CONF-760814--1

LA-UR -76-473

TITLE: STRAINS IN A CANTILEVER PLATE OBTAINED FROM SECOND DERIVATIVES OF HOLOGRAPHICALLY MEASURED MODE SHAPES

AUTHOR(S): Jon E. Sollid and Karl A. Stetson

SUBMITTED TO: International Conference on Applications on Holography and Optical Data Processing, Tel-Aviv, Israel, August 23-26, 1976.

los alamos scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

An Affirmative Action/Equal Opportunity Employer

UNITED STATES
ENERGY RESEARCH AND
DEVELOPMENT ADMINISTRATION
CONTRACT W-7405-ENG. 36

MASTER

STRAINS IN A CANTILEVER PLATE OBTAINED FROM SECOND DERIVATIVES OF HOLOGRAPHICALLY MEASURED MODE SHAPES*

Jon E. Sollid
University of California
Los Alamos Scientific Laboratory
Los Alamos, NM 87545

Karl A. Stetson
Instrumentation Laboratories
United Technology
East Hartford, Connecticut 06108

ABSTRACT

In this paper reliable derivatives up to second order over a surface are obtained from holographic interferograms. This is done on a large scale, handling an amount of data that would be prohibitive if attempted by hand smoothing. This opens the way for bending and shear strain determinations in a non-contacting way over large fields of view. The surface need be treated in no special way, in contrast to the preparation required with brittle lacquers and photoelastic coatings. Strains in the microstrain range are readily measured. Holographic interferometry in conjunction with the numerical methods mentioned herein constitute a non-contacting optical strain gauge. Furthermore it makes possible experimental application of sophisticated modal analysis because the normal modes upon which these techniques are based are directly measured. For example, if the vibratory behavior is known, through a sufficiently large sample of the normal modes, then the static behavior may be calculated. Similarly, if the normal modes of a non-rotating body are known its behavior in a rotating system can be calculated. Thus the centrifugal stiffening of a turbine rotor may be predicted from a knowledge of the spectrum of its normal modes.

*This work was done while at Ford Motor Company, Scientific Research Laboratories, Dearborn, Michigan 48121.

STRAINS IN A CANTILEVER PLATE OBTAINED FROM SECOND DERIVATIVES OF HOLOGRAPHICALLY MEASURED MODE SHAPES*

Jon E. Sollid
University of California
Los Alamos Scientific Laboratory
Los Alamos, NM 87545

Karl A. Stetson
Instrumentation Laboratories
United Technology
East Hartford, CT 06108

INTRODUCTION

Ever since the discovery of holographic interferometry [1,2,3,4,5,6] there have been many successful and interesting applications in nondestructive testing where the information obtained was primarily of a qualitative nature. As an example consider the use of holo-techniques to determine debonds in honeycomb paneling. One wants to know where the debond occurs and its general extent. No further information is necessary. Many such applications are cited in the literature,[7] yet in the interferogram much more information of a quantitative nature is latent. For example, the displacements and deformations of the body under test may be determined. There are a number of examples of interesting and successful applications in this area where quantitative determinations of displacement and deformation were done but they are fewer in number. As an example, consider the determination of one,[8] two,[9] and three[10,11] dimensional deformations under static or dynamic load, Poisson's ratio,[12] vibratory mode shapes and natural frequencies.[13] Very often an engineer is not interested in the displacement field, but rather in the strains which are obtained from the first[14,15,16] and second[17] order derivatives of the displacement field. So far there are even fewer reported applications in which the data reduction and processing has been carried to actual strains. This is primarily due to the labor

*This work was done while at Ford Motor Company, Scientific Research Laboratories, Dearborn, Michigan 48121.

and time involved in successfully producing reliable first and second derivatives from experimentally obtained data.

Leissa[18] in his monumental compendium "Vibration of Plates," states: "Virtually no one in the literature evaluates the bending stresses due to a unit amplitude of motion. This information is obviously important particularly for fatigue studies. The lack of results is undoubtedly due to the fact that the stresses must be obtained from second derivatives of the mode shapes. Not only does this require additional computational work, but also the mode shapes usually are not known with sufficient accuracy to give meaningful results for stresses." In this paper we show that for quantitative work of this nature, holo-techniques are unrivaled. Full field amplitude distributions can be obtained from which reliable bending stress may be calculated.

EXPERIMENTAL

In the work described here a cantilevered plate, which may be considered a zeroth order model of a turbine blade or impellor blade, was driven sinusoidally by eddy current transducers. The natural frequencies were determined using real-time holography. The mode shapes were recorded as time-average interferograms on 4" x 5" photographic plates (Agfa 10E70). Photographic reconstructions were made on 4" x 5" negatives and 8" x 10" prints were used in the data reduction stage. The data reduction was performed from photographic reconstructions simply because the equipment was available. Obviously if much data of this kind is to be processed, a better method would be to proceed without the photographic step by using some form of electronic imaging and direct digitization. Figure 1 shows nine "lowest-order" modes for the cantilever plate made of aluminum with dimensions 1/8" x 6" x 7-1/4". The modes are arranged according to the morphology suggested by Grinsted[19] in which each row corresponds to a constant number n of node lines parallel to the clamped edge, and each column corresponds to a constant number m of node lines perpendicular to the clamped edge. Thus the modes can be characterized by a pair of digits (mn).

The order of appearance of the modes is shown in parenthesis under the frequency at which each mode occurred. Note the 30 mode actually occurs before 22 mode at a frequency of 2150 Hz, hence the quotation marks around "lowest-order" above.

This manner of data presentation is very useful in assessing whether or not any modes have been missed. Furthermore, as long as one deals with homogeneous and isotropic material a relationship can be established between the modal pattern of an unusual shape such as an impellor, and the zeroth order model, in this case, the cantilever. As an example, consider Fig. 2 which shows the lowest frequency modes of an impellor blade. With only a little imagination they are readily identified as the 00, 01, 01, and 11 modes in order of increasing frequency. Where the first (second) digit refers to the number of node lines perpendicular (parallel) to the clamped edge according to the previous convention.

DATA REDUCTION

Once a high contrast 8" x 10" print was obtained on a single weight matte paper the digitization could proceed. Glossy prints were much harder to work with than matte. Several different digitizing devices were used, all of which worked from the prints: two models of Bendix digitizer, a graph pen, and four models of coordinnatograph (Gerber, two Aeros and an old Benson-Lernert). The coordinatographs used TV cameras with 10X objectives to scan the prints. They are designed to work on large (5 feet by 18 feet) mylar drawings of very high contrast, and they are not well suited to interferogram digitizing. An additional inconvenience was the output, punched cards. The more convenient digitizers, the Bendix and the Graph-pen, write directly onto the disk file space of the computer, a DEC-10.

The data reduction program proceeds in two stages. First the distortion in the reconstruction and obliquity factors due to the holocamera geometry are accounted for. Then a program developed initially for use in numerical control machining work is used to provide a smoothing and data compression. This reduces the data set of many discreet points to a handful of parameter. which allow a mathematically continuous and smooth description over the entire surface.

Initially piece-wise continuous cubics or splinefunctions were tried. Taylor and Brandt[17] reported success using this method. They accomplish smoothing by judicious deletion of some of the data. Some kind of smoothing is essential as can be seen from Fig. 3. The solid curve is the second derivative of the sine function over a full cycle which simulates the amplitude distribution of a rectangular plate clamped at both edges. The finely (coarsely) dotted line is the second derivative obtained using piecewise-continuous cubics when a random 1% (10%) error is introduced in the position of points taken so as to correspond to holographically obtained vibration mode data (i.e., at 1/4 wavelength intervals). The percentage error refers to the percentage of the fringe spacing and refers to how well the darkest point in the fringe can be located. Data taken in the manner described above is more nearly 10% than 1%. Yet this data will produce excellent displacement measurements that look very smooth, cf. Fig. 4. Each point is shown with its appropriate error bars. This is data taken along the left edge of the 00 mode as shown in Fig. 1. Figure 5 shows the first derivative of this function taken two ways, the solid (dotted) line is obtained using simple differences (piece-wise-continuous cubics).

Figure 6 is included to show the mountain range produced as a second derivative using splines with no smoothing. This emphasizes the need for some method of smoothing.

Using the vector polygon polynomial, or Bezier polynomial, a surface fit is made, rather than just a curve fit. Hence all three second derivatives are obtained for any point on the surface measured. Therefore, all the bending and shear strains are obtained.

The strains are obtained as:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\ \frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z\ \frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} - 2z\ \frac{\partial^2 w}{\partial x \partial y}$$

where x and y are in the plane of the plate, and z is along the normal in a direction which gives a right-handed coordinate system; u_o and v_o refer to the plane deformation of a particular point parallel to x and y respectively in the surface, and w_o is the amplitude for the same point in the z direction. For the case of the cantilever plate, u_o and $v_o = 0$, and for a surface point $z = t/2$.

Figure 7 is a plot of the bending strain on the same plate in the 00 mode along a vertical slice 1.5 inches from the right edge. The solid line represents the strain one expects assuming the plate is a cantilever beam. The line connecting the closed circles represents the strain obtained using the spline algorithm of Taylor and Brandt.[17] The line connecting the open circles is the strain obtained using the Bezier polynomial surface fitting algorithm. The algorithm of Taylor and Brandt has yielded good results. This plot indicates the Bezier algorithm can be expected to give even better results.

To emphasize that this is a surface fit, Figure 8 shows the bending strain ε_y along a vertical span through the middle of the plate undergoing the 22 mode vibration. Figure 9 shows the bending strain ε_x along a horizontal chord passing through an antinode of the same mode. And Figure 10 shows the actual paths of these sections through the plate. They are not constant y and constant x sections because of the smoothing in the program. Points of maximum Gaussian curvature are indicated by the crosses. The error in the second derivative at the tips is large and the fact that the large second derivatives are seen there is to be expected. On the free edge the bending moment must be zero. For example, at the tip of the plate, where $y = b$, the bending moment M_y must be zero. Therefore,

$$M_y = D(K_y + \nu K_x) = 0$$

and hence the ratio of the curvatures K_y and K_x should give an estimate of Poisson's ratio ν. Unfortunately there is a region of about three-quarters of an inch all around the free edges of the plate in which the second derivatives are suspect. Consequently no reliable measure of Poisson's ratio is possible in this way. Perhaps the smoothing algorithm could be improved by adding such constraints. As it stands this level of sophistication was not considered justified and was not incorporated.

The error in strain to be expected on the basis of a very simple analysis is given by

$$\Delta\varepsilon = t \cdot \frac{z_1 + z_{-1} - 2z_0}{(\Delta x)^3} \, \delta\Delta x$$

Where the values z_1, z_0, z_1 may be thought of as values of the amplitude, and Δx is the spacing between fringes. The error $\delta\Delta x$ actually increases as the spacing decreases below ~ 0.030" which is about the best one can expect human hand and eye to guide a cursor at the rates used here. Note that the error depends rather sensitively on the spacing between fringes Δx, since it goes as the inverse cube. The error to be expected for the spanwise section shown in Fig. 8 is on the order of 0.036 $\mu\varepsilon$ at the tip. The residual may be considered due to oversmoothing.

Shown here were examples of bending strains from the simplest and most complete patterns patterns from Fig. 1. Results for any of ther intermediate modes were also obtained, but not displayed here for brevity.

The Bezier Polynomial

The vector polygon polynomial used in the data reduction is named after P. Bezier, an engineer at Renault. The concepts he developed[22] were further refined at Ford.[20] One of the most useful properties of these polynomials is the data compression; entire surface areas can be accurately represented by as few as sixteen parameters. The surfaces so represented need not be restricted to those that are describable in one of the eleven canonical coordinate systems in which the wave equation is separable. That is, sculptured surfaces may be represented. As a comparison consider a piece-wise continuous cubic or spline fit. Rather than a data compression, a four fold expansion results. For each discreet data point, or knot, four coefficients are generated for the spline fit. A compression can be obtained only by some form of deletion.

Basically the procedure is as follows: One assumes a two dimensional vector function of the form

$$\underset{\sim}{P}(\alpha,\beta) = \sum_{i=o}^{n} \sum_{j=o}^{m} g_n^i(\alpha)\, g_m^j(\beta)\, \underset{\sim}{P}_{ij}$$

where α, and β are the variables in a two dimensional space related to, but not the same as x, y space; the g_n^i are weighting factors defined by

$$g_n^i(\alpha) = \binom{n}{i} \alpha^i (1-\alpha)^{n-i}$$

where $\binom{n}{i}$ is the binomial cofficient, i.e.,

$$\binom{n}{i} = \frac{n!}{(n-i)!i!}$$

An initial correspondence is established between the x, y space of the object and the α, β parameter space, so that the discreet set of data points $\underset{\sim}{Z}\ (x_r, y_s)$ maps into $\underset{\sim}{Z}\ (\alpha_r, \beta_s)$. Then the difference $\underset{\sim}{E}_{rs}$ between the assumed surface and the given surface is formed:

$$\underset{\sim}{E}_{rs} = \underset{\sim}{P}\ (\alpha_r, \beta_s) - \underset{\sim}{Z}\ (x_r, y_s).$$

The dot product of the error is formed and minimized with respect to the $\underset{\sim}{P}_{ij}$, thereby allowing the $\underset{\sim}{P}_{ij}$ to be determined. Hence it is a best fit in the least square sense. A transformation from a three dimensional system $(x, y, z\ (x, y))$ to a two dimensional system (α, β) is accomplished, i.e.,

$$\begin{array}{lll} x = x & & x = x\ (\alpha, \beta) \\ y = y & \rightarrow & y = y\ (\alpha, \beta) \\ z = z\ (x, y) & & z = z\ (\alpha, \beta) \end{array}$$

The art and the skill in the program lies in the iterative procedures which adjust the parameters to allow the smoothing.

CONCLUSION

Demonstrated here was the combination of accurate holographically obtained full-field deformation data, in conjunction with sophisticated numerical methods to measure indirectly bending and shear strains in vibrating plates. The strain levels were quite low, on the order of $\mu\varepsilon$, indicating the sensitivity of the method. one may think of this as an optical, noncontacting, full field strain gauge method.

Furthermore, this may provide a stepping stone to even more sophisticated methods of modal analysis.[21] The application of modal analysis from an experimental basis has been stymied in many applications, due to the difficulty of obtaining reliable first and second order drivatives of them.

One particular application bears description. Suppose the vibratory behavior of a turbine rotor, for example, is known from its holographically determined mode shapes. Further assume that the initial rotor did not have exactly the vibration spectrum desired. Possibly one might use the experimentally determined mode shapes in a first order perturbation theory analysis[23] to predict the changes resulting from modifications to the structure, thus guiding the redesign of the prototype with a resulting savings in the number of prototypes required to produce a satisfactory design.

ACKNOWLEDGEMENTS

The author is pleased to acknowledge the assistance of many people with whom he had contact on this program. Special thanks are due L. G. Gajda, and his assistants C. Schauve, and R. Verbal who were instrumental in adapting the Ford Bezier vector polygon polynomial routine for use on this project, M. Mercado for skillful assistance in the laboratory, M. P. Koskella for his assistance in the laboratory and as a programmer for the two summers he was with us. Last but not least I want to express my appreciation to my colleagues J. A. Levitt, C. L. Giles and G. M. Brown for many stimulating discussions in the course of this work. Critical reading of the manuscript by O. Narayanaswamy, M. Forney and R. Gardon is also gratefully acknowledged.

REFERENCES

1. R. L. Powell and K. A. Stetson, J. Opt. Soc. Am. 55, 1593 (1965).

2. R. E. Brooks, L. O. Heflinger, and R. F. Wuerker, Appl. Phys. Lett. 7, 248 (1965).

3. K. A. Haines and B. P. Hildebrand, Phys. Lett. 19, 10 (1965).

4. J. M. Burch, Prod. Eng. 44, 431 (1965).

5. R. J. Collier, E. T. Doherty, and K. S. Pennington, Appl. Phys. Lett. 7, 223 (1965).

6. M. H. Horman, J. Opt. Soc. Am. 55, 615 (1965).

7. J. E. Sollid, Research Techniques in Nondestructive Testing, pp. 185-223, Ed. R. S. Sharpe, Academic Press, London and New York (1973).

8. J. E. Sollid, Proc. Soc. Photo-optical Inst. Eng. 25, 171 (1971).

9. C. G. Murphy, O. J. Burchett and C. W. Matthews, Proceedings ARPA-TRW sponsored Symposium on Eng. Appl. of Holography, pp. 177-185 (1972).

10. C. A. Sciammarella and J. A. Gilbert, Appl. Opt. 12, 1951 (1973).

11. S. K. Dhir and J. P. Sikora, Exp. Mech. 12, 323 (1972).

12. I. Yamaguchi and H. Saito, Jap. J. Appl. Phys. 8, 768 (1969).

13. R. Apprahamian and D. A. Evensen, J. Appl. Mech. 37, 1083 (1970).

14. J. Wallach, J. M. Holeman and F. A. Passanti, Proceedings ARPA-TRW sponsored Symposium on Engineering Applications of Holography, pp. 167-175 (1972).

15. T. D. Dudderar and R. O. Regan, Exp. Mech. 11, 49 (1971).

16. H. Saito, I. Yamaguchi and T. Nakajima, Applications of Holography, Ed. by E. S. Barrekette, W. E. Kock, T. Ose, J. Tsujiuchi and G. W. Stroke, Plenum Press, New York and London (1971).

17. L. H. Taylor and G. B. Brandt, J. Opt. Soc. Am. 61, A688 (1971).
G. B. Brandt and L. H. Taylor, Proc. ARPA-TRW Sponsored Symposium on Engineering Appl. of Holography, pp. 123 - 131 (1972).
L. H. Taylor and G. B. Brandt, Exp. Mech. 12, 543 (1972).

18. A. W. Leissa, Vibration of Plates, NASA SP-160, Office of Technology Utilization, National Aeronautics and Space Administration, Washington, D. C. (1969).

19. B. Grinsted, Proc. Inst. Mech. Eng. 166, 309-326 (1952).

20. "A Study of the Bezier Vector Polygon Polynomial Form and its Application to Parametric Curve and Surface Definition," Numerical Systems, Manufacturing Development Office, Manufacturing Staff, Ford Motor Company (1970).

21. W. C. Hurty and M. F. Rubinstein, Dynamics of Structures, Prentice-Hall, Inc., Chapter 8 (1964).

22. P. Bezier, Emploi des Machines a Commande Numerique, Masson, Paris (1970).

23. K. A. Stetson, "A Perturbation Method for Mechanical Structures," AAIA Journal 13, No. 6 (1975).