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Cover Sheet

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ABSTRACT

An algorithm for applying advanced control concepts in heating, ventilating, and air conditioning (HVAC) systems has been developed at the Los Alamos Scientific Laboratory (LASL). The algorithm uses optimal control and is adaptive in nature. Simulations for a solar heated and cooled building indicate that use of this algorithm can accomplish a substantial savings in auxiliary energy consumption. In this paper, the algorithm is described and its application is illustrated with a simple example. Simulation results for a more complex system are also presented. The hardware, including the digital computer, needed to implement this algorithm is briefly discussed.

KEY WORDS

Control	Building-Systems
Energy	Modeling
Solar	Research

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Donald R. Farris* and Thomas E. McDonald*

1. ABSTRACT

An algorithm for applying advanced control concepts in heating, ventilating, and air conditioning (HVAC) systems has been developed at the Los Alamos Scientific Laboratory (LASL). The algorithm uses optimal control and is adaptive in nature. Simulations for a solar heated and cooled building indicate that use of this algorithm can accomplish a substantial savings in auxiliary energy consumption. In this paper, the algorithm is described and its application is illustrated with a simple example. Simulation results for a more complex system are also presented. The hardware, including the digital computer, needed to implement this algorithm is briefly discussed.

2. INTRODUCTION

The subject of cost-effective and energy-conserving control strategies and their implementation is widely discussed. It is generally agreed that the way a building is operated has a significant impact on energy use and that a poorly operated building can defeat even the best energy-conserving designs. It is also accepted that the use of controls, advanced when compared to present practice, will lead to improved system efficiencies and the use of these controls requires thorough investigation.

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An implicit element in the dialogue on HVAC control systems is that a systems approach must be used. The entire system, with its dynamic coupling and interplay of subsystems, must be examined to make effective control improvements. It is doubtful that today's "conventional" controls, designed in an era of inexpensive energy, were ever considered to be optimal except perhaps in the sense of their initial cost. Even if each individual subsystem (chiller, air-handling system, etc.) has a control system designed to optimize performance and energy use, it is well known that combining them in a single system does not necessarily result in a system that optimizes overall performance and energy use.

As system complexity increases, the difficulty of designing an optimized control system likewise increases. The difficulty encountered in designing an optimized subsystem controller is compounded when attempts are made to design an optimized overall system controller. Hence, sophisticated design techniques (e.g., optimal control theory) can prove very powerful. It is important to note that these sophisticated techniques would not have to be implemented in a final controller design, but the control strategies they demonstrate could be.

Since 1976, personnel at the Los Alamos Scientific Laboratory (LASL) have been studying the use of adaptive and optimal control techniques in solar heated and cooled buildings.[1-7] This study has led to the development of a specific approach designated as adaptive optimal control (AOC). In this study, theoretical and computer simulation studies using the AOC concept indicate that substantial energy savings can be realized with the systems approach. As the AOC concept has been refined, it has enjoyed increased confidence in its ability to improve energy conservation and system performance. Various stages of development have produced simulations with auxiliary energy savings up to 51% over the controllers implemented in the system. Other experiments in optimizing the control strategies in the solar heated and cooled Los Alamos National Security and Resources Study Center (NSRSC) demonstrate emphatically that a considerable amount of energy can be saved by modifying conventional control hardware and, most importantly, by reviewing the control function from a system level.[8]

The system-level AOC approach is discussed here from both theoretical and practical points of view. The salient features of the resulting control equations are discussed and simulation results are presented.

3. THE AOC CONCEPT

To illustrate the AOC concept, consider the portion of a solar heating system shown in Fig. 1. There the controls to be determined are (1) the hot water flow rate, W_{hw}, (2) the choice of the storage tank or the auxiliary heat exchanger as an energy source, and (3) the temperature, T_{aux}, of the auxiliary energy source if it is chosen. Conventional strategies for determining these controls are also shown in Fig. 1. The hot water flow rate is determined solely on the basis of the room temperature, T_r. In determining the flow rate, no consideration is given to the temperature of the water and therefore, to its ability to heat the room. The choice of the solar storage tank or the auxiliary heat exchanger as the source of energy is based on the storage tank temperature, T_s, and the ambient temperature, T_a. Certainly, the ambient temperature affects the room temperature, but the room temperature is not explicitly considered. Similarly, the auxiliary hot water temperature is determined solely by the ambient temperature and does not consider the actual room heating requirements as reflected in the room temperature.

The AOC approach, however, considers all the significant system variables in determining the controls as schematically illustrated in Fig. 2. In order to determine the controls at the system level, the AOC techniques uses a mathematical model of the system. This model is obtained with a system identification process illustrated in Fig. 3. In the system identification process, certain system variables are measured as a function of time. For any given set of operating conditions, the system may be described by a first-order differential equation such as

$$\frac{dT_r}{dt} = aT_r + bT_s + cW_{hw} + \ldots \tag{1}$$

The system identification technique uses a least squares approach to estimate the parameters, a, b, and c, to produce an equation, like Eq. (1), that fits the measured data.

This dynamic equation is used in conjunction with a mathematical performance index to determine the controller gains. The mathematical performance index assesses a numerical measure to the deviation of both control variables and system variables from their set points. Relative weights are given to each deviation so that the relative importance of deviations may be established. These deviations are integrated in the performance index so that both magnitude and duration of the deviation are important. The linear regulator theory is used to compute controller gains (k_1, k_2, . . .) that minimize the performance index for the dynamic equation determined by the system identification process.

The controller gains are used to implement linear, closed-loop control equations such as

$$W_{hws} = k_1T_r + k_2T_s \ldots , \tag{2}$$

where W_{hws} is the hot water flow rate out of the storage tank. Some of the features of the control equations are depicted in Fig. 4. Fig. 4 (a) shows the multidimensional characteristics of Eq. (2). The hot water flow rate is a function of more than one of the system variables. Both room temperatures and the storage tank temperature, as well as other system variables that would be visualized in a higher order space, determine the hot water flow rate when the storage tank is being used. If all of the variables but one, say T_r, are chosen at some constant value, then W_{hws} becomes the same type control equation the conventional controller uses. Continuing to examine W_{hws} as a function of T_r only, the slope of the line between saturation limits is the controller gain, k_1. The adaptive nature of the AOC approach is manifested when the entire process is repeated periodically, and the resulting gains are subject to change as shown in Fig. 4 (b). A plot of a single gain as a function of time might appear as in Fig. 4 (c). The preceding discussion has concentrated on W_{hws} as a function of T_r, but similar comments apply to all the terms in Eq. (2). In fact, the multidimensional plane of Fig. 4 (a) would change each time the system adapts (or is updated).

One readily sees then that the type of control offered by the AOC is essentially that found in a conventional proportional controller. However, the control parameters are determined using a system-level approach; and further, they are optimally determined to minimize a specific performance index.

4. AOC THEORY

4.1 AOC Block Diagram

The block diagram of an AOC system is shown in Fig. 5. The building, or plant, represents the nonlinear dynamics of the building and entire HVAC system. The model identification block represents the system identification process, which identifies a linear model that accurately reproduces the nonlinear plant behavior for the given set of operating conditions. Once the linearized parameters have been identified, they will be used in conjunction with an integral quadratic cost functional to compute the optimal controller parameters. The computation of the optimal control parameters also uses optimization data from a portion of the algorithm known as the supervisor. The supervisor provides a channel to communicate any human operator's commands to the system. The optimization data it provides contains such information as set points, modifications to the relative weights for the deviations in the performance index, system adjustments based on the time of day, etc. The controller computational algorithm makes use of standard linear regulator/linear servomechanism theory with set points for both the control and state variables. The controller uses these control parameters to generate the appropriate control to minimize the cost functional. The model identification process is an on-line process; after a fixed amount of time it will have an updated linear model available. That updated linear model and the optimization data supplied by the supervisor are then used to recompute the optimal control law. The control system is, therefore, changed (adapted) to remain optimal as the system undergoes changes of various forms.

4.2 Model Identification

The building and HVAC system are mathematically described by the non-linear system

$$x = f(x,u,t) \quad , \tag{3}$$

where x is the plant state vector and u is the plant input vector. In order to utilize the optimal control approach and obtain a closed-loop control law, a linearized system representation is needed in the form of Eq. (4),

$$\Delta x = A\Delta x + B\Delta u \quad , \tag{4}$$

where $\Delta x = x - x_0$, $\Delta u = u - u_0$, and x_0 and u_0 comprise the operating point.

The sequential least squares process is developed[9] from a moving average model of the form

$$y(k) = \sum_{j=1}^{m} \alpha_j \phi_j (k) + \eta(k) \quad , \tag{5}$$

where $y(k)$ is a single observation of some part of the system output at the k^{th} sampling instant, α_j's are parameters to be identified, ϕ_j (k)'s are inputs to the moving average model at the k^{th} sampling instant, and $\eta(k)$ is the noise at the k^{th} sampling instant. Using vector notation, the parameters are denoted by

$$\alpha^T = \left[\alpha_1 \alpha_2 \ . \ . \ . \ \alpha_m\right] \quad , \tag{6}$$

and the inputs are denoted by

$$\phi^T(k) = \left[\phi_1(k)\phi_2(k) \ . \ . \ . \ \phi_m(k)\right] \quad . \tag{7}$$

If we denote the estimate of α based on r observations as $\hat{\alpha}$ (r), the recursive estimate of α becomes

$$\hat{\alpha}(r) = \hat{\alpha}(r - 1) + P(r)\phi(r)\left[y(r) - \phi^T(r)\hat{\alpha}(r - 1)\right] , \tag{8}$$

where P(r) is a matrix defined recursively as

$$P(r) = P(r - 1) - P(r - 1)\phi(r)\left[1 + \phi^T(r)\, P(r - 1)\phi(r)\right]^{-1}\phi^T(r)\, P(r - 1) \quad . \quad (9)$$

The problem of obtaining the initial estimates $\hat{\alpha}(0)$ and P(0) is resolved by assuming that $\hat{\alpha}(0) = 0$ and $P(0) = \sigma I$ with $\sigma >> 0$. Using these initial estimates leads to convergence toward the actual parameter values in a least squares sense.

Application of this approach involves considering a finite difference approximation of the system derivatives as the observations y, the elements of the A and B matrices as the parameter vector α, and the control inputs u and state x as the model input ϕ.

4.3 Optimal Controller Computations

The controller computation algorithm makes use of standard linear regulator/linear servomechanism theory[10,11] with set points for both the control and state variables. The form of the performance index is

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left[(x - x^*)^T Q(x - x^*) + (u - u^*)^T R(u - u^*)\right]dt \quad , \quad (10)$$

where x* and u* are set points for the state and control vectors, respectively. The controller takes the following linear feedback form

$$u = \xi - K(x - x_0) + u^* \quad . \quad (11)$$

The gain matrix K is

$$K = R^{-1} B^T P_{ss} \quad , \quad (12)$$

where R is the symmetric, positive definite control cost matrix and P_{ss} is the steady-state solution of the Riccati equation,

$$\dot{P} = -PA - A^T P + PBR^{-1} B^T P - Q \quad . \tag{13}$$

Here Q is the state variable cost matrix and is symmetric and non-negative definite. The steady-state solution P_{ss} is obtained by integrating Eq. (13) backward in time. The offset vector ξ is given by

$$\xi = K(P_{ss} A + Q)^{-1} \left[Q(x^* - x_o) - P_{ss} B(u^* - u_o) \right] \quad . \tag{14}$$

The adaptive control approach employed is to use a piecewise constant controller. A fixed adaptation interval is used and during each interval the controller structure is held constant. Note that a constant controller structure does not mean constant control. At the beginning of each adaptation interval, a linear model is identified and the controller Eq. (12), (13), and (14) are solved to obtain K and ξ. These values are then used until the beginning of the next adaptation interval.

4.4 Set Points

Since the performance index of Eq. (10) contains set points x^* and u^*, some method must be established to specify them. Some of the set points, such as the room temperature, will be specified at the outset. Others may not be so crucial insofar as comfort and energy use are concerned.

For a general linear system, let x_s and u_s denote the set points. If we assume that the set points of the system describe an equilibrium condition, then the system equation

$$\dot{x} = Ax + Bu \tag{15}$$

becomes

$$\dot{x} = 0 = Ax_s + Bu_s \quad . \tag{16}$$

Certainly, $x_s = 0$ and $u_s = 0$ define an equilibrium condition. But, for the case in which some of the set points (e.g., room temperature) are nonzero, the more general statement of Eq. (16) must be satisfied. Set point conditions may be developed in a more rigorous fashion, but the result is the same. When we take into account the linearized nature of the HVAC problem, appropriate substitutions in Eq. (16) yield

$$Ax^* + Bu^* = Ax_0 + Bu_0 \quad . \tag{17}$$

Equation (17) provides n equations (n is the order of the system) and n + r unknowns (r is the number of control variables). The procedure used is to specify r of the set points based on whatever conditions are appropriate, and solve for the n remaining set point values with Eq. (15). Note that since x_0, u_0, A, and B change, the set points must be recomputed each time the linear system and its operating point are identified.

4.5 A Practical Consideration

The use of hot water to supply energy to the HVAC system from storage and from the auxiliary heat exchanger requires hardware to combine the two sources. A common practice is to use a valve arrangement in which one supply is allowed full flow while the other is completely turned off. This arrangement is referred to here as a mutually exclusive arrangement. This technique is used in the conventional controller simulation. The optimal control approach used here does not readily lend itself to a mutually exclusive arrangement because it considers both supplies as available and establishes the optimum use from each. Thus, earlier reported simulations[2,3] considered a mixing valve arrangement where a continuum of flow rate values could be obtained from both sources simultaneously. Besides being difficult to implement in hardware, this mixing arrangement makes it possible to use auxiliary energy to heat the storage tank. There may be situations where this is desirable (e.g., during off-peak hours), but it is generally considered undesirable.

To provide a mutually exclusive arrangement for the adaptive optimal controller in the heating simulation, the derivative of the room temperature is examined at each adaptation time. This derivative accounts for all the variables affecting the system. The room temperature is projected during the interval until the next adaptation time using a straight-line approximation. If the projected room temperature stops above some comfortable value, the elements in the B matrix that couple auxiliary energy into the system are set to zero. This forces the algorithm to consider only the storage tank as an energy supply and compute controls accordingly. If the projected room temperature falls below the comfortable value, the elements in the B matrix that couple the storage tank to the system are set to zero and the algorithm considers the auxiliary heat exchanger as the only energy supply and computes the controls accordingly. This strategy is used because the storage tank is less capable of heating the building under extreme conditions.

5. SIMULATION AND RESULTS

Operation of an AOC controller has been simulated on a digital computer and compared with operation of a conventional controller, also simulated on a digital computer. The simulations have shown that the AOC can achieve substantial auxiliary energy savings when compared to the conventional controller while regulating the room temperature quite well.

5.1 Simulation Model

The NSRSC at LASL provides the basis for a general model used in this simulation. The NSRSC is a 59,000 ft^2 library and conference facility. It has both solar heating and cooling systems, but only the heating system is discussed here.

A simplified model of the solar heating system for the building is used. A schematic for the system is shown in Fig. 6. The state variables and control variables for the solar heating system are defined in Table I. The system model contains a solar collector, a collector coolant loop, and a one-node storage tank.

Auxiliary energy is provided by a steam-to-water heat exchanger. The steam is supplied by a central steam plant. The hot water flows through coils in the air duct to heat the air flowing over them. The heated space in the building is represented by a single enclosure. The air handling system provides air movement and a capability for recirculation of some of the partially conditioned air and introduces fresh air as appropriate.

5.2 Simulation Results

Heating simulation results obtained using the AOC technique described above are presented in Tables II and III. The simulation runs are for a 48-h period. The "conventional controller" simulation uses the control strategies in the NSRSC with a mutually exclusive arrangement for the two energy sources.

The "AOC" simulation uses the AOC technique with the sequential least square identification procedure. The two energy sources are connected by a mixing arrangement. The "AOC with set point computations" simulation uses the AOC technique with the sequential least squares identification procedure and the procedure for computing the set points. The two energy sources are again connected by a mixing arrangement. The "AOC with mutually exclusive arrangement" simulation uses the AOC technique with the strategy for forcing the algorithm to select one energy source or the other (but not both). The set point computation process is not used here. Table II contains performance data. All the control systems simulated maintain the room temperature within a comfortable range. However, all the AOC technique simulations do so with a significantly smaller amount of auxiliary energy. Table III shows the overall energy usage for each simulation. The total energy used in each simulation is about the same, but the amount of auxiliary energy used by the AOC controllers is significantly reduced. The amount of solar energy collected and placed in the storage tank is significantly increased. This increase in the amount of solar energy collected is attributed to the AOC controllers' operation of the storage tank at a lower temperature, as shown in Fig. 7, to increase the efficiency of the collection of solar energy. The decrease in building shell losses is attributed to the AOC controllers' operation of the building at a slightly lower average temperature.

The data in Table III indicate that heating a building requires a relatively fixed amount of energy. The AOC controllers are able to use the multiple sources of energy and the control points in the HVAC system to operate the building so as to maintain comfort while minimizing the amount of auxiliary energy needed. Emphasis on improved management of energy resources by the AOC controllers is indeed appropriate.

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6. COMPUTATIONAL ASPECTS

The AOC algorithm described here is a general technique that has seen application in other fields such as aerospace flight control systems and supertanker autopilots. Conceptually, the AOC algorithm could be implemented with any computational means available -- even by hand. Computers, however, make implementation of this very sophisticated technique feasible and economically attractive when microprocessors are considered.

The hardware needed to implement the AOC may be considered in three categories.

1. Sensors and actuators
2. Satellite controllers
3. Algorithm processor

The role of each component is shown in Fig. 8.

6.1 Sensors and Actuators

Accurate sensing of system conditions is needed for the system identification process to determine the model and current set of operating conditions. It is important to note that fewer sensors may be needed with the AOC controller than with a conventional controller. This is because the AOC approach bases its model on a state-variable representation, any information needed may be determined from the state, the controls, and the external inputs such as ambient temperature. To illustrate, fan discharge temperature is often measured in a conventional control system. In the simulation results reported earlier, fan discharge temperature does not explicitly enter into the control determination, although its effect is implicity in the state equations.

The actuators must be capable of effecting the control when commanded by digital signals. Electrically controlled valves, dampers, etc. could be used, or electrical-pneumatic transducers could be used with a pneumatic system.

The sensors and actuators are not conceptually different from those found in conventional control systems. However, their placement and use in the HVAC system may be different.

6.2 Satellite Controllers

The satellite controllers receive control parameters from the algorithm processor and implement the control equation. Specifically, the control gains and offsets would be transmitted. Information from the sensors is available and the satellite controllers put into effect the control of Eq. (2).

The satellite controllers are capable of operating without intervention from the algorithm processor. Once the control parameters have been received, they are used until they are updated upon completion of an adaptation interval. This pseudo-independent operation of the satellite controllers allows the HVAC system to continue operation if the algorithm processor should go down. The satellite controllers must have nonvolatile memory in order to provide maximum control capability in the face of control interruption.

6.3 Algorithm Processor

The algorithm processor does what its name implies, it processes the algorithms to provide the gains and offsets to the satellite controllers. It samples the sensor data periodically and computes the system model using the systems identification techniques. When a model is ready (at the conclusion of a fixed interval) it performs the optimal control computations and transmits the data to the satellite controllers. It then begins another interval of sampling data.

6.4 Some Specifics

The discussion about the AOC implementation has been, thus far, centered on functions requirements. Some comments about hardware specifics are in order. Depending on the actuators used, digital-to-analog converters will be needed. They could be implicit in the actuator, built in as a part of the actuator module, or be a part of the satellite controller. The satellite controllers could be made up of integrated circuit logic or a relatively simple microprocessor. A modest amount of memory will be needed to store the control parameters. A very limited amount of numerical processing needs to be done, so the speed and functions complexity required do not generate strenuous demands.

The algorithm processor is primarily concerned with numerical processing, so the demands it generates are somewhat more strenuous. A minicomputer is believed capable of doing the processing in the time necessary, and memory poses the only serious requirement. Precise estimates of memory requirements are not available, but 128,000 words is likely more than enough. This level of computing power can be accomplished with microprocessors and recently announced devices make use of a microprocessor even more attractive.

7. CONCLUSIONS

It has been demonstrated by simulation that significant savings in auxiliary energy (as much as 51%) may be achieved in a solar heated building through the use of adaptive optimal control. The adaptive optimal control algorithm is well suited for implementation using state-of-the-art hardware.

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TABLE I

STATE AND CONTROL VARIABLES FOR THE
SOLAR HEATING SYSTEM MODEL

State Variables

T_r	- room temperature
T_s	- storage tank temperature
T_c	- collector surface temperature
T_{cc}	- collector coolant temperature

Control Variables

W_c	- collector coolant mass flow rate
W_{hws}	- storage hot water mass flow rate
W_{hwx}	- auxiliary hot water mass flow rate
T_{aux}	- auxiliary hot water temperature
W_s	- air mass flow rate
X_r	- air recirculation fraction

TABLE II

HEATING SYSTEM PERFORMANCE

System/ Solution Technique	Average Room Temperature (°F)	Room Temperature Extremes (°F)	Auxiliary Energy Used (10^6 Btu)	% Savings Compared to Conventional Controller
Conventional controller	70.0	69.8 70.4	6.75	----
Adaptive optimal controller (AOC)	69.7	68.4 70.2	4.13	38.8
AOC with set point computations	69.6	67.7 70.5	3.91	42.0
AOC with mutually exclusive arrangement	69.7	67.4 70.5	3.28	51.3

TABLE III

ENERGY TERMS FOR HEATING SYSTEM SIMULATIONS

System	Solar Energy Collected (% inc.)	Internal Heat Generation	Auxiliary Energy Used (% dec.)	Total Energy Used	Building Shell Losses (% dec.)	Building Ventilation Losses (% dec.)
Conventional controller	3.14	12.23	6.75	22.11	10.82	11.30
Adaptive optimal controller (AOC)	5.50 (75.3)	12.23	4.13 (38.8)	21.86	10.75 (0.62)	11.10 (1.77)
AOC with set point computations	5.70 (81.8)	12.23	3.91 (42.0)	21.84	10.73 (0.80)	11.10 (1.76)
AOC with mutually exclusive arrangement	6.37 (103.2)	12.23	3.28 (51.3)	21.89	10.86 (0.52)	11.11 (1.63)

Note: All energies are in 10^6 Btu.

FIGURE CAPTIONS

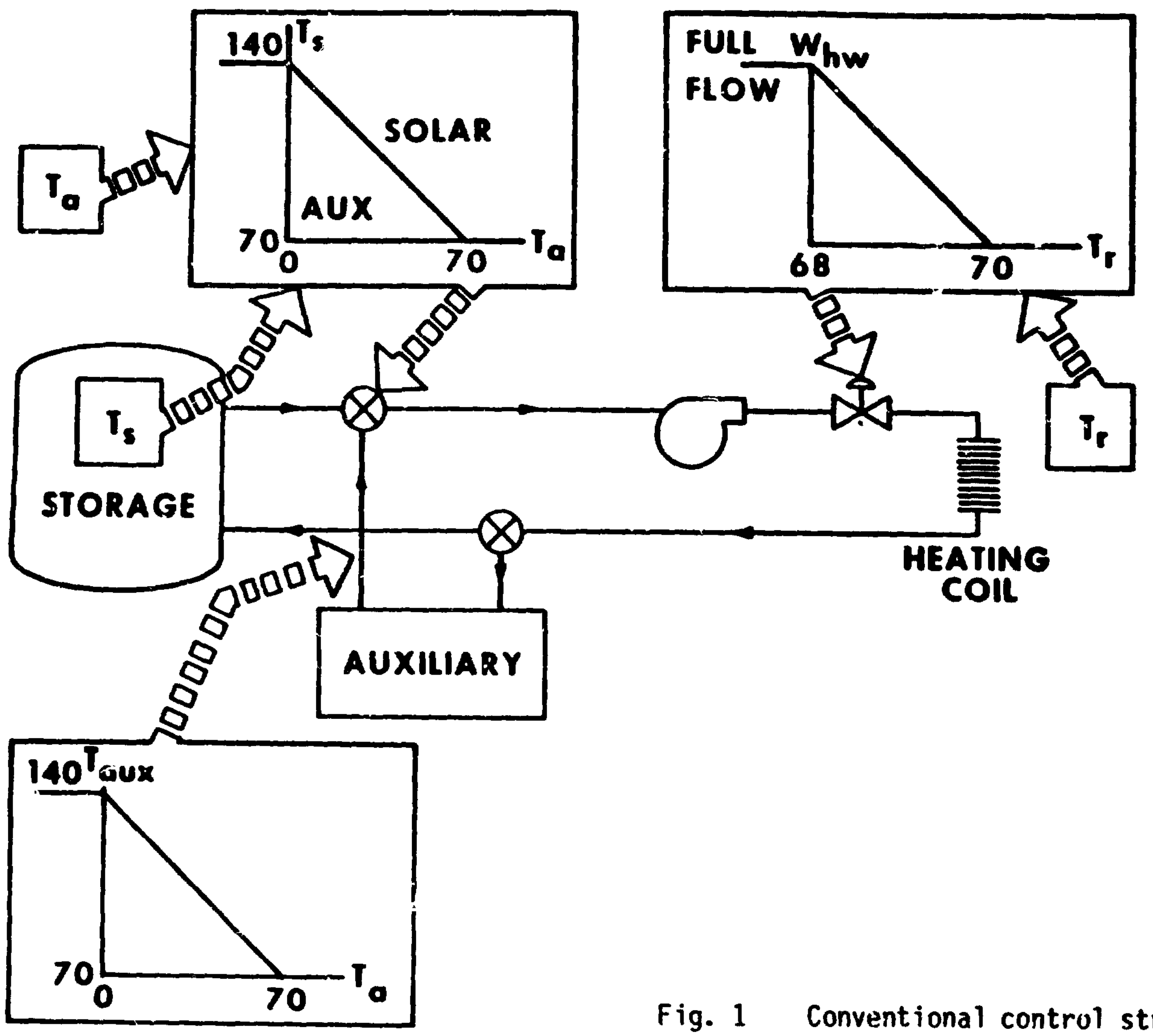

Fig. 1 Conventional control strategy.

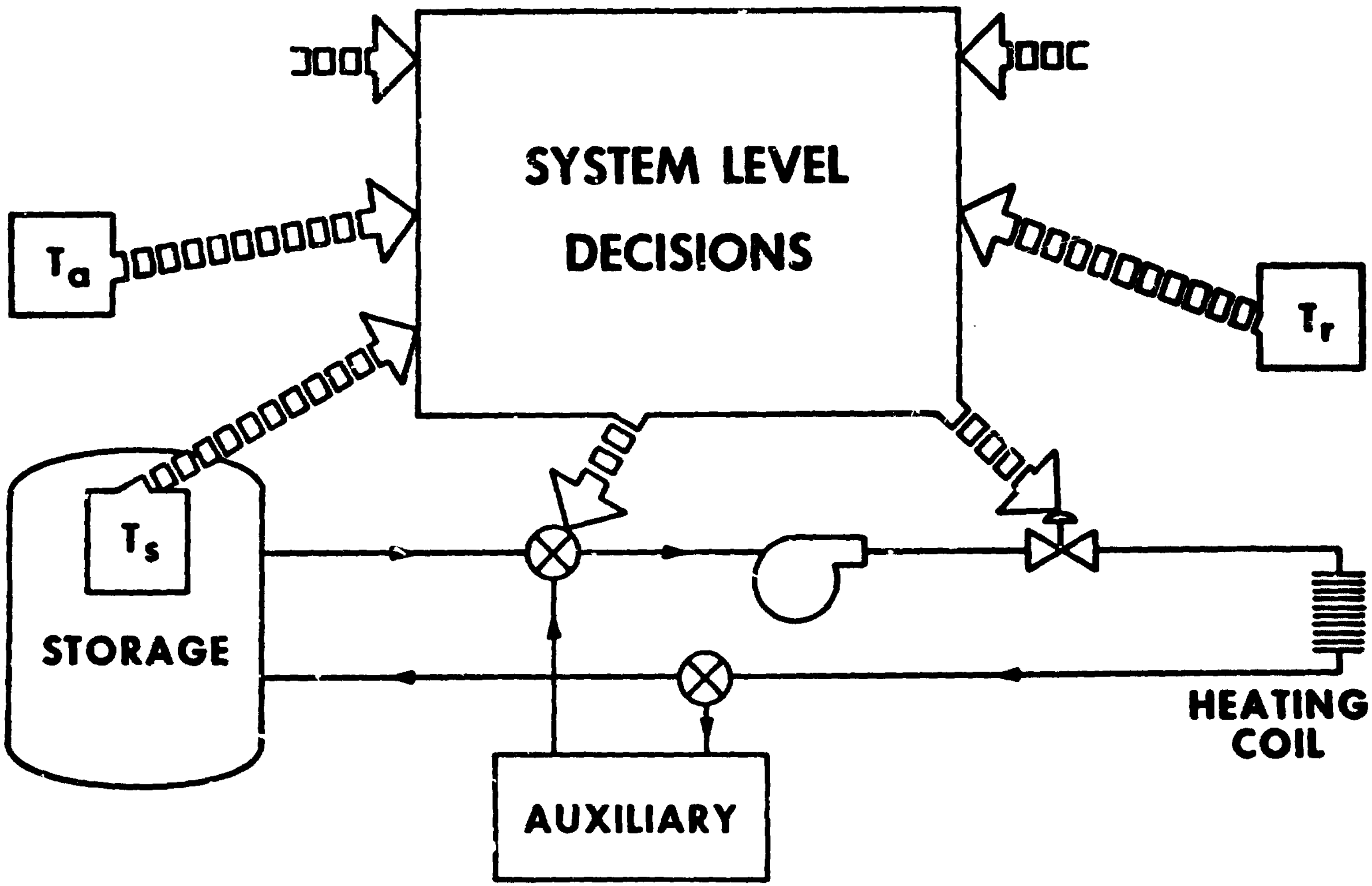

Fig. 2 ACC strategy.

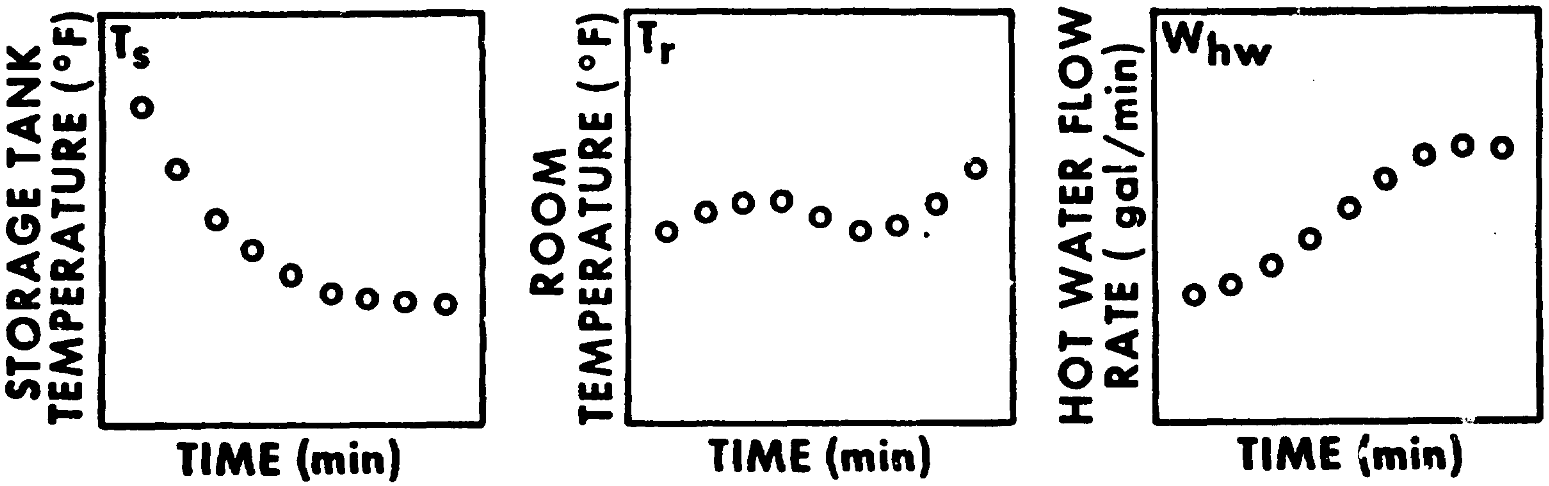

Fig. 3 System identification.

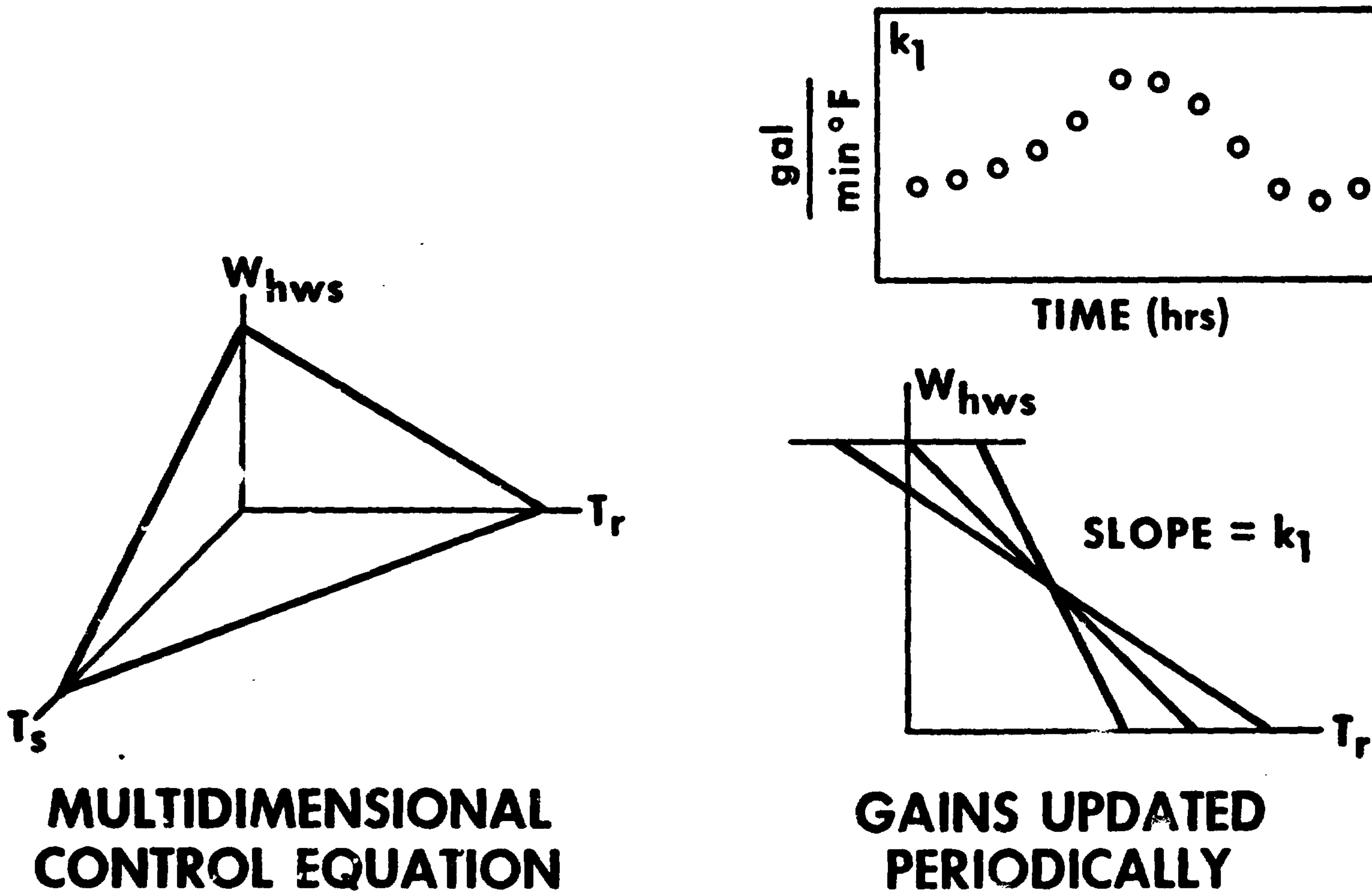

Fig. 4   Control equation features.

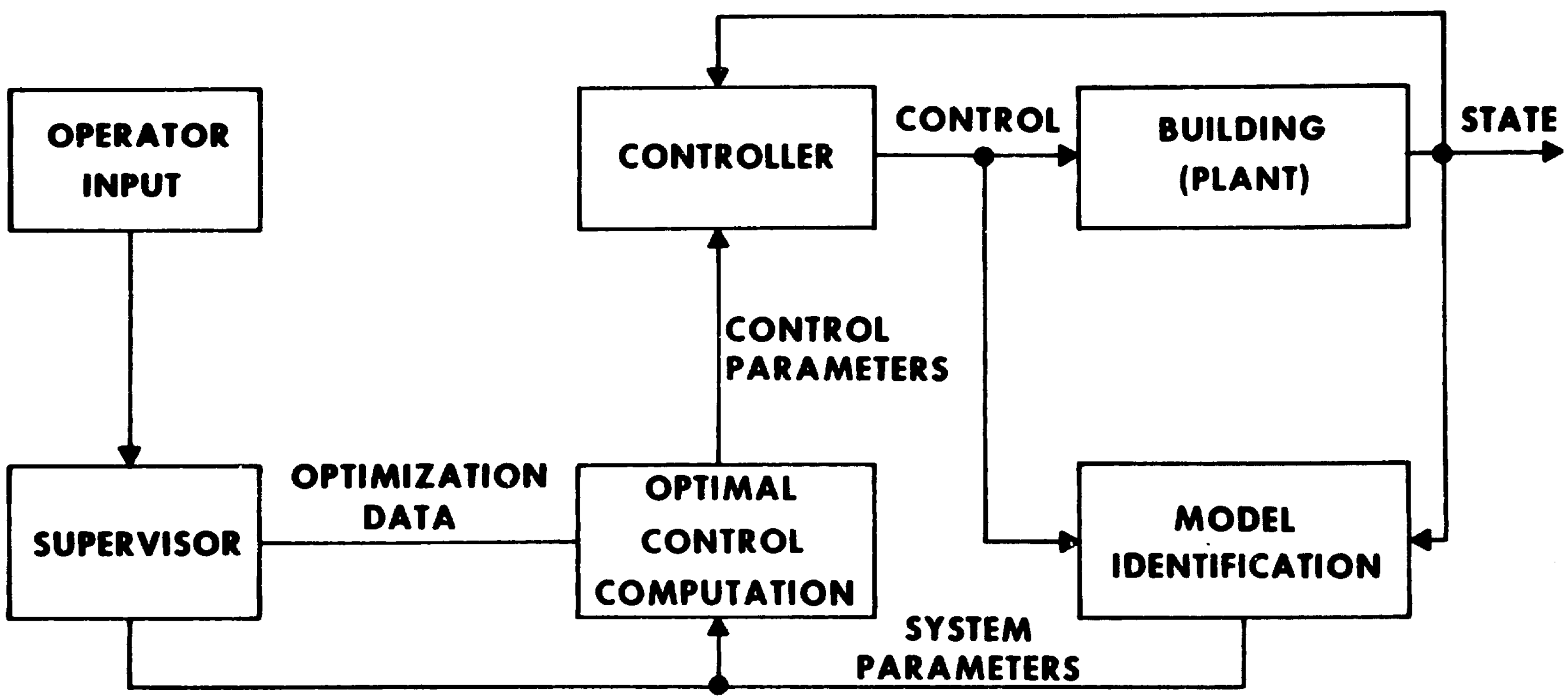

Fig. 5 System block diagram.

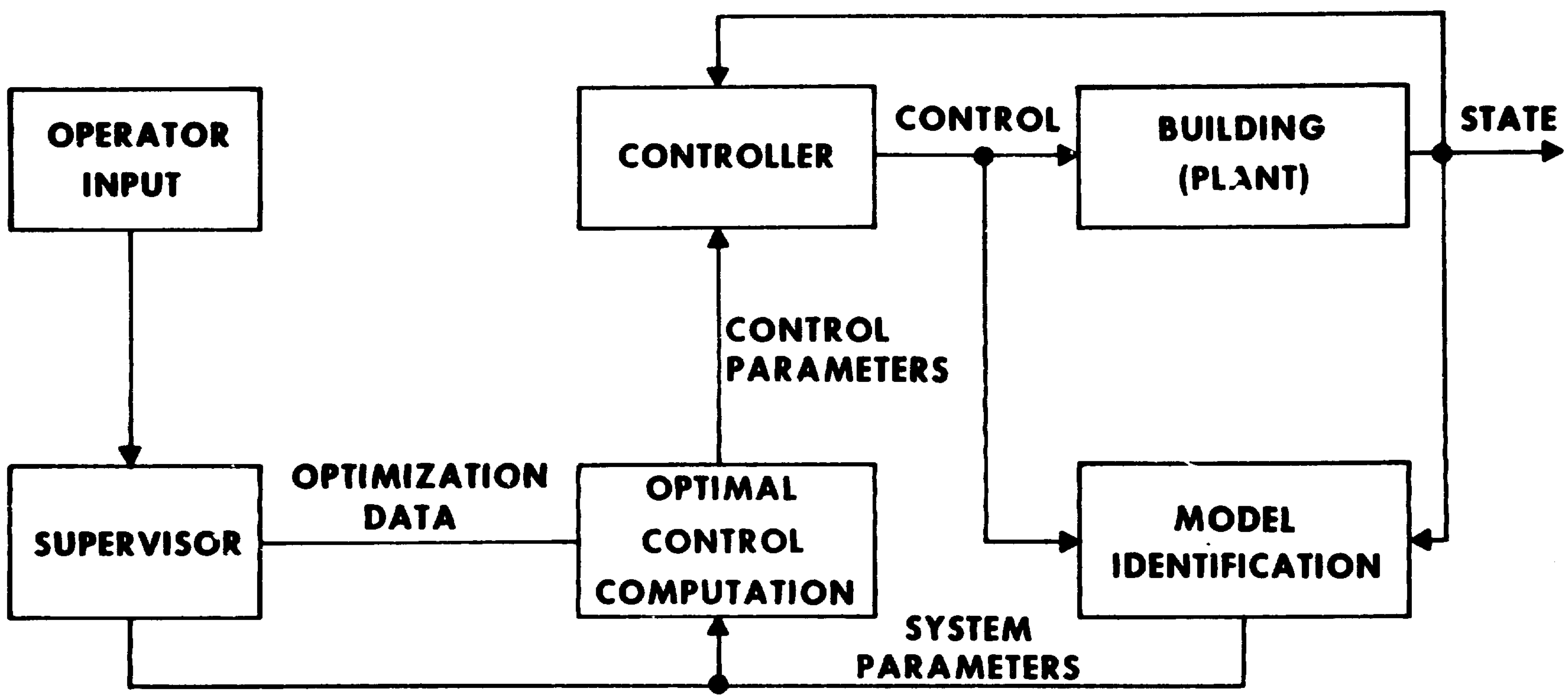

Fig. 5 System block diagram.

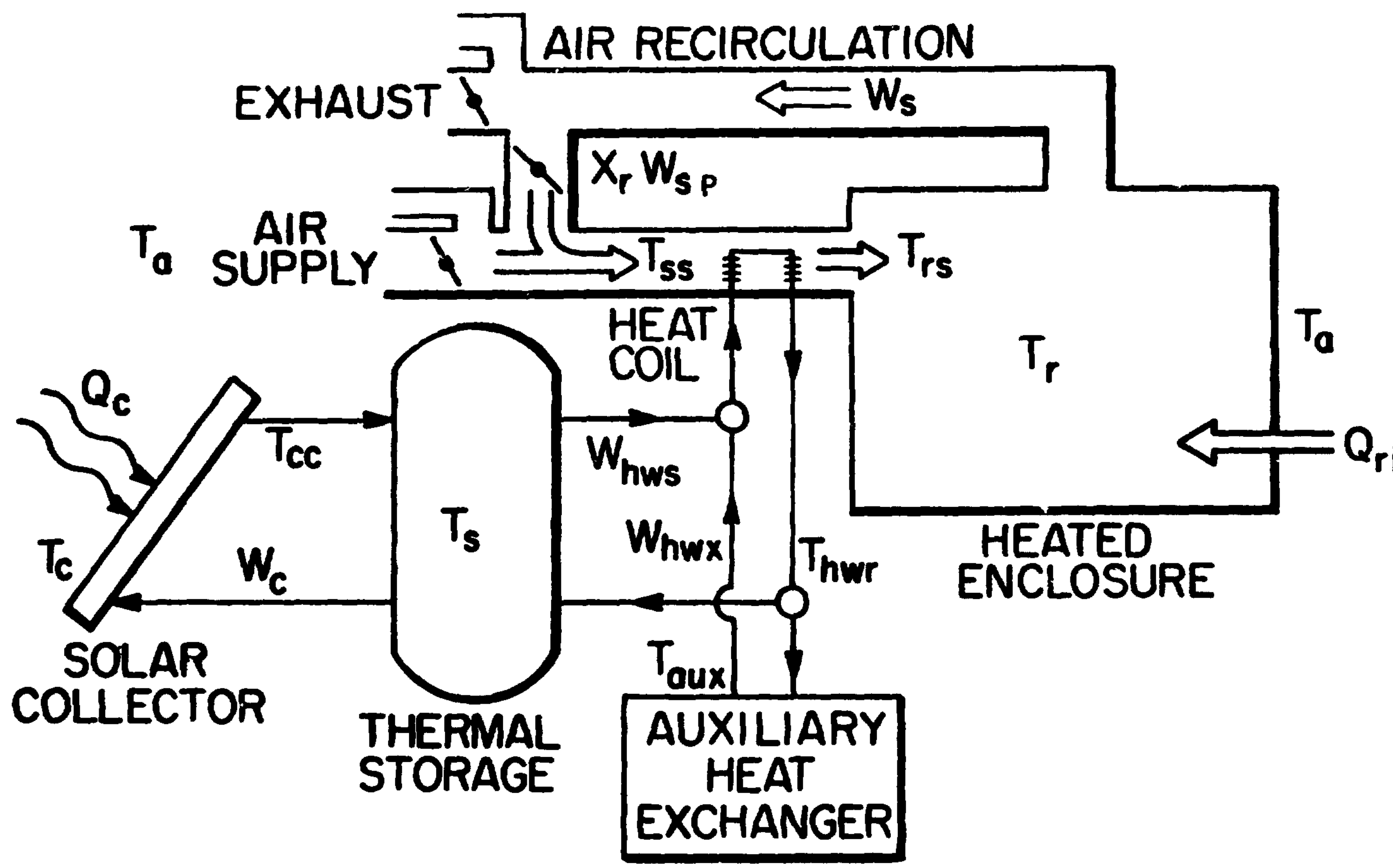

Fig. 6 Solar heating system model.

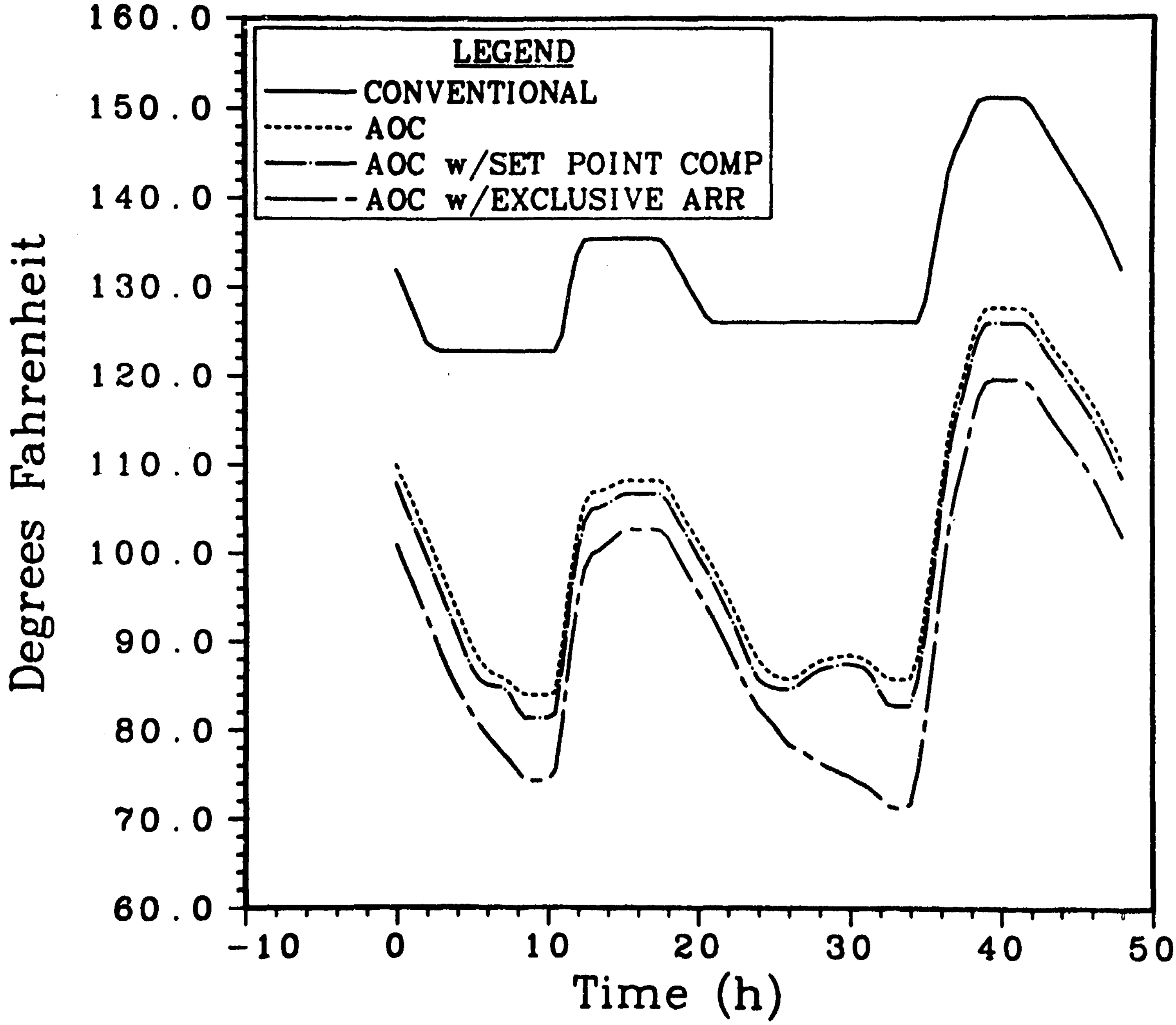

Fig. 7 Storage tank temperatures.

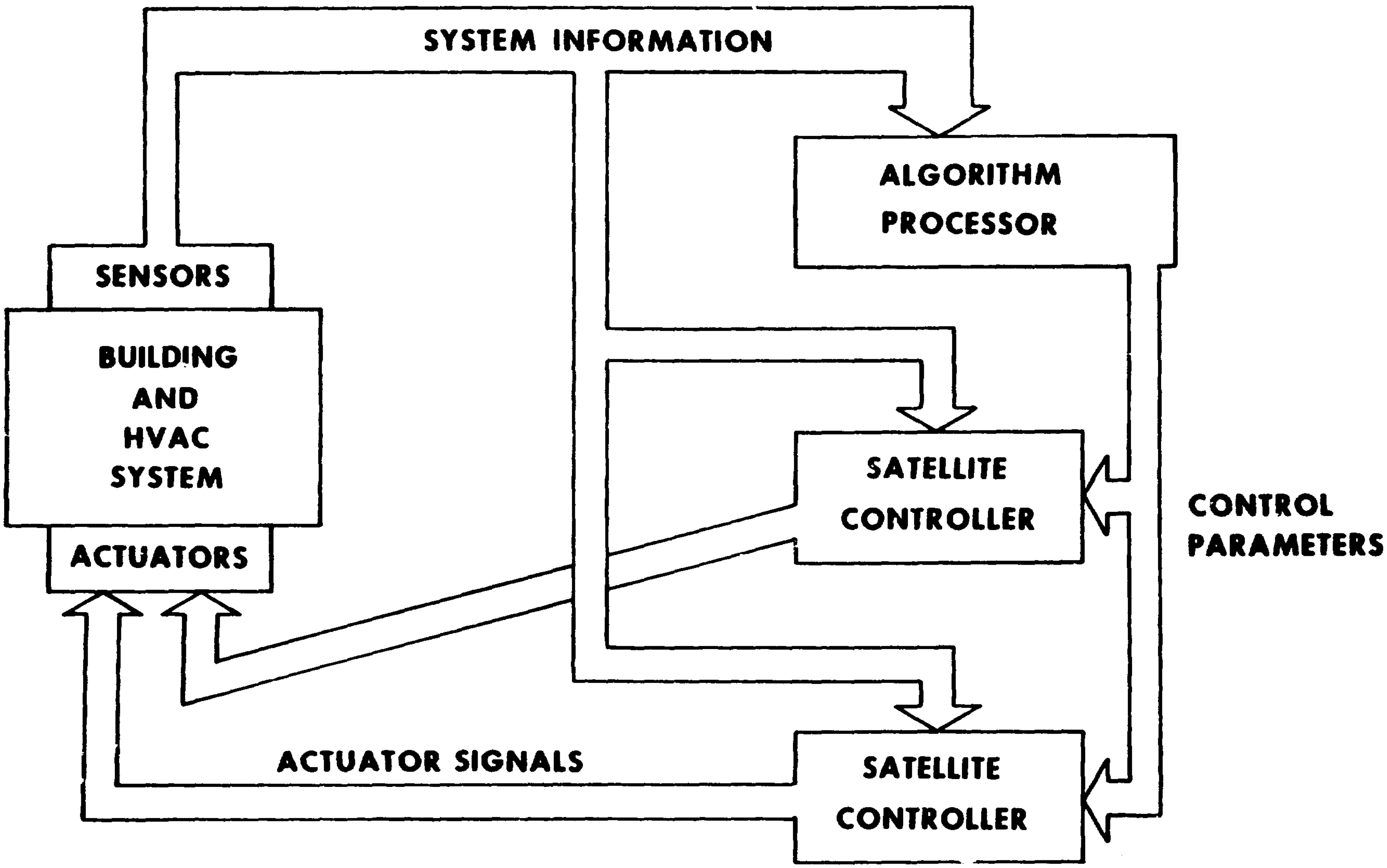

Fig. 5 AOC implementation.