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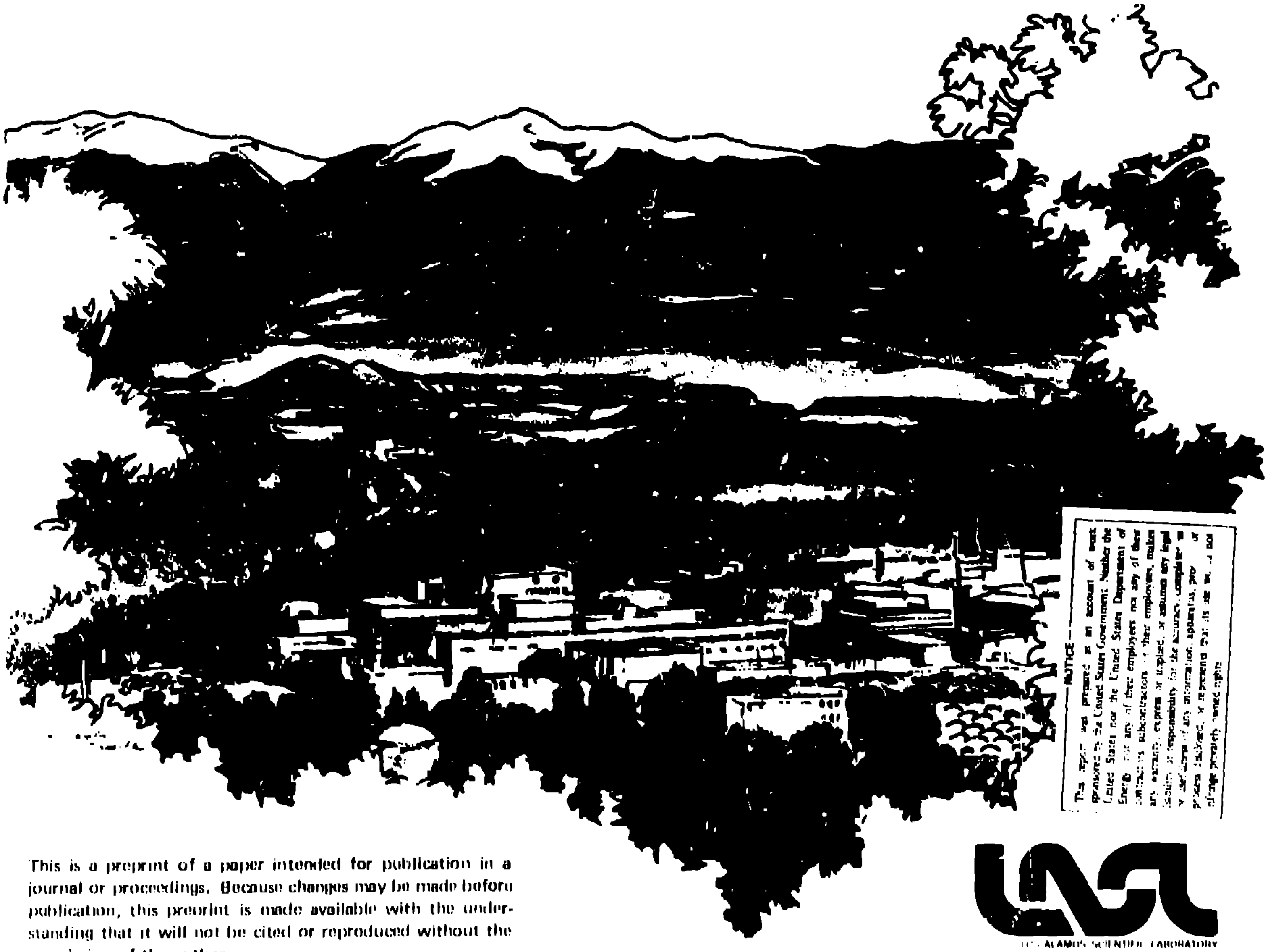

INFLUENCE OF NONUNIFORM EXTERNAL MAGNETIC FIELDS AND ANODE-CATHODE SHAPING ON MAGNETIC INSULATION IN COAXIAL TRANSMISSION LINES

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Abstract

Coaxial transmission lines, used to transfer the high voltage pulse into the diode region of a relativistic electron beam generator, have been studied using the two-dimensional time-dependent fully relativistic and electromagnetic particle simulation code CCUBE. A simple theory of magnetic insulation that agrees well with simulation results for a straight cylindrical coax in a uniform external magnetic field is used to interpret the effects of anode-cathode shaping and nonuniform external magnetic fields. Loss of magnetic insulation appears to be minimized by satisfying two conditions: (1) the cathode surface should follow a flux surface of the external magnetic field; (2) the anode should then be shaped to insure that the magnetic insulation impedance, including transients, is always greater than the effective load impedance wherever there is an electron flow in the anode-cathode gap.

Introduction

Elsewhere in these proceedings, Mike Jones has described both theory and simulation of foilless diodes. The achievement of high voltage (> 5 MeV), high current density (> 500 ka/cm^2), laminar electron beams by such diodes appears at present to require external magnetic fields on the order of 100 kg. The fringing fields from the external magnetic field coils will flare out to low values back in the coaxial transmission line feeding the diode. Also, in this same fringe field region the transmission line anode and cathode radii may taper dramatically in order to provide the proper transition in impedance and size between the diode and the insulator stack. However, previous theory[1-4] and simulation[5] of transmission lines has dealt only with straight coaxial transmission lines and no external magnetic field or with parallel plate transmission lines with a uniform external magnetic field. We are, therefore, in the process of addressing the following three questions. (1) What is the impedance of a straight coaxial transmission line with a uniform external magnetic field, (2) in a tapered coaxial transmission line with a nonuniform external magnetic field, what is the proper cathode shape relative to the external magnetic field lines; (3) What constraints on the impedance profile along the transmission line minimize the loss of magnetic insulation? These questions are being studied using the two-dimensional time-dependent fully relativistic and electromagnetic particle simulation code CCUBE[6] and simple theories. Preliminary results are given below.

Straight Coaxial Transmission Line, Uniform B_0

An analytic theory of magnetic insulation in a straight coaxial transmission line with a uniform external axial magnetic field $\underline{B}_0 = B_0\hat{z}$ appears to require some further simplifying assumption or approximation (e.g., ignoring the axial self-magnetic field or imposing some relation between radius and one or more velocity components). The choice of such a simplifying approximation can perhaps be guided by simulations, but at the time of writing this paper, only the simplest possible theory has been completed and compared with simulations.

This theory first involves generalizing the critical current calculation by Creedon[4] to include the uniform external field $\underline{B}_0$. The result is

$$I_c = \frac{I_\alpha}{\ln(b/a)}\left[\gamma_0^2 - 1 - \left(\frac{\omega_c}{c}\,\frac{b^2 - a^2}{2b}\right)^2\right]^{\frac{1}{2}} \qquad (1)$$

where $I_\alpha \doteq 2\pi m_e c/\mu_0 e \cong 8500$ A, $\gamma_0 \equiv 1 + eV_0/m_e c^2$ with V_0 the anode-cathode potential, $\omega_c \equiv eB_0/m_e$, and a and b are respectively the cathode and anode radii. Our simple theory involves assuming a relation between I_c and the actual total current I_M flowing in the transmission line.

Our motivation for this assumption stems from existing magnetic insulation theory[4] for straight coaxial transmission lines with $B_0 = 0$. In all of these theories a free parameter exists. One possibility is a continuum of magnetic insulation states corresponding to different conditions on the electrons in regions where there is a z-variation along the transmission line. Another possibility is a previously overlooked general principle which would allow the electrons to pick a unique insulation state. We believe that the latter is more likely in a transmission line and that the general principle involved is maximization of the entropy production rate. This translates into maximizing the power flow since in a transmission line the terminating load impedance is equivalent to a resistor.* If the impedance at the input to the transmission line is Z_I and the incoming (or right going) voltage there is V_I, then the voltage V_L across the line is

$$V_L = 2\,V_I\,Z_L/(Z_I + Z_L) \qquad (2)$$

where $Z_L \equiv V_L/I_L$ is the line impedance and I_L is the total line current. The transmitted power $P \equiv V_L\,I_L = I_L(2V_I - Z_I\,I_L)$ has a maximum at $I_L = V_I/Z_I$ and decreases for higher currents. If one includes the effect of insulation loss at an impedance rise, the total or effective Z_I (as viewed from the line) always satisfies $Z_I \geq Z_L$ which implies $I_L \geq V_I/Z_I$.

*Note that this power flow argument cannot in general be applied to the operating characteristics of a diode because a highly ordered electron beam is not equivalent to a resistor.

Thus, maximum power flow requires an electron current distribution that minimizes the total line current I_L. Furthermore, the existing theories[4] with $B_0 = 0$ all have very close to the same value for the minimum I_L. Finally, this value agrees well with simulation results[5,7] for the steady-state magnetic insulation current over a wide voltage range (1-20 MeV), and this value is always approximately 1/0.82 larger than the critical current I_c (with $B_0 = 0$).

Thus, we take the steady-state magnetic insulation current to be $I_M = I_c/\alpha$ even when $B_0 \neq 0$. The corresponding steady state magnetic insulation impedance $Z_M \equiv V_0/I_M$ is then

$$\frac{Z_M}{Z_0} = \alpha\left(\frac{\gamma_0 - 1}{\gamma_0 + 1}\right)^{\frac{1}{2}}\Bigg/\left[1 - \frac{1}{\gamma_0^2 - 1}\left(\frac{\omega_c}{c}\,\frac{b^2 - a^2}{2b}\right)^2\right]^{\frac{1}{2}} \qquad (3)$$

where $Z_0 \equiv 60\ \Omega\ \ln(b/a)$ is the vacuum coaxial transmission line impedance, V_0 is the anode-cathode potential, and $\alpha = 0.82 \pm 0.01$ is determined from a fit to simulation results. As B_0 increases, however, $I_c \to 0$ while we know $Z_M/Z_0 \leq 1$. Hence, Eq. (3) can be correct only for sufficiently small B_0 or large γ_0. This is demonstrated in Fig. 1 where Eq. (2) is used to find $V_0 = V_L$ with $Z_L = Z_M$ obtained from Eq. (3), V_I is fixed at either 1.5 MeV or 6.14 MeV, $Z_I = Z_0 = 37\ \Omega$, a = 1 cm, and b = 1.853 cm. The agreement between simulation and this simple theory, especially at high voltage, is sufficient to help design and interpret the results of the more complex simulations described next.

Field Line Orientation

The next simplest configuration one might try is a straight coaxial transmission line with a nonuniform external magnetic field $\underline{B}_0$. The results are indicated in Fig. 2. Here B_0 increases from 0.5 kg to 100 kg in a length of 60 cm with a = 1 cm, b = 1.853 cm, and $V_I = 6.14$ MeV. The magnetic insulation initially proceeds about the same as with $B_0 = 0$ until the position is reached where $B_0 \approx 20$ kg.

Electrons emitted in the weaker field region cannot pass this position but rather go to the anode. Electrons emitted in the higher B_0 region acquire a negative z-velocity and also go to the anode (for a total current loss of 40%) thereby reducing the actual operating impedance by an additional factor of 1/2 over Eq. (3) with $B_0 = 0$. The negative v_z of the electrons in the high B_0 region is due to the strong $v_\theta \times B_r$ force overtaking the $-v_r \times B_\theta$ force as the B_0 field lines converge toward the axis. Clearly, one should avoid having a component $B_\perp \gtrsim B_\theta v_\perp / v_\theta$ of $\underline{B}_0$ perpendicular to the cathode.

Thus, the next configurations tried have cathodes that follow a flux surface of $\underline{B}_0$. In Fig. 3, the cathode is shaped in this fashion until the straight section is reached where B_0 continues to increase from 18 kg to 80 kg. Also, the anode radius drops linearly from 12.85 cm to 1.85 cm while the cathode drops linearly from 4.4 cm to 1 cm, and $V_I = 8.35$ MeV. There is very little loss of insulation in the tapered section, but approximately 30% of the total current is lost to the anode in the straight section. In Fig. 4, the cathode is shaped to follow $\underline{B}_0$ over the entire length of 33 cm where B_0 goes from 2 kg to 80 kg. The anode tapers roughly linearly from 23.6 cm to 2 cm while the cathode tapers as shown from 8.4 cm to 1.2 cm, and $V_I = 8.35$ MeV. Once again the electrons emitted in the low B_0 region cannot pass a critical B_0 position (here when $B_0 \approx 3.5$ kg). However, the current loss to the anode is only about 7%, and it is spread over about 1.5 cm (along z) for a current density of less than 0.1 kA/cm^2 at the anode surface. In the next section we offer a possible explanation for this loss.

Impedance Revisited

In the shaped transmission lines described above (Figs. 3 and 4), the vacuum impedance $Z_0(z)$ monotonically decreased by about a factor of two with increasing axial position z. In simulations with $B_0 = 0$, this impedance drop insured that only a slight transient insulation loss occurred. The discrepancy between this case and the $B_0 \neq 0$ case (Figs. 3 and 4) might be interpreted by saying that complete magnetic insulation requires $Z_M(z) \geq Z_T$ wherever $Z_M(z) < Z_0(z)$ due to the electron flow, where Z_T is the terminating or load impedance. Otherwise, there will be some steady loss of insulation in the region around the absolute minimum of the impedance.

This would explain the insulation loss in Fig. 3 because Eq. (3) gives $Z_M(z) > Z_T$ only up to near the straight section where $Z_0 = Z_T$. Near the start of the straight section (where B_0 is still small) $Z_M < Z_0 = Z_T$, and as we move into the high B_0 region the electrons are clamped to the cathode and Z_M rises up to $Z_0 = Z_T$.

The explanation of the small insulation loss in Fig. 4 is more subtle because the effective load impedance $Z_E(z,t)$ differs from Z_T due to the rise time of the high voltage pulse. Using the telegraphers equations, with the subscript "T" denoting measurement at the terminating position z_T, and assuming a small time derivative $\dot{V}_T$, gives

$$\frac{Z_E(z,t)}{Z_T} = 1 + \frac{\dot{V}_T}{V_T c} \int_z^{z_T} \left[\frac{Z_0(z)}{Z_T} - \frac{Z_T}{Z_0(z)} \right] dz \quad . \qquad (4)$$

For our case where $Z_0(z) \geq Z_T = Z_0(z_T)$ and $\dot{V}_T \geq 0$, $Z_E(z,t) \geq Z_T$. In spite of this, for the case shown in Fig. 4, Eq. (3) gives $Z_M(z) \geq Z_E(z,t)$ and yet there is still some small loss of insulation. The problem is that in magnetic insulation there are transients where the impedance $Z_M(z,t)$ drops (by as much as 30%) below the final steady state value $Z_M(z)$ given by Eq. (3). Indeed, in the loss region shown in Fig. 4, measurements indicate $Z_M(z,t) < Z_E(z,t)$ by about 7%. Furthermore, once this loss region forms (due to a transient where $Z_M < Z_E$) and propagates to the high B_0 region (where Z_M rises to Z_0) it appears to be difficult to get rid of. Thus, complete magnetic insulation seems to require the stricter condition $Z_M(z,t) \geq Z_E(z,t)$ wherever $Z_M(z) < Z_0(z)$.

Tentative Conclusions

Minimization of magnetic insulation loss appears to require two conditions: (1) the cathode surface should coincide with a flux surface of the external magnetic field $\underline{B}_0$ at least until $B_0 \ll B_\theta\, v_r/v_\theta$; (2) the vacuum impedance $Z_0(z)$ should drop sufficiently and the rise time $V_T/\dot{V}_T$ sould be sufficiently long that $Z_M(z,t) \geq Z_E(z,t)$, including all transients, wherever $Z_M(z) < Z_0(z)$.

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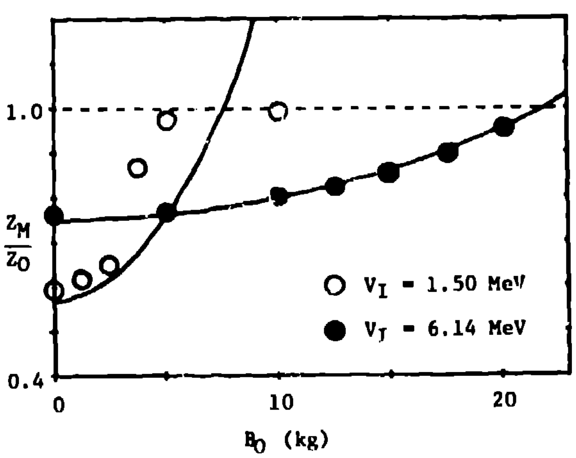

Fig. 1. Impedance of straight coax vs. uniform B_0. Solid lines are theory, Eq. (3), with α = 0.82. Open and closed circles are from simulations.

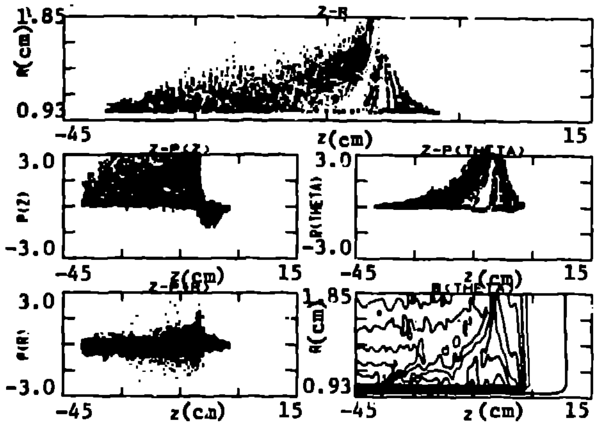

Fig. 2. Straight coaxial transmission line, nonuniform $\underline{B}_0$. Time = 1.72 nsec.

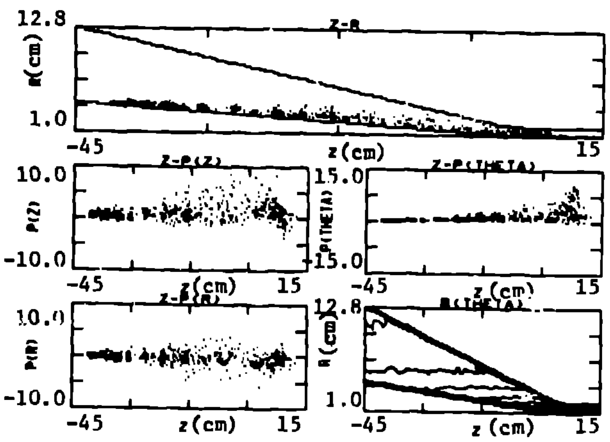

Fig. 3. Shaped coaxial transmission line, nonuniform $\underline{B}_0$. Time = 5.88 nsec.

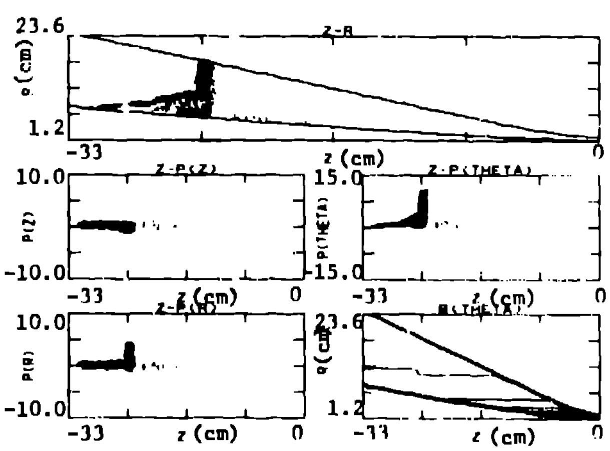

Fig. 4. Shaped coaxial transmission line, nonuniform $\underline{B}_0$. Time = 19.5 nsec.