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PULSE PROPAGATION IN FREE ELECTRON LASERS WITH A TAPERED UNDULATOR*

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Abstract

The one-dimensional theory of short pulse propagation in free electron lasers is extended to tapered undulator devices and is used to study the behavior of an oscillator with parameter values close to those expected in forthcoming experiments. It is found that stable laser output is possible only over a small range of optical cavity lengths. Optical pulse characteristics are presented and are found to change considerably over this range.

Free electron lasers are currently the subject of intensive investigations at a number of universities and research laboratories in the USA and abroad.[1] The interest has been stimulated by the operation of the Compton regime free electron laser (FEL) oscillator at Stanford.[1,2,3] A greater electron efficiency can be expected from Compton regime FELs which utilize tapered wavelength static magnetic field structures.[4,5,6] As yet, such tapered undulator devices have been studied experimentally only in amplifier configurations[7] although oscillator experiments are planned to operate within the next year. The subject of the present work is free electron laser oscillators with tapered undulators. Two other previous works have recently dealt with this topic, but with somewhat different parameters.[8,9]

The physical situation envisaged is shown in Figure 1. A linear accelerator is the source of short pulses of relativistic electrons which are directed along the axis of an optical resonator. The electrons interact with a pulse of light as both pulses traverse a magnetic undulator placed inside the optical cavity. The electrons are then magnetically guided out of the cavity while the light is reflected from the resonator's mirrors and makes a round trip in time to intercept a fresh pulse of electrons at the entrance to the undulator region. The accelerator produces a new pulse of electrons every T seconds, and, since the electron pulse length is assumed to be much shorter the length of the undulator or the optical resonator, it is clear that such a laser will not function unless the round-trip time of the light pulse, $2L_0/c$, is very nearly synchronized with the electron pulse injection interval T. Thus, one expects that the optical cavity length L_0 must be adjusted until the synchronism condition $T = 2L_0/c$ is met or the light pulse will not overlap the gain medium (electron pulse) in the interaction (undulator) region and therefore will decay away at a rate determined by the losses of the optical resonator. Indeed, experiments at Stanford[1,3] and theoretical calculations show that, for a uniform undulator FEL, stable laser output exists only over a very small range of synchronism and different pulse characteristics occur at various locations along the curve of laser output power versus cavity length (the desynchronism or detuning curve).[8,10,11]

Another basic feature of the physical system under consideration is the relative slippage σ between the electron and optical pulses. The accelerator produces relativistic electrons (20-25 Mev) which, of course, travel slower than the velocity of light. Hence, the electron pulse is slightly ahead of the optical pulse at the entrance to the interaction region (undulator), but slightly behind the optical pulse at the exit of the undulator. The electrons slip backward relative to the light pulse which travels slightly faster. This relative pulse slippage, due to the different velocities of propagation of

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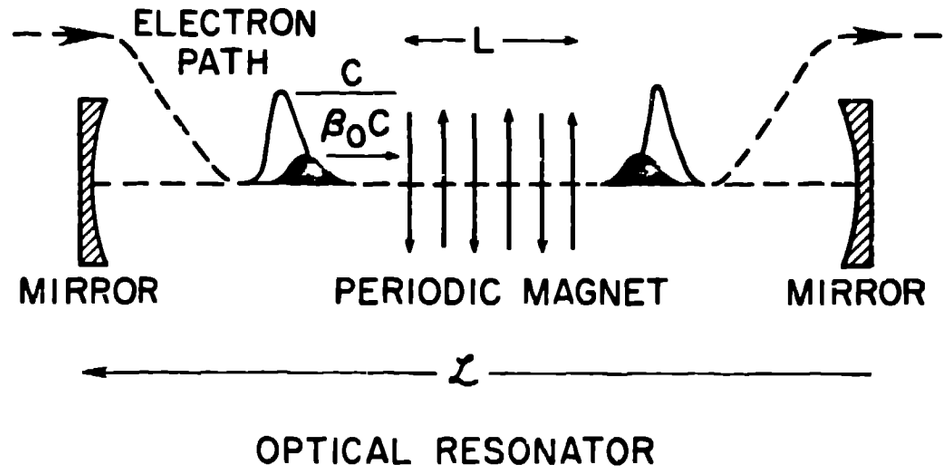

Figure 1: Free Electron Laser Oscillator Configuration.

the light and electron pulses down the axis of the resonator, is an important feature in this problem and distinguishes the FEL from conventional atomic lasers.[18] Since electrons in FELs pass through one magnet wavelength λ_w as one optical wavelength of light λ_0 passes over them, $s = N\lambda_s$ where N is the number of periods in the magnet.

The theoretical model considers one spatial dimension only; the variation of the optical field, electron density, static magnetic field, etc., in any direction transverse to the direction of propagation (resonator axis) is neglected. We shall limit the discussion to systems with helical undulators which therefore result in circularly polarized light. The Compton regime FELs, in which the electron density is small enough to ignore collective (collective plasma) effects, will be considered. The model then self-consistently couples the longitudinal single particle equations of motion of electrons to Maxwell's equations for the optical field.

Some specific parameter values are shown in Table 1. We are attempting to simulate conditions that will be found in the Los Alamos tapered undulator oscillator experiment to be performed in the summer of 1985. In particular, we will assume that the electron beam density is parabolic in shape, and we will present results calculated for two values of the half-width at half maximum, called Δ_1 and Δ_2. Note that $\Delta_1 = \Delta_2$ while $\Delta_2 = 2\Delta_1$. The undulator is taken to be $l = 1$ meter long. Tapering is done in such a way as to keep the dimensionless undulator vector potential, defined in Table 1, constant ($a_w = 0.5$). The magnet wavelength λ_w varies from 2.6 cm at the entrance to 2.0 cm at the exit, a 23% change. The peak current density ρ_0 corresponds to a peak electron current $I_{max} = 40$ amps. The

Table 1: Specific Parameter Values

Electron Beam: $\gamma_0 = 40$

$I_{max} = 40$ amps

$\rho(u) = \rho_0 \left[1 - \frac{1}{2} \left(\frac{u}{\Delta} \right)^2 \right]$

$\Delta_1 = 0.15$ cm ; $\Delta_2 = 0.3375$ cm

$s = N\lambda_s = 0.043$ cm

Undulator: $l = 100$ cm

$a_w = 0.5$

$\lambda_w(z) = 2.6 \left(1 - 0.23 \frac{z}{l} \right)$ cm

$P_w(0) = 200$ [illegible]

Table 2: Single Electron Equations of Motion

$$\zeta = \int_0^z k_w(z')\,dz' + k_s z - \omega_s t$$

$$\frac{1}{\tau}\gamma^2 = 2Lk_s a_w a_s \cos(\zeta + \phi)$$

$$\frac{d}{d\tau}\zeta = \nu = 2\pi L\left(\frac{1}{\lambda_w(z)} - \frac{1 + a_w^2}{2\lambda_s\gamma^2}\right)$$

$$\tau = \frac{ct}{L}$$

$$a_w = \frac{eB_w(z)\lambda_w(z)}{2\pi mc^2} \; ; \; k_w(z) = \frac{2\pi}{\lambda_w(z)}$$

$$a_s = \frac{eE(z)\lambda_s}{2\pi mc^2} \; ; \; k_s = \frac{2\pi}{\lambda_s}$$

initial electron energy is $\gamma_0 = 40$. The undulator magnetic field strength is $B_w(0) = [illegible]$ gauss at $z = 0$. We shall take the total optical cavity loss to be 10% per pass.

The electron equations of motion are shown in Table 2. These are conventional single-particle equations for a tapered undulator system for the energy γ and phase ζ of the electron. The variable τ is a scaled distance such that $\tau = 0$ corresponds to the entrance of the undulator and $\tau = 1$ corresponds to the exit. The right-hand-side of the phase equation gives the electron's resonance condition ν. For the specific system [illegible] discussed below, the resonant ($\nu = 0$) optical wavelength λ_s is about ten microns. The optical field strength $E(z)$ is written as the dimensionless "signal" vector potential a_s.

The optical field has the form $E\cos(k_s z - \omega_s t + \phi)$ and is described in terms of a slowly varying envelope $E(z)$ and phase $\phi(z)$. Under conditions of the SVEA approximation, E and ϕ change appreciably over distances and times large compared to an optical wavelength and period. Maxwell's equations reduce to two first order equations in the variable τ as shown in Table 3. The driving currents on the right-hand-side of these two equations involve averages over electrons labeled by their initial positions ζ_0 and their initial resonance parameters (energies) ν_0. We shall consider examples below for which electrons in the input pulse are monoenergetic.

The indicated averages are to be performed over lengths of the electron pulse which are several optical wavelengths long but which are short compared to the characteristic distance over which the electron density varies appreciably. Although the interaction leads to bunching of electrons on a microscopic (optical wavelength) scale, the macroscopic electron number density ρ is not changed, and thus, the electron pulse propagates undistorted through the undulator region. This is expressed in Table 1 and 2 by making the electron number density ρ a function of the variable u such that the axial position of the density, at the entrance to the undulator, is $u + 1/2n$ while, at the exit of the undulator, it is $u + L - 1/2n$. Here n is the slippage distance which for electrons on

Table 3: Optical Field Equations of Motion

$$\frac{\partial}{\partial\tau}E = -2\pi eLa_w\rho(u)\left\langle\left\langle\cos\left(\frac{\zeta + \phi}{\gamma}\right)\right\rangle_{\zeta_0}\right\rangle_{\nu_0}$$

$$E\frac{\partial}{\partial\tau}\phi = 2\pi eLa_w\rho(u)\left\langle\left\langle\sin\left(\frac{\zeta + \phi}{\gamma}\right)\right\rangle_{\zeta_0}\right\rangle_{\nu_0}$$

$$u = z - L\tau + n(\tau - 1/2)$$

$$n = L(1 - \beta_z) = N\lambda_s$$

resonance ($\nu = 0$), is given by the product of the optical wavelength λ_s and N, the number of periods of the static magnetic field in the length L of the undulator. Hence, one sees directly that the electrons slip back a distance s relative to a point on the optical field envelope.

The method of solution of the model is as follows: The electron number density ρ is taken to be specified on a discrete numerical mesh. The mesh size is supposed to be large compared to an optical wavelength but small compared to the length over which ρ changes appreciably. On a microscopic scale the electrons are initially distributed along a monoenergetic line in (γ, ξ) phase space but uniformly distributed in ξ. The interaction of course changes this distribution, and a different set of simultion electrons is followed dynamically for each different number-density bin of the numerical mesh. Due to the fact that the electron pulse slips with respect to the optical pulse, the simulation electrons of a particular number-density bin will be driven by several different electric field amplitudes and phases during one transit through the undulator. Similarly, the driving current for a particular point in the optical envelope will change as many different number-density bins slip by that point.

We shall look for steady state solutions starting from an initial low amplitude coherent optical pulse. That is, we do not attempt here to calculate the build-up of incoherent radiation in the optical cavity starting from spontaneous emission. Rather, we assume that the amplification process has led to a coherent but low amplitude optical pulse, and we follow its evolution from that point to steady-state. The steady-state solution does not depend on the details of the starting point of the calculation. However, the procedure followed here is directly applicable to a number of proposed oscillator experiments which will begin by injecting a coherent pulse (produced by another laser) into the FEL optical cavity.

The typical evolution of the total optical pulse energy from small-signal conditions to steady-state is shown in Figure 2. In this case, steady-state is reached after about 500 passes. The desynchronism parameter $d = \delta L/\Delta$ where the cavity length is $L_0 + \delta L$, and $\delta L < 0$ corresponds to a shorter cavity. For this case d was relatively large and negative, and the pulse envelope at steady-state is shown in Figure 3. One sees a rather low intensity pulse that is broad in z. The intensity abruptly terminates on the right side because the "window" of the calculation includes only optical fields which can overlap some of the electron density; that is, the light to the right of the last plotted point in Figure 3 would never intercept any electrons and thus would see only the cavity losses. The actual intensity would therefore diminish exponentially from the last plotted point of Figure 3 at a rate determined by the cavity losses.[8] The spectrum of this pulse shown in Figure 4 is a narrow spike peaked at the wavelength of maximum small-signal gain.

If one reduces the magnitude of the desynchronism parameter d (physically by adjusting δL), one can find the type of optical pulse energy evolution shown in Figure 5. Instead of reaching a fixed steady-state, the pulse's energy exhibits limit cycle behavior. In this case, one finds that the pulse shape evolves as a function of the number of passes through the undulator from a high energy single-peaked structure to a lower energy double-peaked one and then back to the single-peaked form, as shown in Figure 6.

If one adjusts the cavity length change δL, or desynchronism d, to very small but negative values, the stable pulse energy is at its maximum value. However, as shown in Figure 7, for our specific choice of parameter values, a very complicated pulse envelope can develop. The degree of complexity of the steady-state pulse envelope seems to be correlated with increasing values of the electron current. That is, smaller currents lead to simpler steady-state pulse profiles. The complex shape shown in Figure 7 has a correspondingly broad Fourier spectrum. Note that the figure corresponds to the "long" pulse case ($\Delta_1 = 4s$); the shorter pulse case reaches a somewhat simpler shape.

The results of the pulse calculations are summarized in Figure 8. We find desynchronism curves for the tapered undulator FEL oscillator similar to that calculated[10,11] and observed[13] for the untapered Stanford case. Qualitatively, the widths of these curves are narrower than for the Stanford case, and one notes from Figure 8 that the longer electron pulse (Δ_1) has the wider tuning curve, as one intuitively expects. The pulse evolution is qualitatively different at different points along the tuning curves; at large negative values of d (or δL), a smooth pulse, broad in time and narrow in spectral width, is found. At smaller $|\delta L|$, a limit cycle behavior is found in which the optical pulse energy and profile change periodically with the pass number. At very small $|\delta L|$, maximum pulse energy is reached, but, for our parameters, very complicated pulse shapes, with temporally narrow features, are generated. These pulses are spectrally very broad.

In conclusion, our results show qualitative similarity between the pulse behavior of an oscillator with a moderately tapered undulator and that of the untapered undulator Stanford oscillator. The results are also in qualitative agreement with a previous calculation on

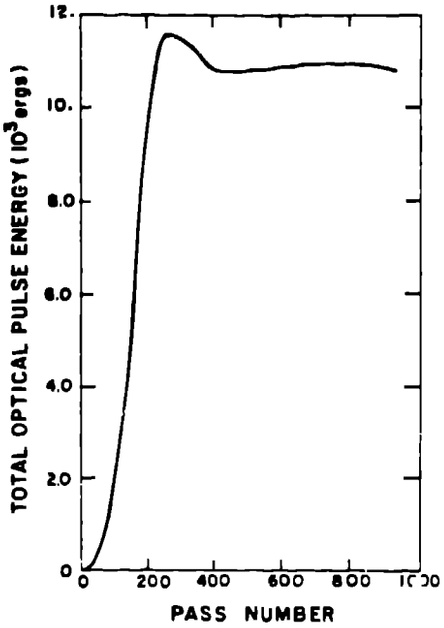

Figure 2: Total Optical Pulse Energy vs Pass Number.

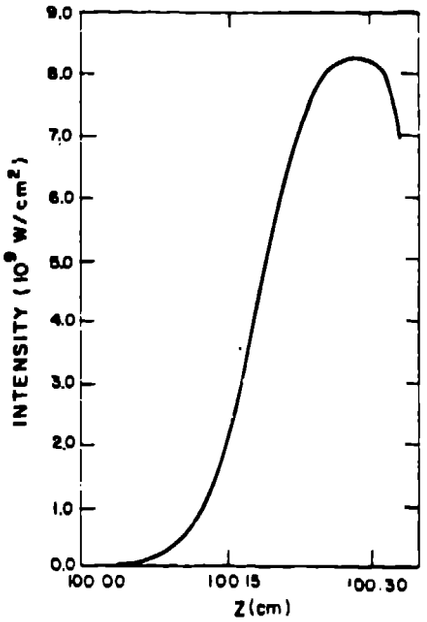

Figure 3: Steady-State Intensity Profile at the End of the Undulator.

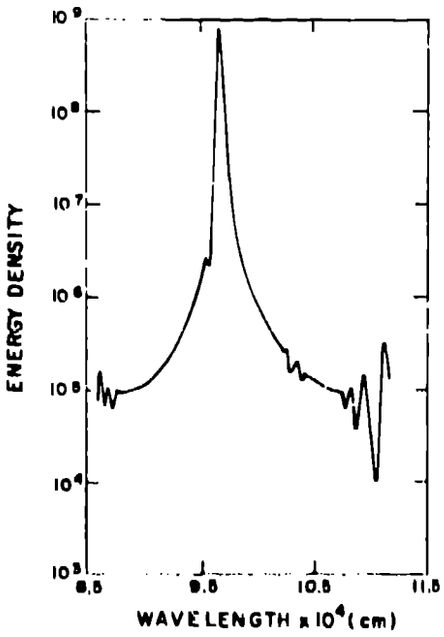

Figure 4: Intensity Spectrum of Steady-State Pulse.

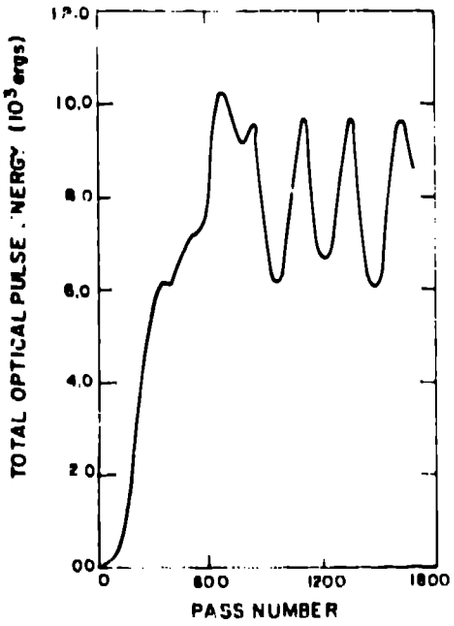

Figure 5: Total Optical Pulse Energy vs Pass Number.

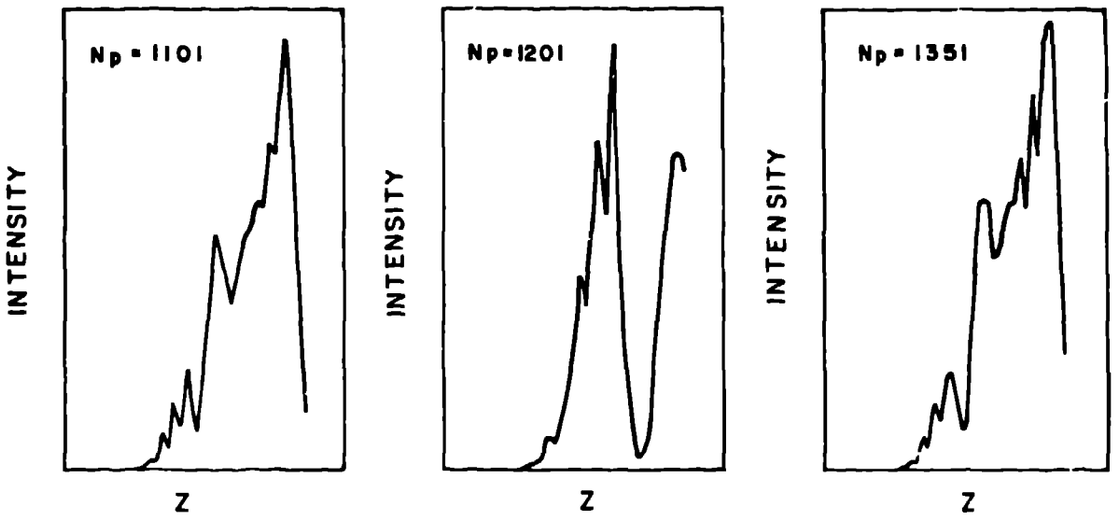

Figure 6: Limit Cycle Intensity Profiles.

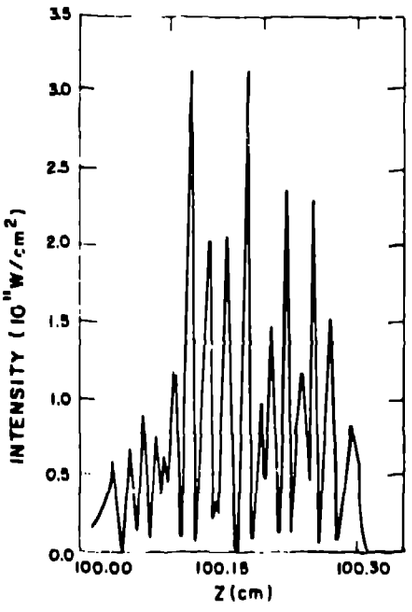

Figure 7: Steady-State Intensity Profile at the End of the Undulator for the Long Pulse Case.

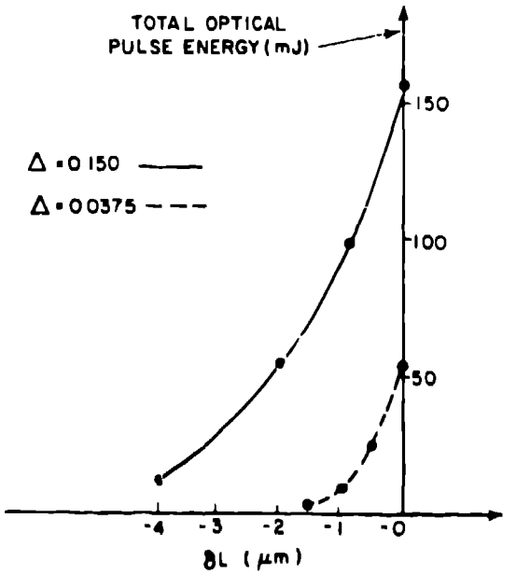

Figure 8: Desynchronism Curves for Tapered Undulator Oscillator.

the tapered undulator.[8] Desynchronism curves exist in both cases in that stable laser output is possible only over a small range of cavity lengths which detune the FEL away from exact synchronism. For large currents, as taken in our examples here, very complex pulse shapes may be generated. The dependence of the results on various other parameters of the problem (such as the cavity losses) is currently under investigation.

Acknowledgement

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