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SYSTEMATIC MODELING RULES FOR REYNOLDS STRESS CLOSURES IN FREE-SHEAR LAYERS

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ABSTRACT

General modeling rules are proposed for constructing Reynolds stress closures. The rules require that the modeled terms in the evolution equations for R_{ij} be represented as a linear combination of certain permissible groups of terms. The rules severely restrict the number of permissible groups of terms. The permissible groups of terms include many, but not all, types of terms previously proposed for turbulence modeling via Reynolds stress closures. It is shown that a lack of experimental information prevents one from concluding that D_{ij} is proportional to δ_{ij} for well-developed turbulence, even though this assumption is commonly made. In consequence, one does not know to which correlation tensor one should assign terms algebraic in R_{ij}.

NOMENCLATURE

A, B, c_o, c_1, C_n, d_n	= constants
A_{ij}, B_{ij}, C_{ij}, D_{ij},	= permissible groups of terms under the proposed modeling rules
D_{ij}	= energy dissipation tensor
$D = D_{kk}$	= trace of energy dissipation tensor
D/Dt	= total time derivative, moving with the mean fluid velocity
$\partial/\partial x^i$	= spatial gradient operator
E_{ij}, F_{ij}, G_{ij}, H_{ij}	= symmetric tensors independent of the Reynolds stress that satisfy modeling rules (1) through (6)
$f(r)$, $g(r)$, $h(r_\alpha)$	= scalar functions appearing in the two point velocity correlation tensor discussed in the text
J_{ij}, K_{ij}, L_{ij}	= supplemental symmetric tensors independent of the Reynolds stress that satisfy modeling rules (1) through (6)
M_{ij}, N_{ij}	= symmetric tensors linear in the Reynolds stress that satisfy modeling rules (1) through (6)
q	= local turbulence energy density
Q_{ij}	= dimensionless Reynolds stress tensor
$Q^2_{ij} = Q_{ik}Q_{kj}$	= square of dimensionless Reynolds stress tensor
r_i	= relative position vector in two point velocity correlation function
r	= length of relative position vector
R_{ij}	= Reynolds stress tensor
$R_{ij}(r)$	= two point velocity correlation function
s	= dissipation length scale
T_{ij}, U_{ij}, V_{ij}, W_{ij}	= supplemental symmetric tensors linear in the Reynolds stress that satisfy modeling rules (1) through (6)
U_i	= mean fluid velocity
δ_{ij}	= Kronecker tensor
$\Lambda(\xi)$	= piecewise linear function of turbulence Reynolds number
ϵ	= local mean energy dissipation rate per unit mass
ψ	= weak function of turbulence Reynolds number
ξ	= turbulence Reynolds number
ν	= kinematic viscosity

Subscripts

α,β = indices of Cartesian coordinates, not summed when repeated in a tensor expression

INTRODUCTION

The ad hoc character of the turbulence models proposed for the Reynolds stress equations has long been a source of discomfort for turbulence modelers. The only previous attempt to provide a systematic derivation of the terms accepted for the equations was made by Lumley (1-3), who used functional methods to develop a systematic perturbation theory of deviations from homogeneous, isotropic turbulence. However, the method performed only moderately well when applied to measurements of the triple correlation tensor (4), presumably because the deviations of laboratory shear flows from homogeneous, isotropic turbulence are substantial.

Here we propose simple, general modeling rules in the form of mathematical requirements on the tensors to be accepted as models for the triple correlation tensor, the pressure-strain tensor, the energy dissipation tensor and the pressure diffusion tensor, as they are used in the Reynolds stress equations. For the model terms which are algebraic in the Reynolds stress tensor, the rules are sufficiently general to encompass all the types of terms that have been previously proposed. These rules thus provide a framework for discussing systematically a class of turbulence theories sufficiently general to be interesting. The coefficients of the terms permitted by the rules must be determined by fitting to experiments and by numerical simulations. The rules are in principle applicable to the terms appearing in the equation for the dissipation scalar, but we restrict attention here to the Reynolds stress equations. Further, we do not consider the influence of walls on the turbulence modeling.

GENERAL MODELING RULES

The modeling rules we propose are:

(1) the evolution equation for the Reynolds stresses should be a tensor equation expressed in terms of the Cartesian tensors of the theory;

(2) the terms of the evolution equation should possess the Galilean invariance of the original Navier-Stokes equations;

(3) the terms of the equation should be expressed in terms of local values of R_{ij} and U_i and should contain no more than two orders of spatial derivatives;

(4) nonlinearities should be no more than quadratic in R_{ij} and U_i;

(5) functions of the turbulent Reynolds number should be no more complex than piecewise linear;

(6) the ratios of components of the energy dissipation tensor and the pressure-strain tensor should not depend on spatial derivatives of R_{ij};

(7) the principle of super-realizability: that terms permitted by the other modeling rules be included only in groups which separately preserve the realizability of the Reynolds stress tensor (namely, that the Reynolds stress tensor remains a symmetric tensor with non-negative eigenvalues under the action of each group of terms separately);

(8) the non-generation hypothesis: that the groups of terms accepted under the principle of super-realizability and which are intended to model (a) the pressure-strain correlation, (b) the triple velocity correlation, (c) the pressure fluctuation induced transport, or (d) the energy dissipation tensor shall not separately cause the total turbulence energy to increase, and shall separately permit the local turbulence energy density to increase only through diffusion processes which disperse turbulence energy more widely in space.

These rules seem simple and natural. They are also quite powerful in delimiting possible forms for turbulence theories.

It was pointed out by Lumley (1) that turbulence models cannot be rationally assessed as to their intrinsic adequacy unless "the closure used is based on a small number of explicitly stated, readily grasped principles, and that all terms, and only those terms, generated by these principles are used." Our proposed modeling rules do provide a small number of explicitly stated principles and we do propose that precisely those terms permitted by the rules be used in constructing Reynolds stress closure models. The coefficients of the groups of terms permitted by the rules would be determined by comparing model predictions with data.

Whether the modeling rules are "readily grasped" might possibly be questioned. They have not, for example, been shown to represent the leading term in an asymptotic solution of an exact turbulence theory. But they can be considered the leading term of a type of approximation scheme quite common in other branches of mechanics and, indeed, in physics and engineering quite generally. That is, the evolution equations for the variables of the theory are assumed to depend only on the current values of the variables, and not on values at previous times (as is true of the underlying Navier-Stokes equations in the present instance). Then, when this dependence is specified as a power series in the variables and their derivatives, the series is truncated at the second order. More general approximations then the one proposed here might truncate the series at higher order. Because there is no general solution technique for the turbulence equations, the terms must be chosen and grouped to insure that they will maintain the appropriate mathematical properties of the variables of the theory. In addition, the coefficients of the series must be determined empirically.

The realizability requirements for Reynolds stress closure models have been stated by Schumann (5) in the form of three types of inequalities (no sum over Greek indices)

$$R_{\alpha\alpha} \geq 0 \quad , \tag{1}$$

$$R_{\alpha\alpha} R_{\beta\beta} - (R_{\alpha\beta})^2 \geq 0 \quad , \tag{2}$$

$$\det (R_{\alpha\beta}) \geq 0 \quad . \tag{3}$$

He points out that for exact solutions $R_{\alpha\beta}$ the Navier-Stokes equations require that when one of the inequalities becomes an equality, then the total time derivative of this equality be zero, namely

$$R_{\alpha\alpha} = 0 \rightarrow \frac{D}{Dt} R_{\alpha\alpha} = 0 \quad , \tag{4}$$

and similarly for the other inequalities. We propose here to weaken these requirements on the time derivatives of the equalities to the maximum permitted by the realizability of the theory, namely

$$R_{\alpha\alpha} = 0 \rightarrow \frac{D}{Dt} R_{\alpha\alpha} \geq 0 \quad , \tag{5}$$

$$R_{\alpha\alpha} R_{\beta\beta} - (R_{\alpha\beta})^2 = 0 \rightarrow R_{\alpha\alpha} \frac{D}{Dt} R_{\beta\beta} + R_{\beta\beta} \frac{D}{Dt} R_{\alpha\alpha} - 2R_{\alpha\beta} \frac{D}{Dt} R_{\alpha\beta} \geq 0 \quad , \tag{6}$$

and similarly for the determinant. The reason for this is that it is common, and sometimes necessary for the numerical stability of a differencing scheme, to have excess diffusion in the solution of fluid dynamics problems. Such excess diffusion will change the equalities. Such theories have been called over-realizable by Schumann.

By the principle of super-realizability we mean that a permissible group of terms for modeling the velocity correlation will preserve the inequalities of Eqs. (5) and (6), and the corresponding one for the determinant, when the group of terms is substituted for DR_{ij}/Dt after the product rule for derivatives has been used to evaluate the time derivative of the equality explicitly, as, for example, in Eq. (6). The total modeling of the evolution equation for the Reynolds stresses is to be a sum of such permissible groups of terms. The modeling will capture another property of the exact solution if, in the evolution equation for the Reynolds stresses the terms resulting from the triple correlation tensor, the total correlation involving the fluctuating pressure, and the correlations proportional to the molecular viscosity are separately represented as a sum of permissible groups of terms. We would strongly recommend such modeling as maximally likely to approximate the exact equations most successfully, but we would not demand this property under the principle of super-realizability until it becomes clear that accurate models may be constructed enforcing this rule. Similarly, we would encourage, but not yet demand, models to be constructed in which the inequalities of Eqs. (5) and (6) are replaced by equalities.

The non-generation hypothesis is reasonable for fully developed turbulence: it agrees with the evidence that is available; it is a statement of something that approaches consensus among turbulence researchers, and it is consistent with past practice in turbulence model building. However, it does disagree with experiment during the transition to turbulence, for then viscous forces can give rise to turbulent energy. We have not built this reservation into our theory because we think that the primary interest of Reynolds stress models is for fairly well-developed turbulence and because it seems over-optimistic to hope that a single workable turbulence theory will be able to describe accurately both fully developed turbulence and the transition to turbulence. We therefore center our attention on the theory for well-developed turbulence and simply build in the most reasonable behavior at low turbulence energies that we can.

RESULTS FOR TERMS ALGEBRAIC IN REYNOLDS STRESSES

We first specify the terms algebraic in the Reynolds stresses that satisfy the modeling rules (1) through (6) and that also preserve the symmetry of the Reynolds stress tensor. The additional requirement of the principle of super-realizability, that the terms be accepted only in groups which separately preserve the non-negativity of the eigenvalues of the Reynolds stress tensor, requires separate discussion, which follows, as does the discussion of limits imposed by the hypothesis of non-generation.

We note that tensors algebraic in the Reynolds stresses will be consistent for use in the evolution equation for the Reynolds stress if they are formed by multiplying the scalar VD by a symmetric dimensionless tensor which satisfies modeling rules (1) through (6). Appropriate dimensionless tensors are built from R_{ij} and U_i and may be enumerated as follows. The tensors independent of R_{ij} are δ_{ij},

$$
\begin{aligned}
E_{ij} &= sq^{-1/2}\left(\frac{\partial}{\partial x^j} U_i + \frac{\partial}{\partial x^i} U_j\right) , \\
F_{ij} &= s^2 q^{-1} \frac{\partial}{\partial x^k} U_i \frac{\partial}{\partial x^k} U_j , \\
G_{ij} &= s^2 q^{-1} \frac{\partial}{\partial x^i} U_k \frac{\partial}{\partial x^j} U_k , \\
H_{ij} &= s^2 q^{-1} \left(\frac{\partial}{\partial x^k} U_i \frac{\partial}{\partial x^j} U_k + \frac{\partial}{\partial x^k} U_j \frac{\partial}{\partial x^i} U_k\right).
\end{aligned} \tag{7}
$$

The terms linear in R_{ij} are

$$
\begin{aligned}
Q_{ij} &= q^{-1} R_{ij} , \\
M_{ij} &= sq^{-1/2}\left(R_{ik} \frac{\partial}{\partial x^k} U_j + R_{jk} \frac{\partial}{\partial x^k} U_i\right) , \\
N_{ij} &= sq^{-1/2}\left(R_{ik} \frac{\partial}{\partial x^j} U_k + R_{jk} \frac{\partial}{\partial x^i} U_k\right) .
\end{aligned} \tag{8}
$$

Finally, the quadratic tensor is

$$
Q^2{}_{ij} = Q_{ik} Q_{kj} . \tag{9}
$$

In addition, we do not count appearances of q, and s in determining the quadratic nonlinearity of the system as a function of R_{ij} and U_i. Thus there are, in principle, two extra vectors available for producing tensors algebraic in R_{ij}, namely $\partial q/\partial x^i$ and $\partial s/\partial x^i$. (The latter vector could be replaced by $\partial \ell/\partial x^i$ if desired.) Because it is not yet certain that these vectors are needed in the algebraic terms of Reynolds stress closures, we would prefer, as a first choice, to restrict the form of the theory by not incorporating them. However, Lumley (1) has argued quite persuasively for their inevitable appearance. So one must be prepared to see that experiment requires their incorporation. When these vectors are added to the theory, one may obtain the additional symmetric tensors algebraic in R_{ij} and satisfying the modeling rules (1) through (6), namely tensors constant in R_{ij}

$$
\begin{aligned}
J_{ij} &= s^2 q^{-2} \frac{\partial}{\partial x^i} q \frac{\partial}{\partial x^j} q , \\
K_{ij} &= \frac{\partial}{\partial x^i} s \frac{\partial}{\partial x^j} s , \\
L_{ij} &= sq^{-1}\left(\frac{\partial}{\partial x^i} q \frac{\partial}{\partial x^j} s + \frac{\partial}{\partial x^j} q \frac{\partial}{\partial x^i} s\right) ,
\end{aligned} \tag{10}
$$

and tensors linear in R_{ij}

$$
\begin{aligned}
T_{ij} &= Q_{ik} J_{kj} + Q_{jk} J_{ki} , \\
U_{ij} &= Q_{ik} K_{kj} + Q_{jk} K_{ki} , \\
V_{ij} &= sq^{-1}\left(Q_{ik} \frac{\partial}{\partial x^k} s \frac{\partial}{\partial x^j} q + Q_{jk} \frac{\partial}{\partial x^k} s \frac{\partial}{\partial x^i} q\right) , \\
W_{ij} &= sq^{-1}\left(Q_{ik} \frac{\partial}{\partial x^k} q \frac{\partial}{\partial x^j} s + Q_{jk} \frac{\partial}{\partial x^k} q \frac{\partial}{\partial x^i} s\right) .
\end{aligned} \tag{11}
$$

Now the two added vectors do considerably enrich the theory, when terms proportional to the first spatial derivative of R_{ij} are considered. However, we shall show here that none of the tensors of Eq. (10) are permitted by the application of both the principle of super-realizability and the hypothesis of non-generation. First consider the constant tensors of Eq. (10). Since the components of L_{ij} do not have a universal algebraic sign, this tensor may violate the inequalities of Eqs. (5) and (6) when L_{ij} is substituted for DR_{ij}/Dt in these equations. Since this is true even when L_{ij} is multiplied by an arbitrary coefficient, L_{ij} is excluded by the principle of super-realizability. Similarly, the principle of super-realizability requires that both J_{ij} and K_{ij} enter the modeling with positive coefficients. But the hypothesis of non-generation requires that the coefficients be negative. Thus J_{ij} and K_{ij} are separately excluded from the theory. Finally, one may show that no linear combination of J_{ij}, K_{ij} and L_{ij} is permitted by modeling rules (7) and (8): these three tensors are fully excluded from the theory, and any enrichment of the theory of the terms algebraic in R_{ij} is provided by the tensors of Eq. (11).

By analyzing in the local principal axes of R_{ij}, one may readily show that all the tensors of Eq. (11) satisfy the principle of super-realizability. However, the hypothesis of non-generation requires that these tensors be added to the right hand side of the evolution equation for R_{ij} only in the linear combination

$$O_{ij} = - A^2 T_{ij} - B^2 U_{ij} + c_o V_{ij} + c_1 W_{ij} \quad , \tag{12}$$

where

$$c_o + c_1 = \pm 2AB \quad , \tag{13}$$

which contains only three arbitrary constants, A, B and one of c_o and c_1. As stated earlier, we would prefer first to attempt modeling the algebraic terms without using the permissible group of terms of Eq. (12).

The constant tensors of Eq. (7) do not form a permissible group of terms, either separately or as a linear combination of terms: to contribute permissibly to the theory, these terms must be combined with the linear and quadratic terms of Eqs. (8) and (9). To see this, consider a general linear combination of the tensors of Eq. (7) and analyze in the local principal axes of R_{ij}. The principle of super-realizability requires that the resulting tensor (of the linear combination) have positive diagonal elements, while the hypothesis of non-generation requires that the sum of the diagonal elements of the resulting tensor be negative. The only way to achieve this for a general mean velocity field is to have the null tensor as the linear combination.

By the same argument one may see that the traceless tensors

$$A_{ij} = Q_{ij} - 2/3\ \delta_{ij} \quad , \tag{14}$$

and

$$B_{ij} = Q^2_{\ ij} - 1/3\ Q^2_{\ kk}\ \delta_{ij} \tag{15}$$

are both permissible groups of terms provided that they are multiplied by negative coefficients. While F_{ij} and G_{ij} would seem to have the positivity properties necessary to replace δ_{ij} in Eqs. (14) and (15), both, in fact, fail to yield a permissible group of terms because the algebraic sign of the diagonal components of the resulting tensor cannot be made definite for general R_{ij} and U_i. The tensors of Eqs. (14) and (15) are the only permissible groups of terms that may be formed by Q_{ij} or $Q^2_{\ ij}$ acting separately with linear combinations of the tensors of Eq. (7).

Finally, we note that both of the tensors M_{ij} and N_{ij} satisfy the principle of super-realizability, but that each separately fails to satisfy the hypothesis of non-generation. In particular, M_{ij} is the negative of the production term in the evolution equation for R_{ij}, which may cause either a gain or a loss of turbulence energy. The fact that both tensors have identical traces does permit a third permissible group of terms to be constructed, namely

$$C_{ij} = M_{ij} - N_{ij} \quad , \tag{16}$$

which is a permissible group of terms even when multiplied by an arbitrary coefficient. The tensors A_{ij}, B_{ij}, and C_{ij} form a complete set of the independent permissible groups of terms that may be constructed from the tensors of Eqs. (7), (8), and (9). The general permissible group of terms is a linear combination of these three tensors, in which the coefficients of A_{ij} and B_{ij} are negative. If necessary, one may add the tensor O_{ij}, which is itself a linear combination of three independent permissible groups of terms.

Finally, we consider the mathematical form demanded of the coefficients of the permissible groups of terms when these groups of terms are required (through the evolution equation for R_{ij}) to reproduce the measured values of R_{ij} for particular turbulent flows.

Previous turbulence modeling has indicated the usefulness of introducing piecewise linear functions of the turbulence Reynolds number

$$\xi = (2q)^{1/2}\ s/\nu \ , \tag{17}$$

such as (6)

$$\begin{aligned} \Lambda(\xi) &= 5 \quad \text{for } \xi < 5 \\ &= \xi \quad \text{for } \xi > 5 \ . \end{aligned} \tag{18}$$

The dissipation length scale s is defined by the equation

$$D = \Lambda(\xi)\ q/s^2 \ . \tag{19}$$

In principle, there might be several such functions, but so far one function of this type seems to be adequate. We adopt the function $\Lambda(\xi)$ of Eq. (18) simply to have a definite example in mind. Nothing that follows is dependent on the particular choice of $\Lambda(\xi)$, which could as well be separately determined through optimization of any concrete turbulence model. We use this function to construct the positive function of

$$\psi = \xi/\Lambda(\xi) \quad , \tag{20}$$

which is unity for large turbulent Reynolds numbers and approaches zero for low turbulence intensities. We then require that the coefficients of any permissible group of terms, say the n^{th} such group, be written in the form

$$c_n + d_n\ \psi \quad , \tag{21}$$

where c_n and d_n are constants, possibly subject to constraints from the requirement that this group of terms be permissible. This allows a permissible group of terms to appear in the evolution equation for R_{ij} with one weight at low turbulence intensities and with another weight at high turbulence intensities, as is sometimes required for accurate modeling.

PHYSICAL INTERPRETATION OF TERMS ALGEBRAIC IN THE REYNOLDS STRESS

Most turbulence models assume local isotropy in the high Reynolds number limit and take D_{ij} proportional to δ_{ij}. Indeed, the strong and persistent feeling that local isotropy has been demonstrated in laboratory flows has been the basis of a criticism of the more general modeling of D_{ij} by Harlow and Daly (6). In fact, the evidence for local isotropy at high Reynolds numbers is seriously incomplete, as discussed in detail' by Mjolsness (7), with the consequence that one strictly does not know from experimental evidence how to apportion terms algebraic in the Reynolds stresses between the pressure-velocity correlations and the viscous correlations between velocities and velocity derivatives.

The common high Reynolds number limit assumed for D_{ij}, adopted, for example, by Lumley and Khajeh-Nouri (1) and by Launder et al. (8), and used to represent the total action of the viscous forces, does not satisfy the principle of super-realizability, even in the less strict sense of the present Eqs. (5) and (6). A fully correct treatment of the viscous forces will, of course (5), satisfy the strictest form of the principle of super-realizability, with equalities in Eqs. (5) and (6). We thus see that the common high Reynolds number modeling of the viscous forces cannot be correct at any Reynolds number. In view of this, it is relevant to review the evidence supporting this common modeling.

This evidence was recently summarized by Corrsin (9) who cites Corrsin (10), Townsend (11) and (12) and (indirectly) Laufer (13) as investigations demonstrating that

$$\left|\frac{D_{12}}{D}\right| << \left|\frac{R_{12}}{R}\right| \tag{22}$$

in laboratory flows. In fact, the investigations measure certain properties of the two point velocity correlation tensor

$$R_{ij}(\underset{\sim}{r}) = < u_i(\underset{\sim}{x})\ u_j(\underset{\sim}{x} + \underset{\sim}{r}) > \tag{23}$$

which are consistent with local isotropy and from which the inference is drawn that local isotropy holds.

To see that Eq. (22) is not forced by the previously cited experiments, we consider a possible generalization of the isotropic expansion of $R_{ij}(r)$ which might be valid under the quasi-steady, quasi-homogeneous conditions of the experiments, namely

$$\begin{aligned} R_{ij}(\underset{\sim}{r}) = {} & 2/3\ q^2 \left\{[f(r) - g(r)]\ \frac{r_i r_j}{r^2} + g(r)\ \delta_{ij}\right\} \\ & + 2/3\ q^2 \left\{h(r_3)\ [\delta_{i1}\ \delta_{j2} + \delta_{j1}\ \delta_{i2}] \right. \\ & \left. + h(r_2)\ [\delta_{i1}\ \delta_{j3} + \delta_{j1}\ \delta_{i3}]\right\} , \end{aligned} \tag{24}$$

where

$$\begin{aligned} f(r) &\simeq 1 - \frac{1}{2\lambda^2}\, r^2 , \\ g(r) &\simeq 1 - \frac{1}{\lambda^2}\, r^2 , \end{aligned} \tag{25}$$

and

$$h(x) = c - \frac{d}{\lambda^2}\, x^2 .$$

Here u, λ, c, and d are weak functions of space and time and c and d are of order unity. The first term, contributed by f and g, represents the contribution due to a local isotropic tensor and the predictions for this term given above are well tested by experiment. The second (h) term must vanish if local isotropy is to hold. But formally this term is of the same general size as the first term if c and d are of order unity. Now none of the experiments cited above is sensitive to the presence of h. Thus the values of c and d must be considered to be arbitrary, and not forced by experimental data to be zero.

From the relation valid in the homogeneous limit

$$D_{ij} \simeq -1/2\ \nabla_r^2\ R_{ij}(\underset{\sim}{r})\Big|_{\underset{\sim}{r}=0} \tag{26}$$

we see that the model tensor of Eq. (20) yields

$$\left|\frac{D_{11}}{D}\right| \simeq \left|\frac{8d}{15}\right| , \tag{27}$$

which is of the order of

$$\left|\frac{R_{12}}{R}\right| \sim 0.4 \tag{28}$$

(in many laboratory flows) if d is of order unity. Thus Corrsin's Eq. (22) does not follow from the data. We recall also experiments on Reynolds stresses (Uberoi, (14)) and on several skewness factors (Gibson et al., (15) and Freymuth and Uberoi, (16)) which show that local isotropy cannot be a completely valid hypothesis in high Reynolds number flows.

We conclude, as in (7), that local isotropy has not been fully demonstrated to hold in laboratory flows (due to the absence of measurements of cross-stream velocity derivatives) and that the absence of a conclusive demonstration of local isotropy implies that there are no firm experimental constraints on the modeling of D_{ij}, save that it be a symmetric tensor with positive trace.

USE OF PERMITTED GROUPS OF TERMS IN TURBULENCE MODELING

We discuss here the extent to which the permitted groups of terms algebraic in the Reynolds stress, obtained earlier, have found application in Reynolds stress closures existing in the literature. The discussion is meant merely to be indicative of the degree of usage. Hence, we do not give a tabulation of the permitted terms used and the non-permitted terms used for each theory.

The tensor A_{ij} was Rotta's (17) original, and inspired, choice for inclusion. It has been employed in nearly every Reynolds stress closure model since that time. The tensor B_{ij} has been used by Lumley and Khajeh-Nouri (1), but is not commonly used in these Reynolds stress closure models.

The tensor C_{ij} was found to be necessary by Daly (18) in modeling the convection phenomena of fluids heated from below. The tensors M_{ij} and N_{ij} were used in a different linear combination than C_{ij} by Launder et al. (8), who were guided by considerations of modeling the pressure-strain correlations. As we indicated earlier, we strongly prefer the combination C_{ij}, because it has more correct mathematical properties. The tensor D_{ij} does not appear to have been used in Reynolds stress closures.

We see that the proposed modeling rules have a sufficiently rich content to include many, but not all, of the tensors used in turbulence modeling via Reynolds stress closures. Yet the number of tensors allowed by the rules, the permitted groups of terms, is not very large.

FINAL REMARKS

We note that the tensors A_{ij} and B_{ij} generate over-realizable turbulence theories. Thus they would not be permissible groups of terms if the inequalities of Eqs. (5) and (6) were replaced by equalities, as would be required by exact solutions to the Navier-Stokes equations. Yet A_{ij} (Rotta's original choice) appears in virtually every attempt at modeling Reynolds stress closures. It is primarily this fact that causes us to wonder whether successful turbulence models can be constructed when the more correct equalities are enforced in Eqs. (5) and (6). This in turn, prompted us to specify only the less stringent requirements of the present version of Eqs. (5) and (6).

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