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TITLE: CALCULABILITY OF THE N-P MASS DIFFERENCE IN GAUGE THEORIES

MASTER

AUTHOR(S): Joe Kiskis

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CALCULABILITY OF THE N-P MASS DIFFERENCE IN GAUGE THEORIES

Joe Kiskis
Theoretical Division, Los Alamos Scientific Laboratory
University of California, Los Alamos, New Mexico 87545

ABSTRACT

The requirement of a calculable N-P mass difference leads to a consideration of unified gauge theories. Future developments in grand unified models may provide a realistic framework for the calculation of the N-P mass difference. The possibility that the relatively soft ultraviolet behavior of QCD softens the divergence in the lowest order electromagnetic mass shift is considered in detail. It is shown that if the bare mass and QCD coupling are constrained to be independent of the electromagnetic coupling, as is natural, then the lowest order electromagnetic shifts of the renormalized mass and QCD coupling are infinite.

I. UNIFIED GAUGE THEORIES

The universe would be a very different place if the neutron were lighter and thus very much more stable than the proton. Let us take the position that a splitting of this importance does not get its observed value by accident, unrelated to other properties of the "complete theory". Thus, we begin with the assumption that the N-P mass difference is calculable.

The further assumption that the mass difference is mainly electromagnetic (EM) leads, with rough calculation, to the conclusion that the neutron is lighter. However, it is now commonly held that the nucleons are made of u and d quarks. So the u-d mass difference is a required imput ot any N-P mass difference calculation. Unfortunately, indications are that the d quark is heavier; the problem has simply been passed down a level. From now on, we will discuss the problem at this level: the u-d mass difference (Δud).

Since the charges of the u and d quarks are different, there is certainly an EM contribution to Δud. In lowest order, this contribution diverges, and we are faced with an ultraviolet (UV) problem. From now on, the discussion will concentrate upon the calculability of the UV contribution to Δud. Can we get a finite result, or equivalently, a result that is not a free parameter after renormalization?

We have seen that the EM contribution is not calculable. A related observation is that EM explicitly breaks flavor symmetry and not in a soft way. Something to soften or cancel the EM UV contribution is needed.

One might hope that the relatively tame UV behavior of QCD could soften the UV divergence. We will analyze this point in some detail in Section II and show that it does not work out. Composite quarks might have softer EM properties in the UV. However, resorting to this argument simply passes the problem down yet another level (a ploy that did not work last time). We will insist upon facing the problem at the quark level.

Consider divergent contributions to Δud that could cancel the EM one. QCD, being flavor symmetric, cannot cancel the nonsymmetric EM contribution. However, the weak interactions are not symmetric and could work. But to do so they would have to have the same strength as the EM contributions in the UV. Thus, we are led to consider electroweak unification!

The standard $SU(2)_L \times U(1)$ electroweak theory, while unified enough to be renormalizable, is not unified enough to make Δud calculable. There are gauge and Yukawa couplings that explicitly break flavor symmetry. Δud is a free parameter.

Electroweak unification schemes with tighter structure could fix the free parameters in the standard model and give a calculable Δud. Unfortunately models of this type seem to have phenomenological difficulties.[1]

We are left with the possibility of including $SU(3)_{color}$ and unifying further: grand unified theories (GUT). The general structure that a GUT must have to give a calculable Δud can be deduced from Weinberg's work.[2] In short: the symmetry and representation content of the fundamental fields must conspire with the requirement of renormalizability in such a way as to rule out a counterterm for Δud. Δud is zero at the tree level for any[2] choice of the Lagrangian parameters. But the residual unbroken symmetry does not prevent the appearance of (necessarily finite) contributions in higher order.

The SU(5) model in its usual form gives a zeroth order relation $m_e = m_d$ and a calculable Δed. Δud remains a free parameter. SO(10) allows one to put all the first generation fermions in a single irreducible representation. This is more interesting. A number of schemes[3] for Higgs scalars, symmetry breaking, and fermion masses have been discussed. With few enough scalars, there are many zeroth order mass relations. However, my impression is that schemes that are restrictive enough to leave Δud calculable are not realistic.

Nevertheless, this work is moving in a promising direction, and future developments may yield a framework for a Δud calculation.

II. $SU(3)_{color} \times U(1)_{EM}$

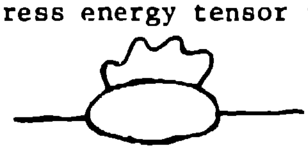
We return now to the idea of softening rather than canceling the EM UV divergence. Recently Brodsky, Schmidt and de Téramond[4] (BST) suggested that the relatively soft UV behavior of QCD may give lowest order EM self-energy integrals that converge on the scale of an eleventh quark flavor mass (if such exists) rather than diverge or converge on some grand unification scale as we have been discussing. This is based on the observation that the integral over the running mass

$$\bar{m}(q^2) = \bar{m}(q_0^2)\left(\frac{\ln q_0^2}{\ln q^2}\right)^{\gamma} \quad , \tag{1}$$

$$\int d^4q\, q^{-4}\, \bar{m}(q^2) \tag{2}$$

will converge if $\gamma > 1$. This condition implies in turn that the number of flavors n_f exceed ten. Dine[5] has analyzed their argument carefully. Craigie, Narison, and Riazuddin[6] have also discussed the subject.

On the other hand, Collins[7] and West[8] have argued generally, using the Cottingham approach, the operator product expansion (OPE), and the conservation of the stress energy tensor that the photon loop integral



must diverge. The two arguments are rather different, but both claim to include QCD to all orders. We have looked at this more carefully.[9]

Consider the $SU(3)_{color} \times U(1)_{EM}$ theory dimensionally regulated ($d=4-\varepsilon$) with minimal subtraction. Let g_B be the dimensionless bare coupling. So $\mu^{\varepsilon/2}\, g_B$ appears in the Lagrangian. When the EM coupling e is zero the connection between the bare (g_B, m_B) and renormalized (g_S, m_S) parameters is

$$g_B = Z_g^S(g_S,\varepsilon)\,g_S \qquad\qquad m_B = Z_m^S(g_S,\varepsilon)\,m_S \quad . \tag{3}$$

If the β function and mass anomalous dimensions are

$$\beta_S(g_S) = -b g_S^3 + \ldots \qquad\qquad \gamma_m^S(g_S) = -a g_S^2 + \ldots \quad , \tag{4}$$

then a renormalization group analysis[9] shows that at fixed g_S and $\varepsilon \to 0$

$$\lim_{\varepsilon \to 0} g_B = \left(\frac{\varepsilon}{2bg_S^2}\right)^{1/2} g_S \qquad\qquad \lim_{\varepsilon \to 0} m_B = \left(\frac{\varepsilon}{2bg_S^2}\right)^{\frac{a}{2b}} m_S \quad .$$

Now turn on EM. It is natural to ask if the renormalized g and m that parameterize the theory remain finite when EM is turned on without changing the bare parameters g_B and m_B. The Cottingham-OPE approach effectively does this and answers "no".[9] We will now give another analysis which does not give the photon loop such a special role. We proceed <u>as if</u> the intent is to develope the theory to all orders in e. The new relationship between the bare and renormalized parameters is

$$g_B = Z_g(g,e,\varepsilon)g \qquad\qquad m_B = Z_m(g,e,\varepsilon)m \quad . \tag{5}$$

Also

$$\beta(g,e) = \beta_S(g) + e^2(cg^3 + \ldots) + \ldots \tag{6}$$

$$\gamma_m(g,e) = \gamma_m^S(g) + e^2(D + dg^2 + \ldots) + \ldots \tag{7}$$

Another renormalization group analysis shows that if we write

$$Z_g = Z_g^S\left[1 + e^2 z_g\right] \quad \text{and} \quad Z_m = Z_m^S\left[1 + e^2 z_m\right] \quad ,$$

then

$$\lim_{\varepsilon \to 0} z_g(g,\varepsilon) = \frac{1}{4}\frac{c}{b} \quad \text{and} \quad \lim_{\varepsilon \to 0} z_m(g,\varepsilon) = \frac{D}{\varepsilon} - \frac{1}{2b}\left(d - a\frac{c}{b}\right)\ell n\varepsilon .$$

As discussed, we now require that g_B and m_B be independent of e and chosen so that g and m are finite and equal to g_S and m_S when $e=0$. In equations:

$$Z_g^S(g_S,\varepsilon)g_S = Z_g(g,e,\varepsilon)g$$

$$Z_m^S(g_S,\varepsilon)m_S = Z_m(g,e,\varepsilon)m \quad .$$

So g and m are fixed in terms of g_S, m_S, e, and ε. Further analysis gives

$$\lim_{\varepsilon \to 0} g = g_S\left[1 + e^2\,\frac{1}{\varepsilon}\left\{\frac{1}{2}\frac{c}{b}\,\beta_S(g_S)\,\frac{1}{g_S}\right\}\right]$$

$$\lim_{\varepsilon \to 0} m = m_S\left[1 + e^2\,\frac{1}{\varepsilon}\left\{\frac{1}{2}\frac{c}{b}\,\gamma_m^S(g_S) - D\right\}\right] \quad .$$

Thus the lowest order EM shifts of g and m diverge. We feel that this calculation most accurately expresses the intuitive concept of "electromagnetic mass shift".

However, there is a different approach that is equivalent to that of BST. Rather than ask for the EM shifts in the renormalized parameters with the bare parameters fixed, one calculates the EM shifts in the bare parameters when the renormalized parameters are held fixed. Then g_B and m_B certainly have an e dependence, and we find

$$\lim_{\varepsilon \to 0} g_B = \left(\frac{\varepsilon}{2bg^2}\right)^{1/2} g \left[1 + e^2 \frac{1}{4} \frac{c}{b}\right] \longrightarrow 0$$

$$\lim_{\varepsilon \to 0} m_B = \left(\frac{\varepsilon}{2bg^2}\right)^{\frac{a}{2b}} m \left[1 + e^2 \frac{D}{\varepsilon}\right] \quad .$$

Thus the shift in the bare mass will be finite only if $\frac{a}{2b} \geqslant 1$, and as BST have observed, this requires $n_f > 11$.

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