}^2 \sum C_i} - 1 \quad (4)$$

It should be pointed out that Equations (2) and (3) are strictly applicable only for so-called open vessels in which the flow is not disturbed as it passes the measuring point. For our case, where the flow pattern changes from plug flow in the injection and production wellbores to dispersed flow in the fracture system, the asymptotic solutions of Brenner (1962) should really be used. However, since the solutions are qualitatively quite similar, the more simple analytical expression of Levenspiel and Smith was used in the present analysis.

Alternatively, Murphy and Cornwell (1982) calculated the E curve which would result from hydrodynamic dispersion due to two-dimensional streamline flow in a circular fracture. Their curve for  $R/L = 0.667$  is plotted alongside the actual E curve and the axial dispersion curve. While neither model adequately simulates the Phase I tracer data, each possesses

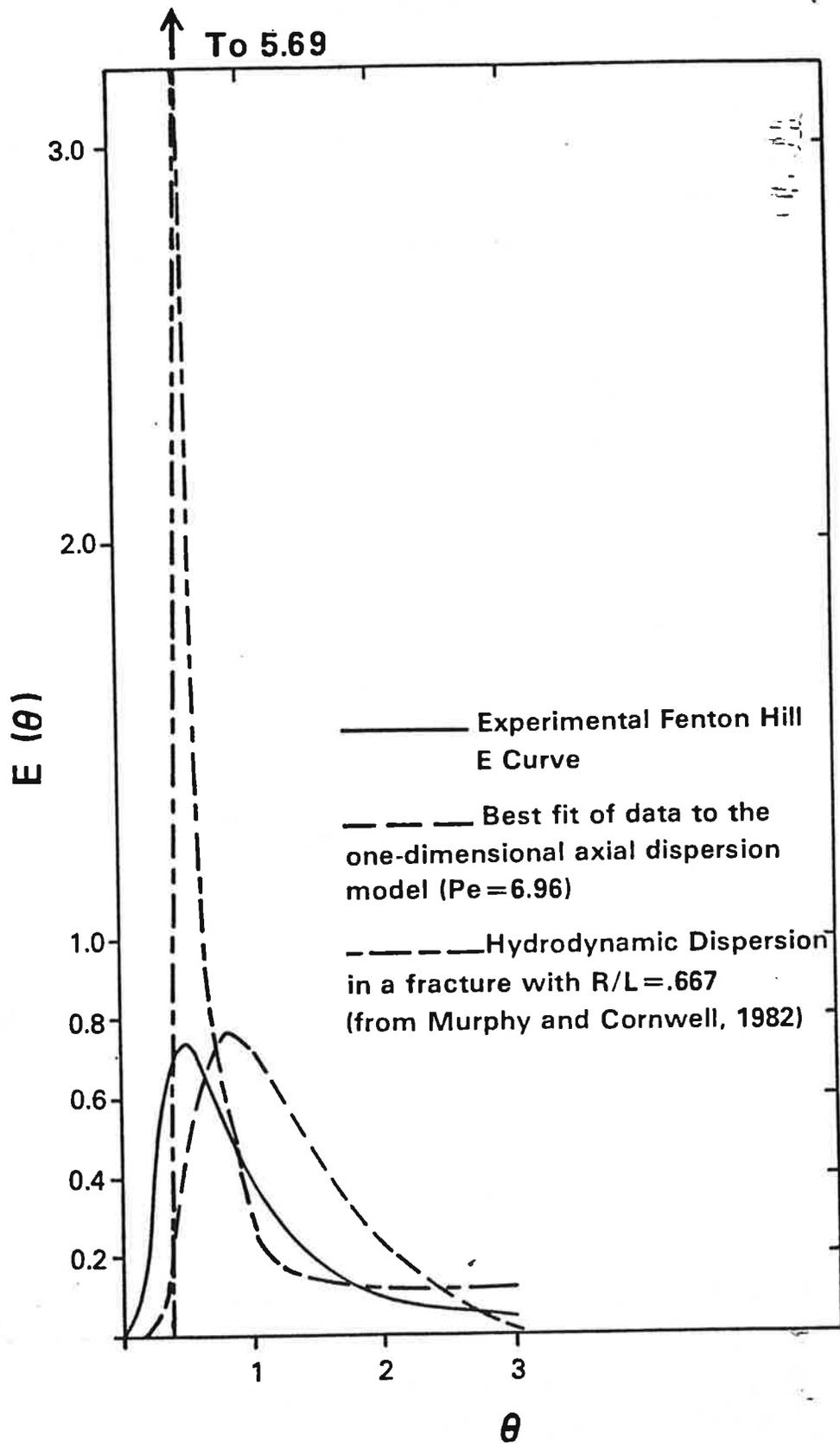


Figure 2. Two attempts to simulate the E curve for the Phase I system with one-parameter dispersion models.

desirable characteristics when compared to the actual E curve. The axial dispersion model predicts the overall shape of the curve reasonably well, while the hydrodynamic dispersion curve predicts the position (but not the height) of the peak.

At first glance, the axial dispersion curve appears to simulate the data more accurately, but we know from previous heat transfer modeling that the modal volume (the volume for which the E curve is a maximum) is the parameter which correlates most readily with effective heat transfer area. For a given fracture aperture, the modal volume may be thought of as a measure of the amount of short-circuiting of the fluid in its path from entrance to exit. Thus, the hydrodynamic dispersion model could perhaps be more useful in heat transfer modeling, because axial dispersion cannot provide a sufficient amount of short-circuiting to predict the modal volume. A two-parameter model involving both turbulent and hydrodynamic dispersion in a single fracture is probably justified and could be used to simulate both the modal volume and the overall amount of dispersion in the system. Alternatively, a multiple fracture or aperture distribution model could be used. In short, the non-uniqueness problem still exists. New diagnostic experiments such as chemically reactive tracers will be required to characterize the Phase II system.

#### Recommendation

In the future, tracer experiments should be run in the fresh water flush mode during the time when the peak of the E curve is being recorded. This would eliminate the uncertainties associated with subtracting out the effect of recirculation in the tracer experiments. In the Phase I system, 80% of the recovered tracer could have been kept from recirculation by

operating in the fresh water flush mode during the period from 68,000 to 248,000 gallons. The radioactive water could be reused after allowing enough time for the  $\text{Br}^{82}$  to decay (about one week).

### Acknowledgements

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### Appendix: Analytical Expression for $C_{\text{out}}(t)$ for an Ideal Reactor of Known

#### RTD

To check the computer code TRACE, we need to solve Equation (1) analytically for a recirculating experiment in an idealized reactor of known  $E(t)$ . Then the values of  $C_{\text{in}}(t)$  and  $C_{\text{out}}(t)$  may be used as input to test the accuracy of the code.

The simplest way to solve Equation (1) is by first taking the Laplace transform (Beyer, 1976):

$$C_{\text{out}}(s) = C_{\text{in}}(s) E(s) \quad (\text{A-1})$$

Since  $C_{\text{in}}(t)$  is the dirac delta function followed by  $C_{\text{out}}(t)$ ,

$$C_{\text{in}}(s) = C_{\text{out}}(s) + 1 \quad (\text{A-2})$$

Thus,

$$C_{out}(s) = \frac{E(s)}{1 + E(s)} \quad (A-3)$$

For a given idealized reactor of known RTD, we can take the Laplace transform of  $E(t)$ , substitute into Equation (A-3), and take the inverse transform to find  $C_{out}(t)$ .

For our test cases we will consider  $N$  CSTR's in series. The  $E$  curve for this idealized system is

$$E(t) = \frac{t^{N-1}}{\bar{t}^N} \frac{1}{(N-1)!} e^{-t/\bar{t}} \quad (A-4)$$

The Laplace transform of (A-4) is

$$E(s) = \frac{1}{(s\bar{t}+1)^N} \quad (A-5)$$

$$C_{out}(s) = \frac{1}{(s\bar{t}+1)^{N-1}} \quad (A-6)$$

The two cases we will consider are  $N=1$  and  $N=2$ . For 1 CSTR,

$$C_{out}(t) = \frac{1}{\bar{t}}, \quad (A-7)$$

while for two CSTR's,

$$C_{out}(t) = \frac{1}{2\bar{t}} (1 - e^{-2t/\bar{t}}) \quad (A-8)$$

Equations (A-7) and (A-8) were used to generate the input data for the computer code. The code calculated  $E(t)$  for the two cases using Equation (1), thus providing a verification of the program logic.

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