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A FINITE BOSON REPRESENTATION OF MONOPOLE AND QUADRUPOLE FERMI PAIRS*

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ABSTRACT

The monopole and quadrupole fermion pairs of a monopole and quadrupole pairing model can be represented by a finite expansion in terms of monopole and quadrupole bosons.

INTRODUCTION

The interacting boson model has been very successful in describing spectra and electromagnetic transition rates for a variety of nuclei.[1] Recently schematic fermion Hamiltonians with monopole and quadrupole pairing have been invented which have a class of eigenstates which are in one-to-one correspondence with the states of the interacting boson model.[2] This class of eigenstates is made up of pairs of fermions paired coherently to angular momentum zero and two only. The monopole pair is given by

$$S = \sum_{jm} (-1)^{j-m} a^{\dagger}_{jm} a^{\dagger}_{j-m} \tag{1}$$

where $a^{\dagger}_{jm}$ creates a fermion in a valence shell model orbital with angular momentum j and projection m. The quadrupole pair is given by

$$D_{\mu} = \sum_{j,j'} \frac{(2)}{jj'} [a^{\dagger}_{j} a^{\dagger}_{j'}]^{2}_{\mu} \tag{2}$$

where the brackets $[\ ]^{J}_{\mu}$ mean that the pair of fermions is coupled to angular momentum J and projection μ. The coefficients $\frac{(2)}{jj'}$ are, in this model,

$$\frac{(2)}{jj'} = (-1)^{k+\frac{3}{2}+j} \sqrt{(2j+1)(2j'+1)} \begin{Bmatrix} j & j' & 2 \\ \frac{3}{2} & \frac{3}{2} & k \end{Bmatrix} \tag{3}$$

where $\begin{Bmatrix} abc \\ def \end{Bmatrix}$ is the Wigner 6-j symbol. In this model k is any positive integer and the allowed valence single nucleon angular momenta j are,[3]

$$k = \frac{3}{2} \quad j = 1 + \frac{3}{2} \tag{4}$$

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DISCUSSION

Since the monopole pair creation operator (1) and quadrupole pair creation operator (2) are composite pairs of fermions, they have complicated commutation relations with the corresponding destruction operators. In fact these pair operators along with the multipole one-body operators,

$$P^{(t)}_{\mu} = 2 \sum_{jj'} \mathcal{L}^{(t)}_{jj'} [a_j^{\dagger} a_{j'}]^{t}_{\mu}, \quad t = 0, \ldots, 3 . \tag{5}$$

where the fermion destruction operator is $a_{jm} = (-1)^{j+m} a_{j-m}$, form a closed algebra under commutation. These commutation relations are given in Reference 2 and are not repeated here for lack of space. The coefficients $\mathcal{L}^{(t)}_{jj'}$ are

$$\mathcal{L}^{(t)}_{jj'} = (-1)^{t+k+\frac{3}{2}+j} \sqrt{(2j+1)(2j'+1)} \begin{Bmatrix} j & j' & t \\ \frac{3}{2} & \frac{3}{2} & k \end{Bmatrix} \tag{6}$$

The complicated commutation relations that the fermion pair operators (1) and (2) satisfy are in sharp contrast to those satisfied by the monopole, s, and quadrupole d_{μ}, boson operators:

$$[s, s^{\dagger}] = 1 \tag{7a}$$

$$[d_{\mu}, d_{\nu}] = \delta_{\mu\nu} \tag{7b}$$

$$[s, d_{\mu}] = 0 \tag{7c}$$

On the other hand there has been a vast amount of research[3] on the expansion of a pair of fermions in terms of bosons. The fermion schematic models[2] provide relatively simple examples to explore the validity of these expansions.

We have found a finite expansion in terms of bosons of the fermion multipole operators (5) and the pair operators (1) and (2). The multipole operators are given by,

$$P^{(0)} = P^{(0)} = \frac{1}{2} N \tag{8a}$$

where N is the boson number operator,

$$N = N_s + N_d = s^{\dagger} s + \sum_{\mu} d^{\dagger} d^{\mu} , \tag{8b}$$

the odd multipoles are

$$P^{(t)}_{\mu} = P^{(t)}_{\mu} = 2 [d^{\dagger} d]^{t}_{\mu}, \quad t = 1, 3 \tag{8c}$$

and the quadrupole operator is

$$P^{(2)}_{\mu} \cdot P^{(2)}_{\mu} = 2(s^{*}\tilde{d}_{\mu} + d^{*}_{\mu}s) \tag{8d}$$

where $\tilde{d}_{\mu} = (-1)^{\mu} d_{-\mu}$. The fermion pair creation operators are given by the boson operators

$$S^{*} \cdot s^{*} \tag{9a}$$

$$D^{*}_{\mu} \cdot d^{*}_{\mu} \tag{9b}$$

However the pair destruction operators have a more complicated structure,

$$S \cdot S = (-2N+N_{s})s - s^{*}d \cdot d \tag{10a}$$

$$D_{\mu} \cdot D_{\mu} = (-2N)d_{\mu} - (-1)^{\mu} d^{*}_{-\mu}(ss - d \cdot d) \tag{10b}$$

where is the number of fermions needed to half fill the valence shells,

$$\frac{1}{2} \sum_{j} (2j+1) \ . \tag{10c}$$

CONCLUSION

Hence the complicated commutation relations between fermion one-body and pair operators for a monopole and quadrupole pairing model can be reproduced by a finite expansion in terms of monopole and quadrupole bosons as given by (8), (9) and (10). This result is a generalization of the result by Dyson in which he showed that the monopole pair operators can be expressed in terms of a monopole boson for a system with monopole pairing only.[4] In fact the Dyson result is obtained by dropping the dependence on the quadrupole boson in (10). Also like the Dyson result, this result is nonhermitian. Although for the monopole pairing alone a finite hermitian expansion is possible, there doesn't seem to be a finite hermitian expansion for both monopole and quadrupole pairing. However it is possible to find a hermitian form in terms of an infinite expansion of monopole and quadrupole bosons.[5]

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