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POSSIBLE DEPARTURES FROM LOCAL ISOTROPY IN THE SMALL SCALE STRUCTURE OF TURBULENT VELOCITY FIELDS

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ABSTRACT

Two weakened similarity hypotheses are proposed as possible replacements for the two Kolmogorov similarity hypotheses as descriptions of laboratory turbulence. It is shown that these hypotheses yield consequences which are in full agreement with the whole range of laboratory, atmospheric, and oceanic measurements bearing on the question of the existence of local isotropy in turbulent flows. However, the weakened hypotheses are fully consistent with the non-existence of local isotropy. It is concluded that a new class of measurements - the measurement of cross-stream variations in the velocity correlations - would be required to establish the existence or non-existence of local isotropy. One consequence of the proposed weakened similarity hypotheses is that the Kolmogorov constant C is not a constant, but is, instead, a flow-dependent quantity, depending on velocity gradients. This is in at least qualitative agreement with the large scatter in values of C reported in the literature.

NOMENCLATURE

$\hat{a}, \hat{a}_i$ = unit vector specified by velocity gradients in mean flow

A_0, A_1, A_2 = proportionality "constants" with weak functional dependences on velocity gradients in mean flow

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$B_o\left(\frac{k^2}{k_\eta^2}\right)$	= general positive function of wavenumber with weak dependence on velocity gradients
$B_{ij}(\underset{\sim}{r})$	= two point velocity correlation
C	= Kolmogorov constant, here taken to depend on velocity gradients
C_{ij}, C_1	= proportionality "constant" for the one-dimensional cross spectra and the spectrum, respectively, depending on velocity gradients in the mean flow
C_2, C_3	= proportionality "constants in non-isotropic part of structure functions, dependent on velocity gradients
$D_{ij}(r), D^o_{ij}, D^1_{ij}$	= the tensor of structure functions, its isotropic component and its non-isotropic component, respectively
$D_{LL}(X)$, $D_{NN}(X)$	= longitudinal and lateral structure functions, respectively
$E(k)$	= the spectrum
$F_{ij}(\underset{\sim}{k}), F_{ij}(k_1)$	= three-dimensional and one-dimensional cross spectra, respectively
$f_{ij}\left(\frac{k_1}{k}, \frac{k_2}{k}, \frac{k_3}{k}\right)$	= dimensionless three-dimensional cross spectra of Hypothesis A
$G_{ij}\left(\frac{k_1}{k_\eta}, \frac{k_2}{k_\eta}, \frac{k_3}{k_\eta}\right)$	= dimensionless three dimensional cross spectra of Hypothesis B
$H\left(\frac{k_i}{k}\right)$	= function of wavenumber component
$I(\theta) = \{\theta : 0 < \theta < \pi/2$ if $k_1 > 0$, $\pi/2 < \theta < \pi$ if $k_1 < 0\}$	= integration domain for polar angle theta

$K(x)$	= function of proportionality in non-isotropic part of structure function
$\underset{\sim}{k}, k_i$	= wavenumber vector
k	= magnitude of wavenumber vector
$k_\eta = (\varepsilon/\nu^3)^{1/4}$	= Kolmogorov number
ℓ	= integration variable
$\underset{\sim}{r}$	= position vector denoting separation of two points in the two-point functions
$\hat{r}, \hat{r}_i$	= unit vectors along $\underset{\sim}{r}$
r	= magnitude of position vector
$\underset{\sim}{x}$	= general position vector
x_1	= local direction of mean flow
$\varkappa$	= general argument of a function of one variable
$u_i(\underset{\sim}{x}, t)$	= mean velocity vector
$v_\eta = (\varepsilon\nu)^{1/4}$	= Kolmogorov velocity
δ_{ij}	= Kronecker delta function
$\delta(x)$	= Dirac delta function
$\frac{\partial}{\partial x^i}$	= spatial gradient operator
ε	= local mean energy dissipation
$\Gamma(1/3)$	= Gamma function of argument 1/3
η	= Kolmogorov length scale
ϕ	= azimuthal angle of polar coordinates in wavenumber space
ν	= kinematic viscosity
θ	= polar angle relative to $\hat{r}$ in wavenumber space

INTRODUCTION

It is ofen presumed that local isotropy of the small scale components of turbulent velocity fields has been demonstrated for several laboratory flows, for atmospheric boundary layers and for oceanic turbulence. There is even a large body of data which is consistent with local isotropy. Thus, the comparatively few and scattered observations which conflict with predictions based on local isotropy are regarded as insufficiently decisive to shake the consensus that local isotropy exists 1. It is the purpose of the present paper to point out that the large number of observations consistent with local isotropy do not logically suffice to establish that local isotropy holds. The many observations all verify limited features of the flow field. In particular, the almost exclusive use of the time dependent signal of a single hot wire anemometer, together with the use of Taylor's "frozen turbulence" hypothesis to interpret the signal as the streamwise spatial dependence of a velocity component, means that the data systematically lacks the cross-stream derivatives of velocity components which are needed to fully establish local isotropy. The crossed wire data permits simultaneous measurement of two velocity components, but, again, does not yield cross-stream derivatives.

Even the numerous data establishing the $k^{-5/3}$ power law for velocity spectra in the atmospheric boundary layer are based only on streamwise spatial structure of the velocity correlations, and, thus, do not fully establish Kolmogorov's second similarity hypothesis 1,2 even though the measurements are certainly consistent with this hypothesis. We will put forward a weakened form of similarity hypothesis, which permits much more general

forms of "universal equilibria" than does the Kolmogorov hypothesis, violates local isotropy, and yet agrees with the experimental data that is used to support the local isotropy hypothesis. We put forward theoretical arguments which suggest that one should at least consider the possibility that local isotropy is violated. Naturally, the present hypothesis, permitting more complex equilibria, cannot have the esthetic appeal of the simpler Kolmogorov hypothesis. But since the more complex equilibria have consequences for turbulence modeling, and they cannot be ruled out by present evidence, it seems most reasonable to decide by conclusive measurements whether local isotropy holds in the inertial range. We mention several measurements that could provide additional evidence on this point.

The theoretical arguments suggesting that local isotropy holds in the dissipation range seem fairly strong, yet it isn't clear that they are necessarily valid. Again, we are able to display an example of a two-point velocity correlation tensor which agrees with the experimental data in the dissipation range porting local isotropy, yet definitely violates local isotropy in the dissipation range. Thus, it is at least logically possible that local isotropy fails to even in the dissipation range; present measurements do not rule out this possibility. Again, the question of whether local isotropy holds is best settled conclusive measurements.

WEAK FORM OF SIMILARITY HYPOTHESES

We formulate two weakened similarity hypotheses for incompressible turbulent flows in terms of the mathematical properties of the two-point velocity relation tensor

$$B_{ij}(\underset{\sim}{r}) = \langle u_i(\underset{\sim}{x},t)u_j(\underset{\sim}{x} + r,t)\rangle = \int d^3k\, F_{ij}(\underset{\sim}{k})e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}} \quad , \tag{1}$$

where the $\underset{\sim}{x}$ and t dependences of B_{ij} have been ignored as usual for the large wavenumber (small r) part of the spectrum.

Hypothesis A

For sufficiently large Reynolds numbers an inertial range exists in the $\underset{\sim}{k}$ space of F_{ij}. When this occurs F_{ij} is determined by the mean energy dissipation rate ε, the mean velocity field and the geometry of the flow field according to the rule

$$F_{ij}(\underset{\sim}{k}) = \varepsilon^{2/3} k^{-11/3} f_{ij}\left(\frac{k_1}{k}, \frac{k_2}{k}, \frac{k_3}{k}\right) \quad . \tag{2}$$

Here f does not depend explicitly on ν but may depend locally, or, perhaps functionally on the mean velocity field and on the flow geometry. The $\underset{\sim}{x}$ and t dependence of F_{ij} are contained implicitly in ε and f.

This is related to Kolmogorov's second similarity hypothesis 1,2, but it assumes only what is necessary to achieve agreement with the data presently supporting the Kolmogorov hypothesis. According to the Kolmogorov hypothesis

$$f_{ij} = \frac{55C}{108\pi\Gamma(1/3)} \left(\delta_{ij} - \frac{k_i}{k}\frac{k_j}{k}\right) \quad , \tag{3}$$

where C, the constant of the longitudinal structure function, is a pure number independent of the particular high Reynolds number turbulent flow. This is one definite function in the infinite class of functions permitted by Hypothesis A.

The principal theoretical reason for thinking that the asymmetries permitted by Eq. (2) might be needed in the inertial range lines in the possibility that some significant fraction of the small scale structure of a turbulent velocity field may be produced by a few discrete steps of nonlinear evolution of instability, further convective distortion, and breakup, rather than via nearly infinite cascading process involving adjacent Fourier components. The arguments that the production process necessarily leads to isotropy via multiple interactions of many degrees of freedom have less cogency in this case. However, even in the case of a cascade proceeding roughly along the lines originally envisioned, it is merely plausible, but not entirely certain, that all phase relations and anisotropic gradients would be lost during the cascade process.

The suggestion of several discrete steps for production of fine scale structure remains a theoretical possibility rather than a well validated scientific hypothesis, but some very impressive recent work 3 is not inconsistent with this suggestion. Corrsin and Kollman 3 give a numerical simulation of a homogeneous shear layer, to which Taylor-Green vortices 4 are added as a turbulence producing finite perturbation. They find that a particular dynamic structure - a stagnation point with the "right" direction of energy transfer - becomes instable and produces a fairly localized disturbance. One may further show 5 that the stagnation point will itself be a source of vorticity amplifications via the localized and highly directional vortex stretching mechanism.

Apparently the first, and only, attempt to account for the fine scale features of a turbulent velocity field, including the inertial range, by

means of a two-step process has been provided by Saffman 6, who has also questioned the existence of small scale isotropy in turbulent flows 7. He is able to achieve agreement with most qualitative features of the fine scale structure by means of a "primary cascade," in which a concentration of vorticity is stretched out into a vortex sheet in the presence of the straining field of a general velocity field, and a "secondary cascade," in which the curved vortex sheet becomes unstable to the formation of Taylor-Gortler vortices 8,9. Of course, this sequence of ideas is speculative and heuristic until the whole theory is developed in much greater detail. Again, it may prove possible to develop a theory of formation of fine scale structure via several discrete steps along rather different lines. Nevertheless, the fact that there is a theoretical possibility of accounting for the fine scale structure of a turbulent velocity field via a production process of several steps seems to be an adequate motivation for formulating the similarity hypotheses from the point of view of clarifying what minimal assumptions need be made to account for the experimental evidence, as done in the present paper, rather than from the point of view of making the greatest simplifications which do not contradict experimental evidence, as done heretofore.

Next, we present a weakened form of Kolmogorov's first similarity hypothesis 1,2, applicable to both the inertial and dissipation ranges. We introduce the Kolmogorov wave number k_η and velocity v_η and formulate

Hypothesis B

For sufficiently large Reynolds number the high wavenumber portion of the spectrum of F_{ij} is determined by ε, ν, the mean velocity field and the flow geometry according to the rule

$$F_{ij}(\underset{\sim}{k}) = k_\eta^{-3}\, v_\eta^{2}\, G_{ij}\left(\frac{k_1}{k_\eta}, \frac{k_2}{k_\eta}, \frac{k_3}{k_\eta}\right) \quad . \tag{4}$$

Here G_{ij} may depend locally or, perhaps, functionally on the mean velocity field, and the flow geometry, but not explicitly on the gradient lengths of the mean flow field. The $\underset{\sim}{x}$ and t dependence of F_{ij} are contained implicitly in ε and G_{ij}.

Of course, the violation of local isotropy seems less likely intuitively in the dissipation range than in the inertial range, but it cannot be excluded by the present evidence. The rough reduction of the data on one-dimensional energy spectra to a universal curve in the dissipation range suggests that the full dependence of F_{ij} on the flow field permitted by Eq. (4) may be excluded by experiment. However, it should be pointed out that the measured values of ε always <u>assume</u> locally isotropy. Corrected values, allowing for measured rather than assumed cross-stream velocity derivatives, could lead to less scatter in the universal curve. Again, Hypothesis B is all that is necessary to achieve agreement with all the existing data, including dissipation range data, which is used to support the hypothesis of local isotropy.

EXPERIMENTAL CONSEQUENCES OF HYPOTHESIS A

We discuss here the possible experimental consequences should Hypothesis A, rather than Kolmogorov's second similarity hypothesis, happen to be valid in the inertial range. We consider a point in a turbulent fluid and orient the coordinate system so that the mean flow is locally in the x_1 direction. We discuss the general implications of Eq. (2) as well as the specific implications of two model Fourier transformed spectral tensors

$$f_{ij}\left(\frac{k_1}{k}, \frac{k_2}{k}, \frac{k_3}{k}\right) = \left(A_0 + A_1 k^{-2}(\underset{\sim}{k}\cdot\hat{\underset{\sim}{a}})^2\right)\left(\delta_{ij} - \frac{k_i}{k}\frac{k_j}{k}\right) , \tag{5}$$

where $\hat{a}$ is a unit vector related to the flow field, A_0 and A_1 are constants determined by the mean velocity field, and secondly,

$$f_{ij}\left(\frac{k_1}{k}, \frac{k_2}{k}, \frac{k_3}{k}\right) = B_0\left(\delta_{ij} - \frac{k_i}{k}\frac{k_j}{k}\right) + \left\{\delta\left(\frac{k_1}{k}\right)\delta\left(\frac{k_2}{k}\right)H\left(\frac{k_3}{k}\right)[\delta_{i1}\delta_{j2} + \delta_{j1}\delta_{i2}] + \delta\left(\frac{k_1}{k}\right)\delta\left(\frac{k_3}{k}\right)H\left(\frac{k_2}{k}\right)[\delta_{i1}\delta_{j3} + \delta_{j1}\delta_{i3}]\right\} \tag{6}$$

where H is a general smooth function decreasing at infinity. The tensors are chosen to illustrate the types of features that may be expected to result from additional measurements, should Hypothesis A be correct.

First, we note that all solutions of Eq. (1) satisfy the observed $k^{-5/3}$ law. Thus Hypothesis A obtains full agreement with the extensive sets of primarily atmospheric and oceanic hot wire measurements which establish the existence of the $k^{-5/3}$ law for one-dimensional longitudinal velocity spectra at high Reynolds number. These measurements are discussed in detail by Monin and Yaglom 1. Here, we merely cite the measurements of Grant, Stewart and Moilliet 10, Pond, Stewart and Burling 11, and of Sandborn and Marshall 12 as instances of measured one-dimensional longitudinal velocity spectra which are in reasonable agreement with each other and with the existence of $k^{-5/3}$ law.

One may immediately verify that the general three-dimensional cross spectrum F_{ij} of Hypothesis A will yield the $k^{-5/3}$ law for the spectrum and for any of the measureable one-dimensional cross spectra. For an arbitrary

one-dimensional cross spectrum, orient the axes so that the $\hat{k}_1$, axis becomes the unintegrated direction and introduce spherical polar coordinates relative to this axis. Then each one-dimensional cross spectrum takes the form

$$F_{ij}(k_1) = \varepsilon^{2/3} \, |k_1|^{-5/3} \, C_{ij} \quad , \tag{7}$$

where

$$C_{ij} = \int_{I(\theta)} d\theta \, \sin\theta \int_0^{2\pi} d\phi \, |\cos\theta|^{2/3} \, f_{ij}(\cos\theta, \, \sin\theta \, \cos\phi, \, \sin\theta \, \sin\phi). \tag{8}$$

Similarly, the spectrum takes the form

$$E(k) = \varepsilon^{2/3} \, k^{-5/3} \, C_1 \tag{9}$$

where

$$C_1 = 1/2 \int_0^{\pi} d\theta \, \sin\theta \int_0^{2\pi} d\phi \, f_{ij}(\cos\theta, \, \sin\theta \, \cos\phi, \, \sin\theta \, \sin\phi) \quad . \tag{10}$$

Clearly C_{ij} depends on the particular function f_{ij} chosen in Hypothesis A and also on the direction chosen for the axis of the one-dimensional spectra, while C_1, which is related to the Kolmogorov "constant" C, depends only on the particular function chosen for f_{ij}.

Of course, the most general function f_{ij} of Hypothesis A would not satisfy the kinematic constraint

$$k_i F_{ij}(\underset{\sim}{k}) = 0 \quad , \tag{11}$$

which is demanded by the equation of continuity and is consistent with the (one-dimensional) spectra that have been measured in turbulent flows.

Clearly, we must choose f_{ij} to be consistent with Eq. (11). We must also restrict f_{ij} to yield a C_1 which is consistent with the measured range of the Kolmogorov "constant" C, namely, within roughly 50% of 2.0, as discussed in Ref. 1.

The principal new content of Hypothesis A in the inertial range is that the Kolmogorov "constant" C of the longitudinal structure function, and the corresponding constants of the other measureable one-dimensional spectra, are not constants, but functions of the velocity gradients in the flow field. Doubts have occasionally been expressed 6,7 about the universality of the Kolmogorov "constant." Here this variability follows as a direct consequence of our similarity hypothesis and is illustrated by reference to the model spectra tensors of Eqs. (5) and (6). We note that the longitudinal structure function

$$D_{LL}(r) = C\,\varepsilon^{2/3}\,r^{2/3} \tag{12}$$

generated by these spectral functions gives the Kolmogorov "constant"

$$C = \frac{108\pi}{17\cdot 11\cdot 5}\,\Gamma(1/3)\,\left\{17A_0 + [12 - 13(\hat{r}\cdot\hat{a})^2]A_1\right\} \quad , \tag{13}$$

and

$$C = \frac{108\pi}{11\cdot 5}\,\Gamma(1/3)\,B_0 \quad , \tag{14}$$

for Eqs. (5) and (6), respectively. In these equations, A_0, A_1, B_0, and $\hat{a}$ may be expected to be weak functions of local velocity gradients. In both Eqs. (13) and (14), the Kolmogorov "constant" is thus a weak function of velocity gradients, but in addition, this "constant" depends on the direction of $\underset{\sim}{r}$ in Eq. (13).

Finally, both Eqs. (5) and (6) yield structure functions that contain deviations from isotropic form. Specifically, these tensors are given by

$$D_{ij}(\underset{\sim}{r}) = D^{o}_{ij} + D^{1}_{ij} \quad , \tag{15}$$

where D^{o}_{ij} is the usual isotropic contribution generated by A_o and B_o terms in Eqs. (5) and (6) respectively,

$$D^{1}_{ij} = C_2\, \varepsilon^{2/3}\, r^{2/3} \Big\{ - [6 + 11(\hat{r}\cdot\hat{a})^2]\hat{r}_i\hat{r}_j + [42 - 7(\hat{r}\cdot\hat{a})^2]\, \delta_{ij} - 9\hat{a}_i\hat{a}_j - 6(\hat{r}\cdot\hat{a})[\hat{r}_i\hat{a}_j + \hat{a}_i\hat{r}_j]\Big\} \tag{16}$$

for Eq. (5), and

$$D^{1}_{ij} = C_3 \Big\{ \varepsilon^{2/3}\, r_3^{2/3}\, [\delta_{i1}\delta_{j2} + \delta_{j1}\delta_{i2}] + \varepsilon^{2/3}\, r_2^{2/3}\, [\delta_{i1}\delta_{j3} + \delta_{j1}\delta_{i3}]\Big\} \tag{17}$$

for Eq. (6). In these latter two equations

$$C_2 = \frac{36\pi}{17\cdot 11\cdot 5}\, \Gamma(1/3)\, A_1 \quad , \tag{18}$$

and

$$C_3 = 3H(1)\, \Gamma(1/3) \quad . \tag{19}$$

The D^{1}_{ij} of Eq. (16) contributes to the Kolmogorov "constant," while the D of Eq. (17) being traceless, does not. As noted earlier, it would be necessary to measure cross-stream components of D^{1}_{ij} to detect the presence of components such as the D^{1}_{ij}, and this has not been done.

EXPERIMENTAL CONSEQUENCES OF HYPOTHESIS B

We discuss here some of the experimental consequences should Hypothesis B, rather than Kolmogorov's first similarity hypothesis, happen to be valid. We illustrate the effects to be expected by means of the model Fourier transformed spectral tensor

$$G_{ij}\left(\frac{k_1}{k_\eta}, \frac{k_2}{k_\eta}, \frac{k_3}{k_\eta}\right) = B_o\left(\frac{k^2}{k_\eta^2}\right)\left(\delta_{ij} - \left(\frac{k_\eta^2}{k^2}\right)\frac{k_i}{k_\eta}\frac{k_j}{k_\eta}\right) + \left\{\delta\left(\frac{k_1}{k_\eta}\right)\delta\left(\frac{k_2}{k_\eta}\right)H\left(\frac{k_3}{k_\eta}\right)\right.$$

$$[\delta_{i1}\delta_{j2} + \delta_{j1}\delta_{i2}] + \delta\left(\frac{k_1}{k_\eta}\right)\delta\left(\frac{k_3}{k_\eta}\right)H\left(\frac{k_2}{k_\eta}\right)[\delta_{i1}\delta_{j3} + \delta_{j1}\delta_{i3}] \qquad (20)$$

where, again, the mean flow is locally in the x_1 direction, B_o is a weak function of velocity gradients and a general function of $(k/k_\eta)^2$, and H is a general smooth function decreasing at infinity. The experimental consequences may be discussed by means of the spectral tensor of Eq. (20) and of the structure functions generated from it. When the two particular structure functions

$$D_{LL}(rk_\eta) = 8\pi v_\eta^2 \int_0^\infty d\ell\ell^2 B_o(\ell^2)\left\{2/3 + \frac{\cos(\ell rk_\eta)}{(\ell rk_\eta)^2} - 2\frac{\sin(\ell rk_\eta)}{(\ell rk_\eta)^3}\right\}, \qquad (21)$$

and

$$D_{NN}(rk_\eta) = 8\pi v_\eta^2 \int_0^\infty d\ell\ell^2 B_o(\ell^2)\left\{2/3 - \frac{\sin(\ell rk_\eta)}{(\ell rk_\eta)} - \frac{\cos(\ell rk_\eta)}{(\ell rk_\eta)^2}\right.$$

$$\left. + \frac{\sin(\ell rk_\eta)}{(\ell rk_\eta)^3}\right\} \qquad (22)$$

are introduced, the tensor of structure functions from Eq. (20) takes the form

$$D_{ij}(r) = [D_{LL}(rk_\eta) - D_{NN}(rk_\eta)]\, k_\eta^2\, r_i r_j + D_{NN}(rk_\eta)\delta_{ij}$$

$$+ 2v_\eta^2 \left\{ K(r_3 k_\eta)\, [\delta_{i1}\delta_{j2} + \delta_{j1}\delta_{i2}] + K(r_2 k_\eta)\, [\delta_{i1}\delta_{j3} + \delta_{j1}\delta_{i3}] \right\} \quad (23)$$

where

$$K(x) = \int_0^\infty d\ell H(\ell)\, [1 - \cos \ell x] \quad . \quad (24)$$

There are thus non-isotropic corrections to the structure functions.

We note that the three-dimensional cross spectra of Eq. (20) and the structure functions of Eq. (23) satisfy all of the experimental relations used to support the assertion that local isotropy holds in turbulent flows, both in the inertial range and in the dissipation range, with the single exception of the $k^{-5/3}$ law in the inertial range, for which the more restrictive assumptions of Hypothesis A are needed. Specifically, the two tests that have been applied experimentally are, for velocity derivatives,

$$\left\langle \left(\frac{\partial u_1}{\partial x_1}\right)^2 \right\rangle = 1/2 \left\langle \left(\frac{\partial u_2}{\partial x_1}\right)^2 \right\rangle = 1/2 \left\langle \left(\frac{\partial u_3}{\partial x_1}\right)^2 \right\rangle \quad , \quad (25)$$

where the mean flow is locally in the k_1 direction and for the one-dimensional cross spectrum,

$$F_{12}(k_1) = 0 \quad . \quad (26)$$

Now Eq. (23) is compatible with Eq. (25), while Eq. (20) is compatible with Eq. (26). Equation (25) has been verified by Townsend in grid turbulence 13 and also, and in much greater detail, in the wake behind a cylinder 14 and by Champagne et al. 15 in homogeneous shear flow. Equation (26) has also been verified experimentally at high wave numbers, or, equivalently, at high frequencies, by Corrsin in a round jet 16 by Klebanoff in a boundary layer 17, by Laufer in channel flow 18 and by Champagne et al. 15 in homogeneous shear flow. Additional experimental confirmations are cited by Monin and Yaglom 1, who also give references to the scattered literature of measurements which contradict the assumption of local isotropy.

The fact that the one-dimensional spectra appear to lie nearly on universal curves in the dissipation range 10-12 suggests that the full generality of Eq. (4) may not be needed in the dissipation range. Instead, the function $B_0\left(\frac{k^2}{k_\eta^2}\right)$ may approach a universal function, apart from a normalization factor, at very large wave numbers. We note that the normalizing of all experimental results to the Kolmogorov scales involves computing the energy dissipation through the assumption of local isotropy. Using a full set of cross-stream velocity derivatives to evaluate ϵ could modify most results and conclusions somewhat. In particular, it could happen that the universal curves referred to above are satisfied with greater accuracy (less scatter) than might appear from present measurements.

Finally, we observe that the deviations of Eq. (23) from the mathematical form of local isotropy, while substantial in size if B_0 and H are comparable, cannot be detected by the standard experimental criteria for the

validity of local isotropy. Measurements of cross-stream variations in velocity correlations, which have not been carried out, would be required to disclose their presence.

POSSIBLE TWO-POINT MEASUREMENT PROCEDURES

In principle, it would seem possible that any measuring technique for two-point velocity correlations, with or without time delay or spectral analysis, might serve as the basis for measurements that could test for the presence of non-isotropic components in the small scale structure of turbulent velocity fields. In particular, the techniques adapted by Favre, Gaviglio, and Dumas in their pioneering series of investigations, summarized in Ref. 19, on two-point, space-time, velocity correlations might be considered for such usage. However, the observations of Tritton 20 suggest that the possible interference effects of the supports of the hot wires may require further investigation before they can be safely dismissed. Possibly the technique suggested by Bradshaw 21, of seeing whether measurements are invariant to small changes in the size of the support structures for a given probe geometry, would suffice for this purpose. Even if the question of possible support interference is resolved satisfactorily, there may be considerable difficulty in making two-point measurements at sufficiently small spatial separations.

A method which might avoid some of these difficulties could be based on a two-point measurement scheme proposed by Poreh, Landa, and Kidron 22. In this scheme two hot wires, with independent electronics and linearized voltage outputs lie in the x_2 - x_3 plane and are oriented parallel to the x_3 axis, while the mean flow is parallel to the x_1 axis. The time average of

the product of the output voltages is measured, noise terms drop out, and a spatial integral of the two-point velocity correlation is obtained. The time derivative of the corresponding average formed when one voltage is time delayed, when used with Taylor's frozen turbulence hypothesis, can be combined with the preceding measurement to obtain much of the information obtainable from measurements at a whole series of spatial separations of the two wires. The fact that the time-averaged product of the output voltages has a simple physical interpretation (proportional to a spatial integral of a two-point velocity correlation) in this scheme is dependent on the output voltage of each hot wire being strictly proportional to the fluid flow velocity transverse to the wire. For this reason it is probably worth taking unusual pains to linearized the anemometer output in this method.

The technique seems interesting because measurements have already been reported 22 at spatial separations of ~0.2 mm for the two wires, and this would seem to be sufficient for testing possible non-isotropy. Moreover, interference effects appear to be small in the reported measurements.

The lack of definite theoretical predictions for the mathematical form of the nonisotropic part of the two-point velocity correlations has the consequence that no definite predictions for the expected form of the output signal can be given, if non-isotropic effects are significant. However, it would be possible to use mathematical models of possible two-point velocity correlations to explore the possible consequences of non-isotropy on the spatially integrated two-point velocity correlation measured by experiment. For example, in cases where the wire separation and the wire lengths are small compared to the Taylor microscale, functions such as those of Eqs. 5

and 6 (more specifically, the Fourier transforms of such functions) might be used to affect the spatial integrations. When the physical lengths are large compared to the Taylor microscale, functions more adequately modeling the asymptotic form of the two-point velocity correlation at large spatial separation should be used.

CONSEQUENCES FOR TURBULENCE MODELING

It has been pointed out 23 that the usual 24-25 high Reynolds number modeling of the viscous terms in the Reynolds stress equations, namely, ignoring the viscous diffusion and replacing the energy dissipation effects by the factor

$$2/3\ \varepsilon\ \delta_{ij} \tag{27}$$

on the right hand side of the Reynolds stress equation, has the potentiality for violating the realization conditions 26 of the Reynolds stress tensor. This is not to say that Reynolds stress tensors computed with the aid of Eq. (27) will often be unphysical, but only that this equation violates the principle of super-realizability formulated in Ref. 23 - that modelings of the physical groups of terms (the triple correlations, the pressure-velocity correlations, and the viscous terms) should separately satisfy the realizability criteria that the exact solutions of the Navier-Stokes equations do satisfy - and, thus, may on occasion give trouble.

It is usually felt that Eq. (27) is fully supported by experiment. Indeed, Corrsin 27 cites Ref. 13, 14, 16, and 18 as evidence that local isotropy has been shown to hold in laboratory flows and that, a fortiori, Eq.

(27) is valid. We have shown in previous sections of this paper that these, and other, works do not suffice to demonstrate that local isotropy holds in turbulent flows. In consequence, Eq. (27) need not be assumed, and no existing measurements suffice to specify the modeling of either the pressure--velocity correlations or the energy dissipation tensor. It should be regarded, at present, as an unsolved problem for Reynolds stress closure models of how to portition the terms of the model which are algebraic in the Reynolds stresses between the two physically distinct types of terms - the pressure-velocity correlations and the energy dissipation tensor.

The situation could be resolved by direct measurements of the pressure-velocity correlations or of all the components of the energy dissipation tensor. One could then see what terms of the turbulence model best represents the measured tensor. Measurements of the pressure-velocity correlations are thought to be extremely difficult. But measurements of the components of the energy dissipation tensor would involve nothing more intricate than measuring cross-stream variations of the velocity correlations - measurements which are needed in any case to establish the existence of local isotropy in turbulent flows

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