CONF-810115--4

LA-UR-81-189

TITLE: REMARKS ON THE S-SHELL Λ-HYPERNUCLEI

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SUBMITTED TO: NUCLEAR AND PARTICLE PHYSICS WORKSHOP
LOS ALAMOS NATIONAL LABORATORY
JANUARY 5-8, 1981

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St. No. 2629
12/78

UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG. 36

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ABSTRACT

The complexities of the s-shell Λ-hypernuclei ($A \leq 5$) are explored. Difficulties associated with attempts to describe the hyperon-nucleon (YN) interaction in all such Λ-hypernuclei by simple, effective Λp and Λn potentials are examined. The explicit A dependence of the effective YN interaction due to ΛN-ΣN coupling and isospin differences among the 'nuclear core' states is investigated. The necessity of using exact four-body theory to calculate small charge-symmetry-breaking effects (in the A=4 system) using ΛN potentials fitted to free ΛN scattering data is emphasized. Possible use of s-shell hypernuclear binding energies to help distinguish among candidate YN potential parameterizations is discussed.

I. INTRODUCTION

The light (s-shell) hypernuclei provide a unique opportunity for the in-depth study of few-body bound states of baryons other than just the neutron and proton as well as a rich source of information about the basic hyperon-nucleon (YN) force. The hypertriton (${}^{3}_{\Lambda}H$) binding energy places important restrictions upon the strength of the dominant spin-singlet component of the ΛN-ΣN interaction.

The A=4 isodoublet ground state energies are not consistent with a charge symmetry hypothesis for the YN interaction. The A=4 (spin-flip) excited states are very sensitive to the ΛN-ΣN coupling in the spin-triplet channel. The anomalously small ground state binding energy of ${}^{5}_{\Lambda}$He provides important information about the strength of the basic ΛN component of the YN force as well as the size of the tensor coupling in the triplet channel. (The ground and excited states of ${}^{6}_{\Lambda\Lambda}$He, the only tripley closed 1s-shell nucleus known, should provide useful knowledge about the ΛΛ force and the possible existence of a di-Λ; unfortunately the data are very limited.)

In this brief report, we wish to emphasize a few of the interesting aspects of 'exact' calculations for the A=2,3,4,5 Λ-hypernuclei: 1) Simple effective force models of the ΛN potential (neglecting explicit Λ-Σ conversion) fitted to free ΛN scattering data are not valid except (with minor caveats) for the A=3 and 4 ground states. 2) The small size of the charge-symmetry-breaking (CSB) energy difference in the A=4 ground state isodoublet requires exact 4-body calculations in order to utilize or extract information about the nature of the CSB aspect of the YN force. 3) The tensor nature of the nucleon-nucleon (NN) spin-triplet force is important and should be included in meaningful hypernuclear calculations. 4) The 'suppression', that results from the reduced strength of the ΛN-ΣN coupling potential when the trinucleon core is restricted to isospin T=1/2, is significant in understanding the excitation energy between the ground and first excited states in the A=4 system. 5) Extension of this idea to the A=5 hypernucleus, which is built upon a strongly bound T=0 nuclear 'core', indicates why one should expect an anomaly in the ${}^{5}_{\Lambda}$He binding. 6) A combination of model calculations for A=3,4,5 Λ-hypernuclei should help one discriminate among various proposed OBE model parametrizations of the YN force.

We discuss first the YN two-body interaction as a simple ΛN effective force model and as a coupled ΛN-ΣN system. Our use of the separable potential approximation is explained. We then discuss in order the A=3, 4, and 5 hypernuclear systems. We close with a brief summary.

II. THE YN INTERACTION

Lack of precision data on YN scattering is a severe limitation in our characterization of that interaction. Courageous efforts have been made to parametrize potentials using 1) a combined analysis of all of the existing YN data and the extensive NN data and 2) various symmetry assumptions concerning meson coupling in an OBE potential model of the YN and NN interactions.[1-4] We shall consider the consequences of some of these models in the following sections, but first we examine the model that results when the YN force is assumed to be independent of explicit ΛN-ΣN coupling. This model has been extensively employed in the literature in s-shell hypernuclear studies.

Such a phenomenological approach is based upon the following spin-isospin decomposition of the effective ΛN central potential (neglecting for the moment any CSB difference between Λp and Λn interactions):

$$\Lambda N: \quad V_{YN} = \bar{V}^{s}_{\Lambda N}\ ,\ \bar{V}^{t}_{\Lambda N}$$

$$^{3}_{\Lambda}H: \quad V_{YN} = \frac{3}{4}\,\bar{V}^{s}_{\Lambda N} + \frac{1}{4}\,\bar{V}^{t}_{\Lambda N}$$

$$^{4}_{\Lambda}H: \quad V_{YN} = \frac{1}{2}\,\bar{V}^{s}_{\Lambda N} + \frac{1}{2}\,\bar{V}^{t}_{\Lambda N}$$

$$^{4}_{\Lambda}H^{*}: \quad V_{YN} = \frac{1}{6}\,\bar{V}^{s}_{\Lambda N} + \frac{5}{6}\,\bar{V}^{t}_{\Lambda N}$$

$$^{5}_{\Lambda}He: \quad V_{YN} = \frac{1}{4}\,\bar{V}^{s}_{\Lambda N} + \frac{3}{4}\,\bar{V}^{t}_{\Lambda N}\ ,$$

where it has been assumed that the singlet interaction is stronger than the triplet interaction.[5,6] Here, the YN subscript indicates that the potential describes the general hyperon-nucleon (ΛN-ΣN) interaction. Implicit in the above effective potential description is the assumption[7] that the ΛN-ΣN coupling in the YN interaction is identical in each system regardless of the isospin of the (A-1) nucleons forming the nuclear 'core'; i.e., one has assumed that the 2x2 matrix potential

$$V^{i}_{YN} = \begin{pmatrix} V^{i}_{\Lambda N} & V^{i}_{XN} \\ V^{i}_{XN} & V^{i}_{\Sigma N} \end{pmatrix}\ , \quad i = s,t$$

can be represented by a unique effective one-channel potential $\bar{V}^{i}_{\Lambda N}$ for A=2,3,4,5. Such is not the case.

Let us define the free interaction to be of the form

$$V^{s}_{YN} = \begin{pmatrix} V^{s}_{\Lambda N} & V^{s}_{XN} \\ V^{s}_{XN} & V^{s}_{\Sigma N} \end{pmatrix}\ , \qquad V^{t}_{YN} = \begin{pmatrix} V^{t}_{\Lambda N} & V^{t}_{XN} \\ V^{t}_{XN} & V^{t}_{\Sigma N} \end{pmatrix}\ .$$

(We note that the ΛN elastic scattering is dominated by the triplet interaction, since $\sigma = (\sigma^{s} + 3\sigma^{t})/4$.) For the $^{3}_{\Lambda}H$ system, where the np pair is restricted to be in the S=1,T=0 'deuteron' state, the relevant potentials are

$$V^s_{YN} = \begin{pmatrix} V^s_{\Lambda N} & 0 \\ 0 & 0 \end{pmatrix} \quad , \qquad V^t_{YN} = \begin{pmatrix} V^t_{\Lambda N} & 0 \\ 0 & 0 \end{pmatrix} ;$$

i.e., there is no Λ-Σ conversion unless one allows for the np T=1 'excited' state in the formalism. This is a consequence of the T=0 nature of the ${}^3_\Lambda$H ground state (the Λ and the deuteron each being T=0 objects); the Σ has T=1 and must couple to the T=1 singlet np state to produce a hypernucleus with total T=0. For the A=4 hypernuclei, the $J^\pi=0^+$ ground state potentials are

$$V^s_{YN} = \begin{pmatrix} V^s_{\Lambda N} & -\frac{1}{3} V^s_{XN} \\ -\frac{1}{3} V^s_{XN} & V^s_{\Sigma N} \end{pmatrix} , \quad V^t_{YN} = \begin{pmatrix} V^t_{\Lambda N} & V^t_{XN} \\ V^t_{XN} & V^t_{\Sigma N} \end{pmatrix}$$

and the $J^\pi=1^+$ excited state potentials are

$$V^s_{YN} = \begin{pmatrix} V^s_{\Lambda N} & V^s_{XN} \\ V^s_{XN} & V^s_{\Sigma N} \end{pmatrix} , \quad V^t_{YN} = \begin{pmatrix} V^t_{\Lambda N} & \frac{1}{5} V^t_{XN} \\ \frac{1}{5} V^t_{XN} & V^t_{\Sigma N} \end{pmatrix}$$

(see for example, Refs. 8 and 9). In neither case is the coupling of the Λ-Σ system to a composite T=1/2 object the same as is the coupling to an elementary nucleon constituent. The <u>singlet</u> potential differs from the free interaction in the A=4 <u>ground</u> state. The <u>triplet</u> potential differs from the free interaction in the A=4 <u>excited</u> state. In each case the ΛN-ΣN coupling strength is reduced, weakening the YN interaction relative to its free strength. For the ${}^5_\Lambda$He system, the situation is similar to that encountered with the hypertriton. A T=0,S=0 assumption for the four-nucleon 'core' (the alpha particle is bound by 28 MeV) leads to potentials of the same form as in the case of ${}^3_\Lambda$H:

$$V^s_{YN} = \begin{pmatrix} V^s_{\Lambda N} & 0 \\ 0 & 0 \end{pmatrix} \quad , \qquad V^t_{YN} = \begin{pmatrix} V^t_{\Lambda N} & 0 \\ 0 & 0 \end{pmatrix} ;$$

i.e., there is again no ΛN-ΣN coupling unless one allows for even parity, T=1 'excited' states of the alpha-like core in the formalism.[10] (Note that this does <u>not</u> mean that we assume a rigid, non-distorted alpha-core model; however, the formalism must be extended if coupling of T=1 and T=0 four-nucleon states is to be permitted.)

It is clear that <u>in principle</u> the YN interactions acting in each of the five systems (ΛN, ${}^3_\Lambda$H, ${}^4_\Lambda$H, ${}^4_\Lambda$H*, and ${}^5_\Lambda$He) cannot be represented by single, unique $\bar{V}^s_{\Lambda N}$

and $\bar{V}^t_{\Lambda N}$ effective potentials. In practice, one finds experimentally[11] that $V^s_{XN} \approx 0$, so that effective potential representation of the free YN interactions is 'reasonable' when dealing with the A=4 ground states, where $V^t_{YN}({}^4_\Lambda H) \equiv V^t_{YN}(\Lambda N$ scattering). However, the triplet interactions involved in ${}^3_\Lambda H$, ${}^4_\Lambda H^*$, and ${}^5_\Lambda He$ calculations differ from the free case (i.e., the coefficient of V^t_{XN} is not unity as in free scattering), and the free effective triplet potential $\bar{V}^t_{\Lambda N}$ should not be used in those calculations.[7] ${}^3_\Lambda H$ is a possible exception since the ΛN interaction in that ground state is 3/4 singlet and $V^s_{XN} \approx 0$ experimentally. The importance of including ΛN-ΣN coupling in calculations involving these hypernuclei has been previously noted; see, for example, Refs. 8,9,12,13, and 14.

In the numerical calculations referred to below, we assume that effective ΛN interactions $\bar{V}^{s,t}_{\Lambda N}$ (i.e., one-channel ΛN potentials determined from the free ΛN scattering parameters) can be used to describe the coupled ΛN-ΣN hyperon-nucleon system. Thus, we are restricted to estimates of the ${}^3_\Lambda H$, ${}^4_\Lambda H$ and ${}^4_\Lambda He$ ground-state energies. As just noted, this is not an entirely correct procedure in the case of ${}^3_\Lambda H$; however, since the average ΛN interaction is 3/4 singlet and only 1/4 triplet, and since the binding is weak, we shall assume that the error produced by this procedure is small. We shall also neglect in the ${}^3_\Lambda H$ case the tensor nature of the ΛN triplet force, which tends to compensate for our neglect of explicit ΛN-ΣN coupling in that channel.[14]

We use a separable potential representation of both the NN and YN interactions in all of our numerical calculations in order to have a consistent model with which to carry out the exact 4-body calculations. We use rank one potentials of the form

$$V_i = -\frac{\lambda_i}{2\mu} g_i(\vec{k})\, g_i(\vec{k}') \quad , i = s,t ,$$

where $g_i = (k^2 + \beta_i^2)^{-1}$ if there is no tensor component and where

$$g_t = g_c + \frac{S_{ij}}{\sqrt{8}} g_T$$

$$g_c = (k^2 + \beta_c^2)^{-1}$$

$$g_T = \zeta_T k^2 (k^2 + \beta_T^2)^{-2}$$

$$S_{ij} = 3\, \vec{\sigma}_i \cdot \hat{k}\, \vec{\sigma}_j \cdot \hat{k} - \vec{\sigma}_i \cdot \vec{\sigma}_j$$

in the case of a tensor force in the spin-triplet channel. The quantity μ is the appropriate two-body reduced mass. The low-energy ΛN scattering parameters which we use to determine our separable-potential parameters are listed in Table I.

TABLE I.

The ΛN scattering lengths and effective ranges in fm for the YN potential models A-F.

Model	Ref.	$a^s_{\Lambda p}$	$r^s_{\Lambda p}$	$a^t_{\Lambda p}$	$r^t_{\Lambda p}$	$a^s_{\Lambda n}$	$r^s_{\Lambda n}$	$a^t_{\Lambda n}$	$r^t_{\Lambda n}$
A	1	-2.16	2.03	-1.32	2.31	-2.67	2.04	-1.02	2.55
B	2	-2.11	3.19	-1.88	3.16	-2.47	3.09	-1.66	3.33
D	3	-1.77	3.78	-2.06	3.18	-2.03	3.66	-1.84	3.32
F	4	-2.18	3.19	-1.93	3.35	-2.40	3.15	-1.84	3.37

TABLE II.

Separable potential parameters and properties for the NN interactions.

Spin	Model(ref.)	a_i(fm)	r_i(fm)	P_D	Q(fm^2)	λ_i(fm^{-3})	β_i(fm^{-1})	ξ_T	β_T(fm^{-1})
t	GL(16)	5.423	1.761	0.	-	0.3815	1.406	0.	-
t	P_4(15)	5.397	1.727	0.04	0.282	0.24310	1.3134	1.6894	1.5283
t	P_7(15)	5.397	1.722	0.07	0.283	0.14297	1.2412	4.4949	1.9476
s	GL(16)	-17.0	2.84	-	-	0.1323	1.130	-	-

TABLE III.

Hypertriton Λ-separation energy in MeV for YN models A-F as a function of P_D in the np triplet interaction.

YN Model	GL	P_4	P_7
A	0.90	0.56	0.35
B	0.37	0.22	0.13
D	0.12	0.06	0.03
F	0.37	0.23	0.13

These are taken from the meson exchange theoretic potentials developed by Nagels, Rijken, and deSwart.[1-4] Mass differences in the isomultiplets as well as symmetry breaking exchanges were included in a combined analysis of NN, Λp, $\Sigma^{\pm}p$, etc. data. The NN low energy scattering parameters as well as resulting potential parameters are listed in Table II.[15,16]

III. THE HYPERTRITON

${}^3_\Lambda H$ ($J^\pi = 0^+$, $T=0$) is the lightest of the bound hypernuclei having a Λ-separation energy $B_\Lambda = B({}^3_\Lambda H) - B({}^2H) = 0.13 \pm 0.05$ MeV.[17] Because the Λ-binding is weak, it was originally assumed that the loose structure would make B_Λ insensitive to the short range (high-momentum) character of the YN force and the tensor nature of the triplet component.[12] As noted above, we also assumed that explicit ΛN-ΣN coupling could be omitted since it was included implicitly by using the physical low-energy ΛN scattering parameters to construct the potentials. It was later pointed out that, while repulsion in the YN force and explicit ΛN-ΣN coupling were not large effects, neglecting the tensor nature of the np triplet force was a significant omission.[18] Because the average ΛN interaction is 3/4 singlet, we have neglected the tensor nature of the ΛN triplet interaction. This tends to slighty overestimate B_Λ but should be compensated for by our neglect of explicit ΛN-ΣN coupling in that channel which tends to underestimate B_Λ.[12,14] The fact that there is little or no ΛN-ΣN coupling in the dominant singlet YN interaction should ensure that our model calculations are reasonable.[11] We summarize in Table III values of $B_\Lambda({}^3_\Lambda H)$ for the various YN potentials models with and without including the explicit tensor force nature of the np spin-triplet force. (For details of the equations used, see Refs. 12 and 19.)

Model A clearly overbinds ${}^3_\Lambda H$ regardless of the np triplet force used. This is a result of the comparatively small values ($\lesssim 2.5$ fm) for the effective ranges of the ΛN potentials in that model, as noted in Ref. 12. Although the value of B_Λ differs among models B,D, and F by 0.1-0.2 MeV, none of these models is obviously incorrect. (B_Λ for $P_D=0$ is not considered to be realistic, and we do not consider B_Λ for model D to lie significantly outside the experimental limits.) The B_Λ from model D are systematically smaller than those of models B and F, because the average (3/4 singlet plus 1/4 triplet) effective range is larger; $r_0 > r_0'$ implies $B_3 < B_3'$.[20] Models B and F produce very similar values of B_Λ because their average singlet scattering lengths and effective ranges are similar; they would produce different values of ΔB_Λ in the ${}^4_\Lambda He$-${}^4_\Lambda H$ isodoublet system where differences in the Λp and Λn triplet scattering lengths and effective ranges are significant.

A recent estimate of B_Λ using a sum of local Yukawa forms (including short range repulsion) to represent the model F ΛN interaction and the Reid-soft-core potential for the np triplet interaction by Narumi, Ogawa, and Sunami gave a value of 0.17 MeV.[21] This agrees very well with our 0.13 MeV estimate for model F using an np potential model with $P_D = 7\%$, and it lends credence to the accepted use of rank one separable potentials to represent baryon-baryon interactions phenomenologically. The agreement here and for the well known case of the triton illustrates the point that the important aspects of the interactions for relatively weakly bound systems are the low-energy scattering parameters and not the short range behavior of nor the off-shell behavior generated by the potentials.

IV. THE A=4 ISODOUBLET

The latest experimental estimates of the Λ-separation energies for these $J^\pi = 0^+$ ground states are[17]

$$B_\Lambda({}^4_\Lambda\mathrm{He}) = B({}^4_\Lambda\mathrm{He}) - B({}^3\mathrm{He}) \simeq 2.42 \pm 0.04 \text{ MeV}$$
$$B_\Lambda({}^4_\Lambda\mathrm{H}) = B({}^4_\Lambda\mathrm{H}) - B({}^3\mathrm{H}) \simeq 2.08 \pm 0.06 \text{ MeV} .$$

Because we do not solve the complete set of tensor force equations for each model (we treat the YN triplet potentials in a central force approximation and use the truncated t-matrix approximation[22] for the NN triplet force), we consider the Λ-separation energy difference $\Delta B_\Lambda \simeq 0.34 \pm 0.07$ MeV to be a better measure of model consistency. This ΔB_Λ reflects true charge symmetry breaking in the YN interaction; simple considerations of Coulomb energies in the A=3 and 4 nuclear systems suggest that ΔB^C_Λ, the additional Coulomb energy in ${}^4_\Lambda$He due to compression of the '^{3}He core', is small and of opposite sign.[23] It is this Coulomb corrected quantity $\Delta B_\Lambda \simeq 0.36$ MeV that we estimate for each of the YN potentials defined by the low-energy scattering parameters in Table I.

The exact coupled two-variable integral equations that must be solved for the A=4 hypernuclear problem when the NN and YN interactions are represented by separable potentials are described in detail in Ref. 24. The integral equations are solved numerically without resort to separable expansions of the kernels. The resulting solutions possess the characteristics of true few-body calculations: for an attractive potential with a negative scattering length, $|a| > |a'|$ implies that V is more attractive than V' in two-body, three-body, and four-body calculations, whereas $r > r'$ implies that V is more attractive than V' in a two-body calculation, but less attractive in three-body and four-body calculations. Even though this picture is an oversimplification in terms of scattering length and effective range, it is possible to understand ΔB_Λ from each of the models in Table I qualitatively in terms of the low-energy scattering parameters of the various models.

In our numerical calculations, we assume that effective ΛN interactions $\bar{V}^{s,t}_{\Lambda N}$ (i.e., one channel ΛN potentials determined from the free ΛN scattering parameters) can be used to describe the coupled ΛN-ΣN hyperon-nucleon system. As noted above, this can be justified for the $J^\pi = 0^+$ ground state (but not for the $J^\pi = 1^+$ excited states), where the triplet interaction is unmodified from its free form

$$V^t_{YN} = \begin{pmatrix} V^t_{\Lambda N} & V^t_{XN} \\ V^t_{XN} & V^t_{\Sigma N} \end{pmatrix} \simeq \bar{V}^t_{\Lambda N} \quad .$$

Since $V^s_{XN} \simeq 0$ in the singlet interaction,

$$V^s_{YN} = \begin{pmatrix} V^s_{\Lambda N} & -\frac{1}{3}\, V^s_{XN} \\ -\frac{1}{3}\, V^s_{XN} & V^s_{\Sigma N} \end{pmatrix} \simeq \bar{V}^s_{\Lambda N}$$

is also a good approximation. Thus, the effects of Λ-Σ conversion upon the ΛN potential parameters, including charge symmetry breaking due to meson mixing, $\Sigma^{\pm,0}$ mass differences, etc., are taken into account implicitly, but there are no explicit Σ-channels in the calculation.[24]

The Λp and Λn potential averages appropriate to ${}^4_\Lambda He$ and ${}^4_\Lambda H$ are

$$
\begin{aligned}
{}^4_\Lambda\mathrm{He}:\quad & V^t_{\Lambda N} = V^t_{\Lambda p} & \qquad {}^4_\Lambda\mathrm{H}:\quad & V^t_{\Lambda N} = V^t_{\Lambda n} \\
& V^s_{\Lambda N} = \frac{1}{3}\, V^s_{\Lambda p} + \frac{2}{3}\, V^s_{\Lambda n} & & V^s_{\Lambda N} = \frac{1}{3}\, V^s_{\Lambda n} + \frac{2}{3}\, V^s_{\Lambda p} \quad .
\end{aligned}
$$

Instead of using the two potential formula to obtain the required potentials, we used the excellent approximation of scattering length and effective range averages

$$a^{-1}_{\Lambda N} = \frac{1}{3}\, a^{-1}_{\Lambda p} + \frac{2}{3}\, a^{-1}_{\Lambda n}$$

$$r_{\Lambda N} = \frac{1}{3}\, r_{\Lambda p} + \frac{2}{3}\, r_{\Lambda n}$$

to parametrize the ΛN singlet interaction, etc. The resulting potential parameters are listed in Table IV. The NN potential parameters for the model calculations were chosen to be the P_7 model; the triton binding energy is 7.05 MeV in the truncated t-matrix approximation which is only 7% below the complete model result.[22]

The results of our ${}^4_\Lambda$He-${}^4_\Lambda$H binding energy difference calculations are tabulated in Table V.[25] Because the singlet potentials are averages of Λn and Λp potentials, most of the charge symmetry breaking results from the triplet interaction differences (see Table IV). It is clear that differences between triplet scattering lengths and effective ranges for the ${}^4_\Lambda$He and ${}^4_\Lambda$H systems are very similar for models B and D. Thus one anticipates similar values of ΔB_Λ for models B and D, and these values are not inconsistent with experiment. Model A has an even larger difference in scattering length values ($\Delta a \sim -0.3$ fm vs. -0.2 fm for models B and D) and effective range values ($r \sim -0.25$ fm vs. -0.15 fm). Hence ΔB_Λ for model A is expected to be larger than that for models B and D, as is the case; it is probably outside the limits set by the experimental values. The perhaps surprisingly large model A value of ΔB_Λ results from the small values of the effective ranges in that model, which produce large values of $B_\Lambda({}^4_\Lambda\text{He})$ and enhance CSB differences. We pointed out above that these small effective ranges of the model A singlet interactions are primarily responsible for the value of $B_\Lambda({}^3_\Lambda\text{H})$ being inconsistent with experiment. It is clear from the effective ranges in Table I that model F is a much more charge symmetric model than models A,B, or D. In fact, the model F ${}^4_\Lambda$He and ${}^4_\Lambda$H scattering lengths and effective ranges in Table IV show very little difference between the two singlet sets or the two triplet sets. Thus, one anticipates a small value of ΔB_Λ, one which is too small to be consistent with the experimental binding energy difference.

Since we have used a central potential approximation in representing the ΛN triplet interaction, we have overestimated ΔB_Λ for each of the models. Although this is a non-negligible effect, we have previously shown[24] that it would not alter the conclusions drawn above and that it would bring our model D result into closer agreement with the experimental value of $\Delta B_\Lambda = 0.36$ MeV. We constructed a tensor force YN triplet potential (of the same form as that of our np triplet potential) fitted to the model D triplet phase shift and mixing parameter up to laboratory momenta of 300 MeV/c. We made the same truncated t-matrix approximation in the complete set of 4-body equations as noted above for the NN channel. Our estimate of ΔB_Λ for model D was reduced from 0.43 MeV to 0.37 MeV; see Ref. 24

TABLE IV.

Potential parametrizations and their low energy properties for the interaction averages appropriate to each A=4 hypernucleus.

Model	System	Spin	$\lambda(fm^{-3})$	$\beta(fm^{-1})$	a(fm)	r(fm)
A	$\Lambda N(^4_\Lambda He)$	s	0.4787	1.8891	-2.48	2.04
		t	0.4348	1.9660	-1.32	2.31
	$\Lambda N(^4_\Lambda H)$	s	0.4957	1.9217	-2.31	2.03
		t	0.3819	1.9608	-1.02	2.55
B	$\Lambda N(^4_\Lambda He)$	s	0.1578	1.3634	-2.34	3.12
		t	0.1670	1.4229	-1.88	3.16
	$\Lambda N(^4_\Lambda H)$	s	0.1532	1.3527	-2.32	3.16
		t	0.1542	1.4128	-1.66	3.33
D	$\Lambda N(^4_\Lambda He)$	s	0.1099	1.2549	-1.94	3.70
		t	0.1581	1.3846	-2.06	3.18
	$\Lambda N(^4_\Lambda H)$	s	0.1093	1.2607	-1.85	3.74
		t	0.1484	1.3785	-1.84	3.32
F	$\Lambda N(^4_\Lambda He)$	s	0.1532	1.3527	-2.32	3.16
		t	0.1421	1.3531	-1.93	3.35
	$\Lambda N(^4_\Lambda H)$	s	0.1525	1.3558	-2.25	3.18
		t	0.1428	1.3632	-1.84	3.37

TABLE V.

The A=4 binding energy difference ΔB_Λ for each of the YN models discussed in the text in the central potential approximation for the ΛN interaction.

Model	ΔB_Λ
A	1.32
B	0.47
D	0.43
F	0.19

for details.

In the ${}^{3}_{\Lambda}H$ and ${}^{4}_{\Lambda}He$-${}^{4}_{\Lambda}H$ calculations discussed, we have used exact few-body equations based upon separable potential approximations to the YN and NN interactions. Could one have done as well for ΔB_{Λ} with a simpler effective 2-body model? The answer is no. We have explicitly demonstrated this for one standard 2-body formalism:[24] in the procedure outlined by Dalitz and Downs[26] the 2-body ΛN potentials are folded with the nuclear core density to produce a Λ-${}^{3}He$ (or Λ-${}^{3}H$) effective 2-body potential which is then inserted into the Schrodinger equation to determine the Λ-separation energy. (Radial compression of the nuclear core is easily accommodated by altering the radius of the core density.) Using this formalism, we found ΔB_{Λ}(2-body) to be between 0.21 and 0.24 MeV, depending upon the core compression permitted, for model D.[24] This is about 1/2 that obtained (0.43 MeV) for model D using the exact 4-body theory and the identical ΛN potentials. This can be understood in terms of the characteristics of true few-body calculations outlined above and the scattering lengths and effective ranges listed in Table IV. For model D the ${}^{4}_{\Lambda}He$ and ${}^{4}_{\Lambda}H$ singlet scattering lengths and effective ranges are very similar and contribute little to the CSB difference. On the other hand, $|a_t({}^{4}_{\Lambda}He)| > |a_t({}^{4}_{\Lambda}H)|$ implies that ΔB_{Λ} (with $\Delta r^t_{\Lambda N}=0$) > 0, whereas $r_t({}^{4}_{\Lambda}He) < r_t({}^{4}_{\Lambda}H)$ implies that ΔB_{Λ}(with $\Delta a^t_{\Lambda N}=0$) < 0 in an effective 2-body formalism but > 0 in an exact 4-body formalism. Thus, the model D $\Delta a^t_{\Lambda N}$ and $\Delta r^t_{\Lambda N}$ produce compensating effects in an effective 2-body calculation but reinforce each other in a true 4-body calculation.

The spin-flip $J^{\pi}=1^{+}$ state energies are not calculable in terms of the free interactions unless one has coupled ΛN-ΣN potentials with which to work. While the singlet potential is the same in this case as the free V^s_{YN} (whether or not $V^s_{XN}\approx 0$), the equality does not hold for the triplet potential where

$$V^t_{YN} = \begin{pmatrix} V^t_{\Lambda N} & \frac{1}{5}\,V^t_{XN} \\ \frac{1}{5}\,V^t_{XN} & V^t_{\Sigma N} \end{pmatrix} \simeq \bar{V}^t_{\Lambda N}(A=4^*) \quad .$$

One must explicitly alter the coefficient of the V^t_{XN} coupling potential and recompute the effective $\bar{V}^t_{\Lambda N}(A=4^*)$ potential to use in our one-channel, effective potential formalism. The resulting $\bar{V}^t_{\Lambda N}(A=4^*)$ will be considerably weaker than the free $\bar{V}^t_{\Lambda N}$. In fact, the use of $\bar{V}^t_{\Lambda N}$ in calculating the binding energy of the $J^{\pi}=1^{+}$ states would result in the conclusion that these were the ground states of the $\Lambda=4$ system and not the 0^{+} states. Therefore, it is not possible in simple model calculations to use the $0^+ \rightarrow 1^+$ transition energies to determine the spin dependence of the effective ΛN potential; a unique single-channel potential representation is not an adequate description of the physics. Σ-suppression in the A=4 excited states is a very important effect.

V. THE ${}^5_\Lambda$He ANOMALY

The possibility that Σ-suppression (actually suppression of the Λ-Σ conversion) is responsible for the anomalously small Λ-separation energy in ${}^5_\Lambda$He has been the subject of speculation for some time.[27] Shell model and variational[5,6] estimates of $B_\Lambda({}^5_\Lambda\text{He})$ are of the order of 5-6 MeV compared to an experimental[17] value of approximately 3.1 MeV when one uses effective ΛN spin-dependent potentials fitted to the binding energy of ${}^3_\Lambda$H and the average of ${}^4_\Lambda$He and ${}^4_\Lambda$H. However, the wave function is actually of the form[10]

$$a\{\ |{}^4\text{He},T=0\rangle \times |\Lambda\ ,T=0\rangle\ \}^{T=0} \qquad + \qquad b\{\ |{}^4\text{He}^*,T=1\rangle \times |\Sigma,T=1\rangle\ \}^{T=0} \quad .$$

The even parity T=1 states of ^{4}He have large excitation energies relative to the ground state which should strongly suppress the ΛN-ΣN coupling. The isospin structure is very reminiscent of the hypertriton, where conversion of the Λ to a Σ requires that the Σ couple to the d^*(T=1) state of the np pair and not the d(T=0) state in order that the total isospin of the ${}^3_\Lambda$H system be T=0.

A first estimate of this is possible in a simple A=5 calculation. If one assumes that the T=0, four-nucleon core is the only allowed isospin state, then one need only use the $V^t_{\Lambda N}$ element of V^t_{YN} in the calculation. The difference in $B_\Lambda({}^5_\Lambda\text{He})$ in that approximation compared to the same calculation using the effective potential approximation of the free interaction $\bar{V}^t_{\Lambda N}$ would provide an upper limit on the effect of Σ-suppression for a given potential model.

VI. CONCLUSIONS

In summary, we have tested separable potential approximations to four of the hyperon-nucleon potential models of Nagels, Rijken, and deSwart in exact 3-body calculations of $B_\Lambda({}^3_\Lambda\text{H})$ and exact 4-body calculations of ΔB_Λ for the ${}^4_\Lambda$He - ${}^4_\Lambda$H isodoublet. We find model A, which overbinds ${}^3_\Lambda$H, to overestimate ΔB_Λ. Models B and D appear to be consistent with the experimental value of ΔB_Λ (and give reasonable ${}^3_\Lambda$H binding energies). We find model F, which is consistent with $B_\Lambda({}^3_\Lambda\text{H})$, to underestimate ΔB_Λ for the A=4 system; this result is understood in terms of the small differences between the singlet Λp and Λn scattering lengths and effective ranges in that model.

We emphasize that exact formalisms are required when dealing with small quantities such as ΔB_Λ; effective 2-body calculations have been shown to underestimate exact 4-body results by a factor of 2. Formalisms which treat properly the ΛN-ΣN coupling are required to account for the Σ-suppression that separates the $J^\pi=1^+$ states from the 0^+ ground states by an MeV and that produces the anomalously small Λ-separation energy $B_\Lambda({}^5_\Lambda He)$.

Finally, we point out that similar ΛN-ΣN coupling effects should be apparent in the ${}^4_\Sigma He$ and ${}^5_\Sigma He$ decay widths. The Λ-Σ conversion should be uninhibited in the former case leading to a broad width, whereas the T=1 nature of ${}^5_\Sigma He$ will require a 'core' state transition from T=0 to T=1 when the Σ converts to a Λ which should lead to an inhibited transition and correspondingly narrower width.

ACKNOWLEDGMENTS

The work of BFG was performed under the auspices of the U. S. Department of Energy, that of DRL was supported in part by the U. S. Department of Energy. We wish to thank the Clinton P. Anderson Meson Physics Facility (LAMPF) for making available their VAX computing facilities.

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