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MULTIDIMENSIONAL MHD COMPUTATIONS FOR THE FIELD-REVERSED THETA PINCH AND THE REVERSED-FIELD PINCH

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I. INTRODUCTION

The study of alternative approaches to the tokamak for the design of a magnetic fusion reactor is an area of active research in both the United States and Japan. Among the most promising of these concepts are the Field-Reversed Theta Pinch (FRTP) and the Reversed-Field Pinch (RFP). This paper briefly describes some recent large-scale numerical MHD simulations of these devices.

The FRTP is one of a class of Field-Reversed Configurations (FRCs) that are characterized by the presence of a separatrix that extends to the major axis. Unlike the spheromak, the FRTP is highly prolate (elongations of 4 to 1 or more are typical) and contains no toroidal field. A field null thus exists. Additionally, both the safety factor and the shear vanish everywhere. The validity of the MHD model in such devices may thus be questioned. However, such calculations are known to give a pessimistic view of stability, and certainly become more valid for larger devices where kinetic effects may become less important. Examples of these calculations and their relevance to particular experiments are given in Sec. II

Like the tokamak, the RFP is a toroidal pinch that contains both poloidal and toroidal fields. The tokamak attains stability against large-scale ideal MHD modes by satisfying the Kruskal-Shafranov condition. This requires the imposition of a strong toroidal magnetic field such that $B_p/B_T \sim \epsilon << 1$, where ϵ is the inverse aspect ratio. The resulting safety factor profile $q(r)$ is greater than unity and monotone increasing. The RFP, on the other hand, is characterized by fields such that $B_p/B_T \sim 1$, and a profile $q(r) << 1$, which is monotone decreasing and changes sign in the outer regions of the plasma. The resulting shear grossly stabilizes the pinch. Thus, high values of β may be attainable. Also, since the toroidal current is not limited by the Kruskal-Shafranov condition, significant ohmic heating may occur.

It is now accepted that much of the characteristic behavior of tokamak plasmas can be described in terms of the 3-D nonlinear evolution of resistive MHD instabilities. However, as yet the fundamental MHD processes that occur in the RFP remain unknown. Such processes are probably responsible for attainment and maintenance of field reversal. In particular, simple estimates of the resistive diffusion time in the ZT-40M device at Los Alamos indicate loss of field reversal in a time short compared to the observed lifetime. This dynamo effect may result from the turbulent generation of mean magnetic field, or from large scale modes. Accurate simulations of these 3-D nonlinear processes are vital to understanding the basic physics of the RFP.

The advances in 3-D calculations for tokamaks depend crucially on expanding the primitive MHD equations

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla\times\left(\underset{\sim}{v}\times\underset{\sim}{B} - \frac{\eta}{S}\nabla\times\underset{\sim}{B}\right) \tag{1a}$$

$$\frac{\partial(\rho \underset{\sim}{v})}{\partial t} = -\nabla\cdot\left[\rho\underset{\sim}{v}\underset{\sim}{v} - \underset{\sim}{B}\underset{\sim}{B} + \frac{1}{2}(p+B^2)\underset{\sim}{I}\right] \tag{1b}$$

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\underset{\sim}{v}) \tag{1c}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1}\right) = -p\nabla\cdot\underset{\sim}{v} - \nabla\cdot\left(\frac{p\underset{\sim}{v}}{\gamma-1}\right) + \frac{2\eta}{S}(\nabla\times\underset{\sim}{B})^2 \tag{1d}$$

in the small parameter $\varepsilon = B_p/B_T$, discussed earlier, to obtain the reduced set of equations

$$\frac{\partial A}{\partial t} + \underset{\sim}{v} \cdot \nabla_{\perp} A - \frac{\eta}{S} \nabla_{\perp}^2 A = B_0 \frac{\partial v}{\partial z} \tag{2a}$$

$$\frac{\partial \omega}{\partial t} + \underset{\sim}{v} \cdot \nabla_{\perp} \omega - \underset{\sim}{B} \cdot \nabla_{\perp} j = B_0 \frac{\partial j}{\partial z} \tag{2b}$$

where $\underset{\sim}{B} = \nabla_{\perp} \times \hat{e}_z A$, $\underset{\sim}{v} = \nabla_{\perp} \times \hat{e}_z v$, $\nabla_{\perp}^2 v = -\omega$, and $\nabla_{\perp}^2 A = -j$. However, since the field components in the RFP are all of the same order, and since these devices may possess finite β, no universally small expansion parameter exists for this device. Thus, instead of solving the two scalar equations (2) as is the case for the tokamak, computations for the RFP require the solution of the eight primitive equations (1).

Additionally, the computational speed of codes based on the tokamak reduced equations (2) is greatly enhanced by the fact that in this ordering, the plasma is incompressible. This eliminates the fast compressional wave from the calculation. This wave evolves on a time scale that is on the order of the minor radius divided by the Alfvén velocity. The remaining time scale is defined by the shear Alfvén wave that, because of the strong toroidal field, evolves on a time scale that is on the order of the major radius divided by the Alfvén velocity. This time scale may be more than an order of magnitude longer than that of the compressional Alfvén wave. In the RFP, on the other hand, even the (unjustified) assumption of incompressibility does not eliminate the fast time scale, since now a shear Alfvén wave traveling near the field reversal surface evolves on a time scale that is on the order of the minor radius divided by the Alfvén velocity, i.e., the same order as that of the compressional wave.

To summarize the above remarks, calculations for the tokamak require the advancement of two equations that evolve on a slow time scale, while those for the RFP require the advancement of eight equations that evolve on a fast time scale. Because such calculations are difficult and time consuming, most simulations of the RFP have been in two dimensions or have assumed helical symmetry. A recent example of such a calculation is discussed in Sec. III. There we also speculate on a possible dynamo mechanism involving the interaction of a few large scale resistive modes, and briefly describe a 3-D code that is presently under development.

II. FIELD-REVERSED THETA PINCH

The MHD stability of compact toroids was first investigated analytically by Rosenbluth and Bussac[1] who used a modified energy principle to determine marginal stability to all ideal modes and most tearing modes. They found stability against magnetically driven modes for all toroidal mode numbers $n > 1$, but found an unstable $n = 1$ mode for prolate spheroidal plasmas. This mode, which is characterized by a rotation of the major axis inside the separatrix, is termed the tilting mode. Highly elongated configurations characteristic of FRTP plasmas were investigated by Barnes[2] who found instabilities for all n if the flux surfaces where elliptical. In this case the unstable $n = 1$ displacements are more axial than rotational.

Computationally, these predictions have been investigated in a numerically generated equilibrium that includes plasma on open field lines and closely resembles the FRX-B experiment at Los Alamos.[3,4] These studies have employed the linearized code RIPPLE VI[5] and the nonlinear ideal 3-D code MALICE.[6] Both solve the primitive MHD equations and are initialized with perturbations characterized by $n = 1$ and $k = 1$ for the toroidal and axial mode numbers. They both develop unstable tilting instabilities and the growth rates are in good agreement when the codes use comparable grids.

Using the MALICE code in the pure Lagrangian mode, the instability has been followed into the nonlinear regime. When the tilt has proceeded about 20° a damping of growth and a corresponding increase in internal energy produced by compressive heating effects are observed. At this point the shortened grid spacings of the Lagrangian mesh produces unacceptably small time steps. Attempts to remedy this by increasing the amount of rezoning have lead to numerical diffusion of the magnetic field, which causes much of the reversed-field region to be lost.

The results of the nonlinear simulation are shown in Fig. 1 where contours of constant plasma density, both initially and at the end of the run, are shown. Note that the displacement is primarily axial, in agreement with the theory of Barnes.[2]

The growth rate $\omega t_H \approx 0.5$ as determined numerically corresponds to an e-folding time of ~2 μs. However, experimental results indicate that such configurations can persist in a stable state for times approaching 30 μs, the discharge being lost due to a rotational n = 2 interchange instability. Recently Schwarzmeier,[7] using the Vlasov-fluid kinetic model, has shown that parallel kinetic effects can stabilize the tilting mode. Additionally, he has shown that shaping the flux surfaces in a racetrack manner can significantly reduce the MHD growth rate. Both effects are probably important experimentally, perhaps accounting for the lack of observation of the tilting mode in FRTP devices.

The n = 2 rotational instability has also been simulated with the MALICE code. The results are detailed elsewhere.[8]

III. REVERSED-FIELD PINCH

For the reasons cited in Sec. I, studies of global MHD activity in the RFP are in approximately the same state as similar studies for tokamak plasmas about five years ago. Much of the activity currently underway is limited to the case of helical symmetry. The thrust of these studies is as follows.

1. Classify Possible Modes. Indications are that the RFP does not simply behave as a modified tokamak, but rather exhibits MHD behavior uniquely its own.

2. Can MHD Activity be Induced by Transport and How Does MHD Activity Affect Transport? Studies with transport codes have shown that initially stable profiles will naturally evolve to unstable states. The major question is: how does the nonlinear evolution of these destabilized modes modify the profiles, and can these effects be modeled in transport calculations?

3. <u>Is Self-healing of Profiles Possible?</u> In tokamaks, sawtooth oscillations result when a profile that is destabilized by transport is stabilized by nonlinear MHD activity. There is evidence that analogous activity can occur in the RFP.

4. <u>Can Dynamo Action Result From Large Scale Modes?</u> The usual theories relate dynamo action to small scale turbulence. In either case, fully 3-D simulations are necessary.

An example[9] that addresses part or all of the above questions will now be given.

A stabilized tokamak profile typically has $q > 1$ on axis. Transport processes peak the current on the axis thus lowering q and eventually introduce the $q = 1$ resonance into the plasma. The nonlinear evolution of the resulting $m = 1/n = 1$ mode (which does not enter the Rutherford regime and saturate at a low level) flattens the current outward, raises q on axis, and removes the $q = 1$ resonance. This self-healed profile then evolves again due to transport process leading the familiar sawtooth oscillation.

In the RFP, on the other hand, since $q \ll 1$ many $m = 1/n \gg 1$ resonances exist even in the stable state (they are shear stabilized, for example). Transport processes again peak the current on the axis causing a decrease in q there and a loss of shear stabilization. This process has been observed in 1-D transport studies, and the nonlinear evolution of the resulting $m = 1/n = 10$ tearing mode has been followed with a 2-D helical resistive MHD code that solves the primitive equations.[10] It is found that the mode undergoes two successive reconnections resulting in a stabilized profile. Like the tokamak, the first reconnection removes the $q = 1/10$ rational surface from the plasma. However, since in this case q is monotone decreasing, this must decrease q on axis. Thus, instead of flattening the current, the first reconnection enhances the overpeaking. The plasma then undergoes a second reconnection that flattens the current, increases q_{axis}, and restores the original resonance to an axisymmetric profile that is stable to the original mode.

The sequence of double reconnection is shown in Fig. 2, where we plot the helical flux surfaces and flow patterns at various times during the evolution of the mode. Time is measured in units of Alfvén transit times. The first reconnection is completed by $t = 37$, and the second reconnection results in an axisymmetric state. This convoluted, unlocalized, nonsymmetric tearing has been independently verified by another code.[11]

One can now envision the self-healed profile again evolving due to transport, resulting in relaxation oscillations reminiscent of the tokamak sawtooth behavior. Indeed, bursts of $m = 1$ activity lasting for tens of microseconds separated by periods on the order of 100 microseconds have been observed in the density signals from ZT-40M. These time scales are consistent with the time scales of the MHD and transport calculations described above.[9]

In light of these results, one can now speculate as to a possible mechanism for the maintenance of field reversal due to large scale modes.[9] It is likely that more than one $m = 1/n \gg 1$ mode will be unstable simultaneously due to the close proximity of the rational surfaces in the RFP. (In the tokamak, the only $m = 1$ mode allowed corresponds to $n = 1$ since $q \geq 1$ everywhere.) It is also likely that these $m = 1$ modes will be preferred over other poloidal mode numbers because they do not enter the Rutherford regime. These modes will interact nonlinearly. For example, the $m = 1/n = 10$ and $m = 1/n = 11$ modes will interact as follows:

0th interaction	1/11	1/10			
1st interaction	0/1	2/21			
2nd interaction	1/12	1/10	1/11	1/9	from 0/1
	3/32	1/10	3/31	1/11	from 2/21

It is likely that the 2/21 mode will be stabilized by FLR effects due to its large toroidal mode number. Thus, the only avenue to the nonlinear generation of $m = 1$ modes (they are "preferred") is by the presence of an active $m = 0/n = 1$ mode that requires a reversal of the toroidal field for resonance. We hypothesize that the q = 0 rational surface may be necessary for the "preferred" spectrum and may be created and maintained by the nonlinear interaction of m = 1 modes.

It is interesting to note that the observed m = 1 density oscillations mentioned previously are accompanied by m = 0 oscillations, and that m = 0/n = 1 flux loop signals are characteristic of the latter stages of the ZT-40M discharge.

To either refute or verify the above hypothesis requires an accurate 3-D resistive MHD code in realistic geometry. Such a code must solve the primitive equations, and must include the effects of compressibility since density fluctuations are observed to be large experimentally. To handle a large number of modes accurately and efficiently, spectral or pseudospectral techniques should be employed. This is crucial since one cannot rule out the importance of small scale turbulence. With the advent of vector computers, and for ease of debugging, such a code should be explicit. Additionally, unconditionally stable explicit pseudospectral numerical algorithms can be constructed.[12] Such a code is currently being written at Los Alamos.

IV. ACKNOWLEDGEMENT

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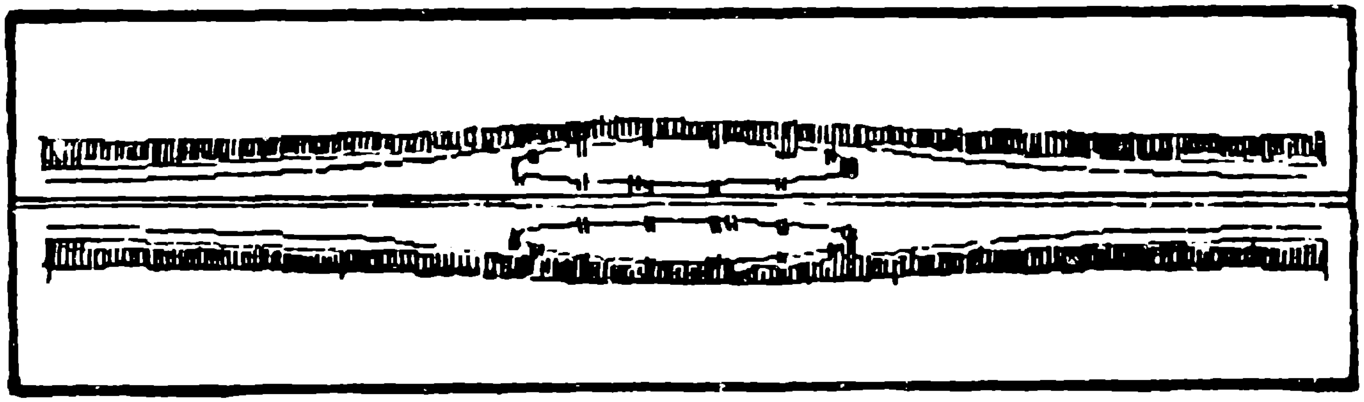
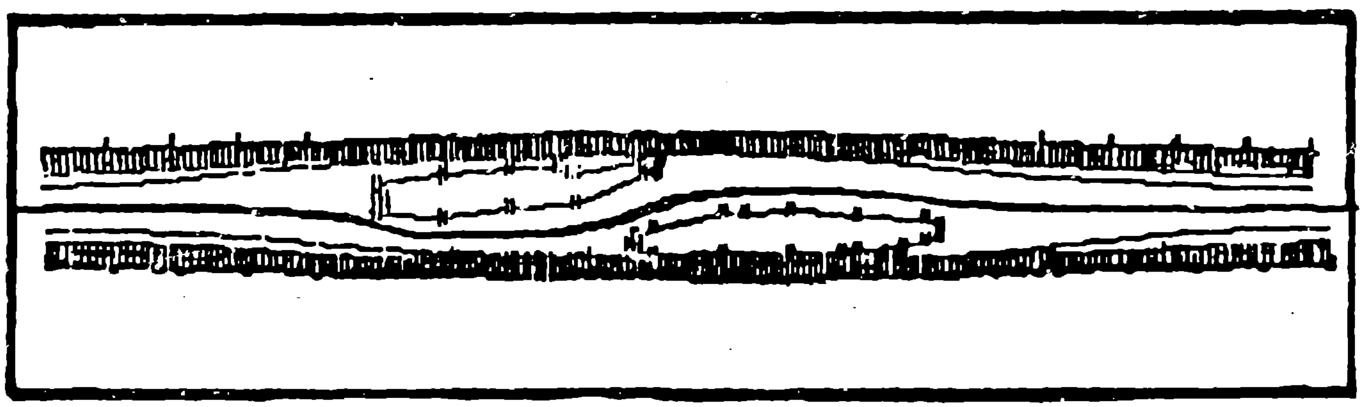
NONLINEAR TILTING MODE IN THE FIELD REVERSED THETA PINCH

T = 0



T = 170



2-D COMPRESSIBLE MHD CODE (D. SCHNACK)[8]

m = 1, n = 10

Helical Flux and Flow Patterns

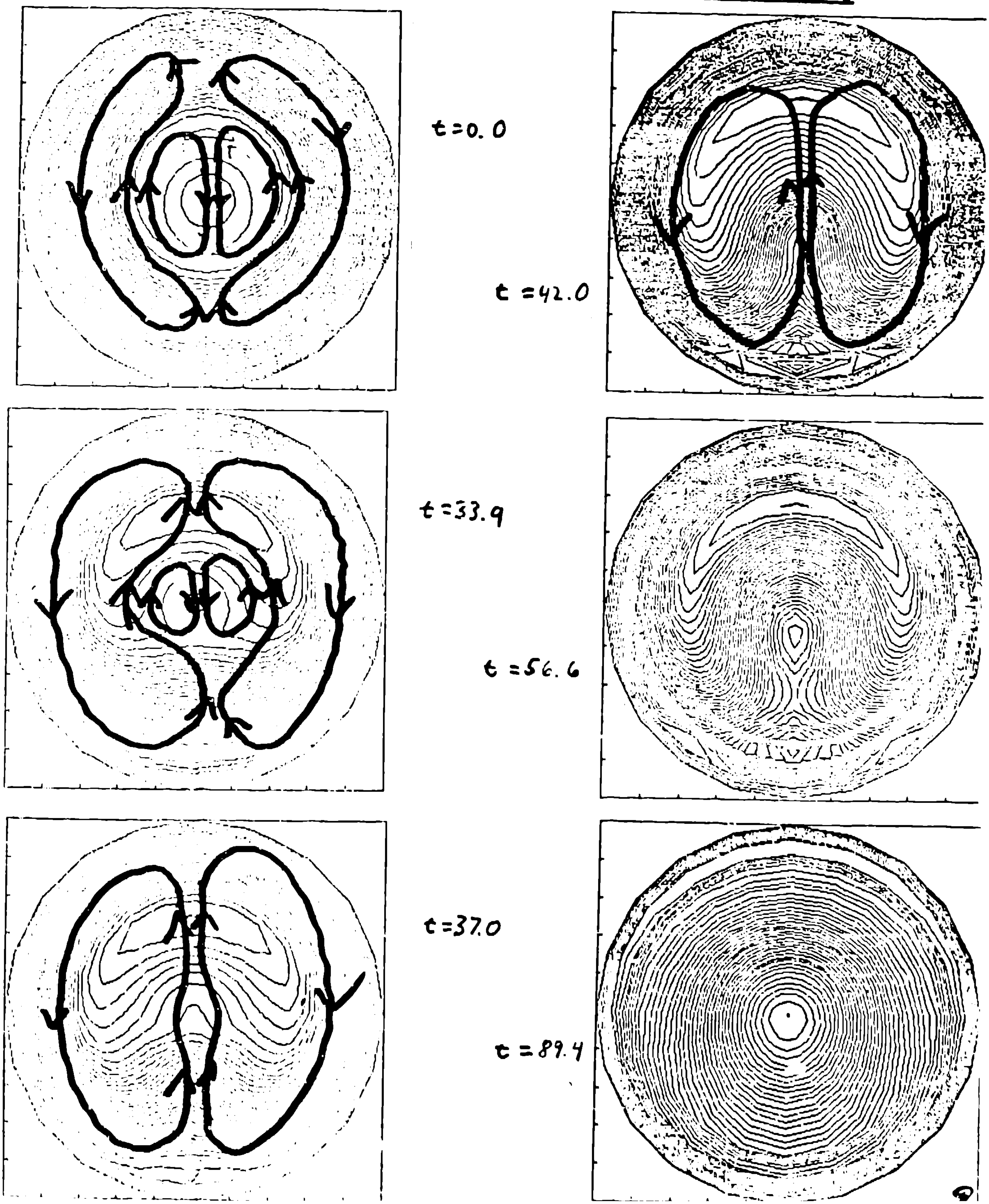


FIG. 4.