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DIPOLE APERTURE AND SUPERCONDUCTOR REQUIREMENTS*

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Summary

The cost of an accelerator is not proportional to the aperture. A change in aperture by a certain percentage results in an overall accelerator cost change by only a fraction of that percentage; the fraction may be between 0.1 and 0.5 and is almost independent of the bending field. This estimate is obtained by analyzing the superconductor requirements as a function of aperture and by making rough estimates of the largest cost items of the accelerator such as magnets and ring tunnel.

Introduction

The aperture of a Superconducting Super Collider (SSC) should be small to keep the accelerator cost down, but large enough to accommodate the beam without undue inconvenience. The main component of the increase in cost with aperture is the cost of superconductor. We can reasonably predict the amount of superconductor needed as a function of aperture. From this information we can make a rough estimate (guess is perhaps the better word) as to the effect of an aperture change on overall accelerator cost.

Amount of Superconductor Needed for Bending Magnets

Although it is not possible to develop reliable cost estimates for the magnets without specifying a concrete design, we can gauge the relative cost changes with aperture by considering the amount of superconductor required. The superconductor is a significant component of the bending dipoles; it may account for as much as one-fourth to one-third of the cost of the complete magnet system and may well be the largest single cost item for an SSC. Other components of the magnets are iron; cryostat; and fabrication, including quality control.

Single-Layer Approximation

Consider a typical dipole cross section as shown in Fig. 1. A winding space of thickness d, between radius r_i and r_o, with a current density $j = j_0 \cos\varphi$ produces a field[1]

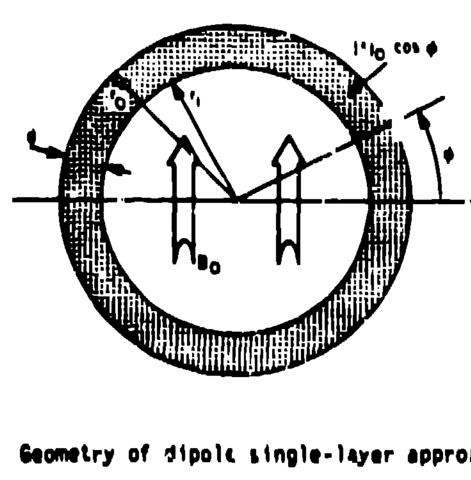

Fig. 1. Geometry of dipole single-layer approximation.

$$B_0 = \frac{\mu_0}{2} j_0 d \quad . \tag{1}$$

The current density is related to the critical current density in the superconductor at the field B_0:

$$j_0 = \lambda \cdot j_c(B_0) \quad ,$$

where the factor λ accounts for the space in the winding that is not filled with superconductor, but with things such as insulation, necessary force-containing structure, and (mainly) the copper (or aluminum) used for stabilizing purposes. The range $0.2 < \lambda < 0.4$ is available in modern designs. The thickness of the winding necessary to produce B_0 with a given superconductor is

$$d = 2\frac{B_0}{\mu_0 j_0} = 2\frac{B_0^2}{\mu_0 \lambda F_p} \quad , \tag{2}$$

where $F_p = j_c \times B$ is the flux pinning strength, an important critical property of the superconductor; F_p usually has a flat maximum near half the critical field.

The volume of winding per unit dipole length is

$$V_w = 2\pi(r_i + \frac{d}{2})d \quad , \tag{3}$$

but the volume of the superconducting material itself is smaller by λ and a factor $2/\pi$ because of the wire density necessary to produce the $\cos\varphi$ distribution in current density; thus, we find that

$$V_{sc} = 4(r_i + \frac{d}{2})d\lambda = 8\frac{B_0^2}{\mu_0 F_p}\left(r_i + \frac{B_0^2}{\mu_0 \lambda F_p}\right) \quad . \tag{4}$$

The superconductor volume consists therefore of two terms: the first is due to the aperture r_i, and the second is due to the current density (or flux pinning strength) determining the thickness of the winding.

To obtain the total superconductor volume for the SSC, we multiply V_{sc} times $2\pi\rho$, where $\rho = (E/c)/B_0$ is the bending radius for the beam energy E. Thus,

$$V_{sc(total)} = 4.10^{7}\left(\frac{r_i}{j_c} + \frac{B_0}{\mu_0 \lambda j_c^2}\right) \quad , \tag{5}$$

with E in eV, B in tesla and other units also in the mks system.

To illustrate this relationship, we may choose some representative superconductors:

A: NbTi at 4.2 K, $B = 5$ T, $j_c = 2 \times 10^9$ A/m^2;

B: NbTi at 1.9 K, $B = 8$ T, $j_c = 2.3 \times 10^9$ A/m^2;

C: Nb_3Sn at 4.2 K, $B = 10$ T, $j_c = 1.25 \times 10^9$ A/m^2;

and plot $V_{sc(total)}$ for $E = 20$ TeV versus r_i in Fig. 2.

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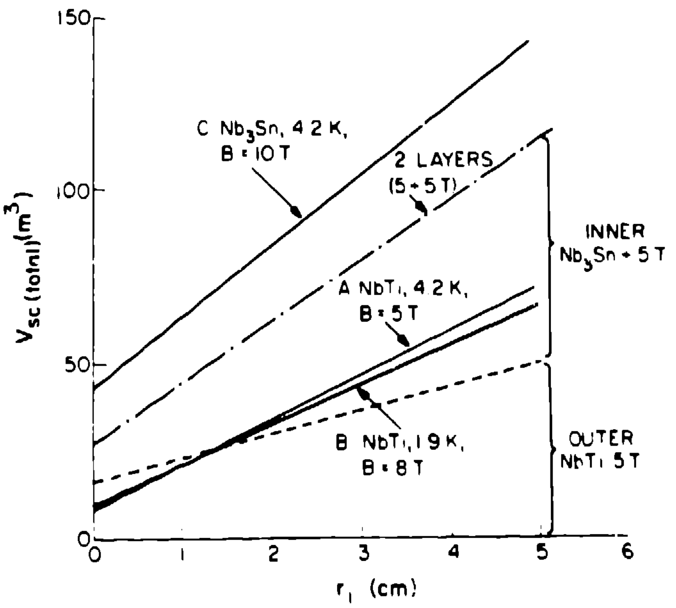

Fig. 2. Plot of the total volume of superconductor versus aperture for the bending magnets of a 20-TeV accelerator. Examples are for 5-, 8-, and 10-T single-layer windings and for a 10-T two-layer winding, each layer producing a 5-T increment (see below), all with λ = 1/3.

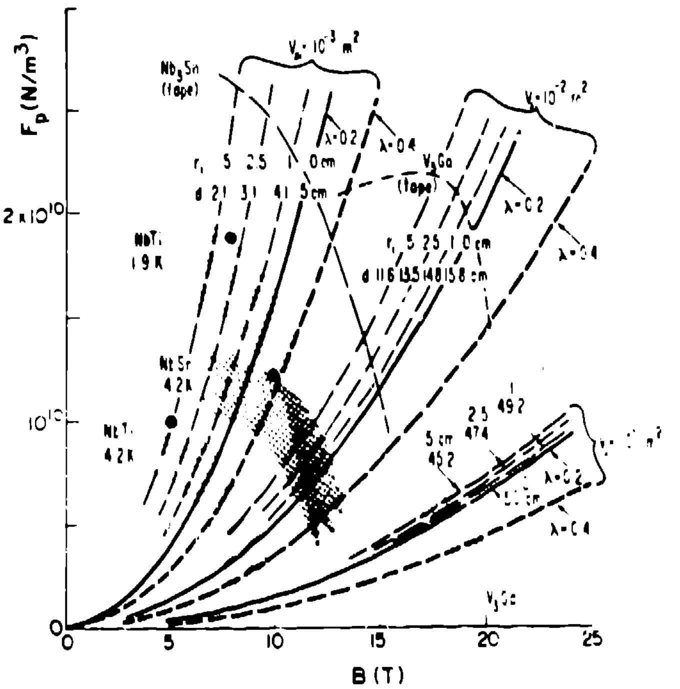

Fig. 3. Required flux pinning strength F_p for dipole windings of field B with a given superconductor volume V_{sc} per length of dipole. Several apertures for λ = 0.2 are indicated; for λ = 0.4, only r_i = 0 is given for comparison. The shaded regions give F_p (averaged over the noncopper cross section) of available multifilament conductor and F_p of a tape conductor for comparison. The points are the three examples of Fig. 2.

The general features are obvious: zero aperture is not gratis, and the superconductor volume is not just proportional to the aperture. Thus, we see from the figure that a change from a comfortable r_i = 3 cm to an uncomfortable r_i = 1.5 cm may save rather less than 30% in superconductor volume.

The other components that contribute to the magnet cost have the same feature: a significant cost is incurred even for r_i = 0 and then the additional cost is approximately proportional to r_i. This is notably so for the iron shield (whose cost may be between 10 and 50% of the superconductor cost, depending on field and construction), and also for the cryostat (having a high fixed cost and a very small increase with r_i). For superferric magnets, the iron cost dominates.

Influence of Current Density

From Eq. (4) or (5), we see that the relative importance of aperture depends on current density. A high current density reduces the overall superconductor requirement by reducing the winding thickness term inside the bracket and the multiplier in front of it; in Fig. 2, the intercept for r_i = 0 decreases like j_c^{-2}; the slope of the curves decreases like j_c^{-1}. In a good magnet design, the winding thickness is kept as low as possible, however, the limits are set by the highest flux pinning strength in available superconductors.

The dependence of V_{sc} on current density is best illustrated with the help of Fig. 3, where F_p versus B_0 is given. The loci of coils containing V_{sc} = 10^{-3} m^3/m and also those with 10 and 100 times larger V_{sc} are plotted. The solid line is for λ = 0.2, the dashed line for λ = 0.4, both for r_i = 0. The three thin, dashed lines are for λ = 0.2 and r_i = 1, 2.5, and 5 cm. The winding thickness d corresponding to the given V_{sc} is entered for each curve. In addition, we also enter the available ranges of F_p versus B for various superconductors.[2]

From this plot, we can estimate the practicability of reaching high fields in superconducting dipoles. The coil costs and the design complications increase with V_{sc} and with d; in fact, one would like not to exceed V_{sc} = 10^{-3} by more than a factor 2 or 3. The importance of high current density (high F_p) is evident.

The bands on F_p versus B in Fig. 3 of multifilament conductors indicate available material; it may be that in the future the upper limit can be raised somewhat,[3] but is unlikely to reach the performance levels obtainable with tape. F_p is averaged over the noncopper (that is, nonstabilizer) cross section of the wire, and the multifilamentary process for A-15 material always leaves a substantial amount of inert material in the form of bronze and diffusion barriers behind; for tape these components are not necessary. The current densities and fields chosen for the examples in Fig. 2 reflect present state of the art.

Superconductor Cost

The cost of the superconductor is

$$C = V_{sc}\, p_s = 8\, \frac{B_0^2}{\mu_0}\, \frac{p_s}{F_p} \left(r_i + \frac{B_0^2}{\mu_0 \lambda F_p} \right) \quad , \qquad (6)$$

where p_s is the price per volume of superconductor including fabrication and stabilizing copper, etc. At present, the approximate (large quantity) values of p_s are for NbTi: 3 M\$/m^3; for Nb_3Sn multifilament: 5 M\$/m^3; (V_3Ga might cost 10-15 M\$/m^3).

It is seen from the formula that the cost is improved by reducing the ratio p_s/F_p. This means that an improvement in F_p by methods that also increase p_s is not of great help; this is why NbTiTa need not be considered. The values of p_s/F_p are for NbTi at 4.2 K, 5 T: 3×10^{-4} \$/N; at 1.9 K, 10 T: 2×10^{-4} \$/N; for Nb_3Sn multifilament: 5×10^{-4} \$/N. The cost unit \$/N indicates that the job of the magnet windings is to restrain the Maxwell tensor of the produced field.

Manufacturers often give the conductor price p_c in \$/kAm at a given field B. Multiplying p_c by j_c (critical current density in the noncopper cross section of the conductor, in kA/m2) one obtains p_s in \$/m^3; dividing by B [T] gives p_s/F_p in \$/kN.

The winding thickness should not be too large. With r_i = 2.5 cm, the three examples chosen for Fig. 2, have according to Eq. (4), the following V_{sc}: A: 4.93×10^{-4} m^2; B: 7.4×10^{-4} m^2; C: 2.25×10^{-3} m^2.

The actual winding cross section is larger by a factor 4.7 (π/λ); that is, the ratio of winding cross section to aperture becomes 2.4×10^3 V_{sc} and is for A: 1.2; for B: 1.8; for C: 5.4.

When the winding is too thick, the single-layer approximation becomes inadequate. The superconductor being chosen for the highest field is overdimensioned. There is substantial savings in splitting the winding into high- and low-field sections.

Dipoles with Several Layers

The winding space is subdivided into n layers. Counting the layers, starting from the outermost, layer #1 is between r_o and r_1, and layer #k is between r_{k-1} and r_k with $r_n = r_i$. Each layer creates a field increment ΔB_k. The maximum field (that is, B_o in the aperture) is the sum of all the increments. The thickness of layer k is

$$r_{k-1} - r_k = 2 \frac{\Delta B_k \cdot B_k}{\mu_o \lambda F_{pk}(B_k)} \quad , \tag{7}$$

where F_{pk} is characteristic for the superconductor in layer k. (In this example the field distribution is simplified and taken as independent of angle φ. In reality the field is larger toward the poles and smaller at the equator.) Taking the simple case where all increments ΔB_k are equal and F_{pk} the same in all layers, we get

$$B_k = k\Delta B \ , \quad \text{and} \quad \Delta B = \frac{B_o}{n} \quad ; \text{therefore,}$$

$$d = \sum_{k=1}^{n} (r_{k-1} - r_k) = \frac{2(\Delta B)^2}{\mu_o \lambda F_p} \Sigma k$$

$$= \frac{B_o^2}{\mu_o \lambda F_p} \frac{n(n+1)}{n^2} = \frac{B_o^2}{\mu_o \lambda F_p} \left(1 + \frac{1}{n}\right) \quad . \tag{8}$$

In the limit of large n, the thickness is only one-half of the single-layer thickness from Eq. (7). In practice, one may reach a reduction of 0.6 or 0.7, and n = 3 or even n = 2 is sufficient. The example of a two-layer coil illustrates this.

Two-Layer Winding

The thickness of the outer and inner layers are

$$d_1 = r_o - r_1 = 2 \frac{B_1^2}{\mu_o \lambda_1 F_{p1}(B_1)} \quad ,$$

and (9)

$$d_2 = r_1 - r_i = 2 \frac{\Delta B \cdot B_o}{\mu_o \lambda_2 F_{p2}(B_o)} \quad .$$

For the respective superconductor volumes, we have [see Eq. (4)]:

$$V_{sc1} = 4(r_i + d_2 + \frac{1}{2} d_1) d_1 \lambda_1 \quad ,$$

and

$$V_{sc2} = 4(r_i + \frac{1}{2} d_2) d_2 \lambda_1 \quad .$$

Taking NbTi for the outer and Nb_3Sn for the inner layer with the following values:

$F_{p1} = 10^{10}$ N/m^3; $F_{p2} = 1.25 \times 10^{10}$ N/m^3;

$\lambda_1 = \lambda_2 = 0.333$; $B_1 = 5$ T;

$B_o = 10$ T; $\Delta B = B_o - B_1 = 5$ T;

we obtain the volumes as entered in Fig. 2. For r_i = 2.5 cm the Nb_3Sn layer has a thickness of d_2 = 1.9 cm, the NbTi layer of d_1 = 1.2 cm, and the outer radius is r_o = 5.6 cm. The ratio of winding thickness to aperture is now down to 4 (from 5.4) for the single-layer, all-Nb_3Sn winding, that is, a 25% savings in superconductor volume. However, the cost savings is larger because approximately half of the winding is now replaced with the less expensive NbTi conductor. The cost reduction is approximately 40%.

More on Costs

Having discussed the influence of the aperture on the total amount of superconductor needed for the accelerator, we now ask: how large a fraction of the total cost of the accelerator goes for superconductor? In answer to this question, the cost of bending magnets and of the ring tunnel has to be considered; both, individually, are dependent strongly on field strength but their sum is much less so.

Magnets

Any accurate estimate of total magnet cost needs more detailed design information than is now available. The two discusssions of this subject by Hassenzahl differ so widely from each other that they can only be taken as a very pessimistic (1981)[4] and very optimistic (1982)[5] prediction. My own guess is that the cost per T.m for 2 T < B_o < 10 T dipoles will be roughly proportional to B_o and, for r_i = 2.5 cm, will be approximately 0.6 k\$/T.m at 2 T and 2.5 k\$/T.m at 10 T. (For a 20-TeV accelerator, 4.2×10^5 T.m are needed.) Alternately, the magnet cost can be taken as 3-4 times the cost of superconductor, using the volume of superconductor from Fig. 2. Between the two recipes, we arrive at the following brackets for the total bending magnet costs: at 10 T: 1.0-1.9 G\$; at 8 T: 0.3-0.8 G\$; at 5 T: 0.35-0.5 G\$; at 2 T: 0.25 G\$.

Tunnel

The other major cost item of the accelerator is the tunnel. It is inversely proportional to B. Assuming a tunnel length of $1.3 \times 2\pi\rho = 8.2\ (E/c)/B$, and a price bracket of 2-8 M\$/km (the upper limit being probably closer to a realistic cost), we arrive at costs of tunnel + bending magnets in the following brackets: for 10 T: 1.1-2.3 G\$; for 8 T: 0.45-1.35 G\$; for 5 T: 0.6-1.4 G\$; for 2 T: 0.8-2.4 G\$.

Aperture Relative to Total Accelerator Cost

It should be understood that the cost brackets given here are purposely quite wide. No attempt is made to include any refinements or complications, such as the more esoteric technology of 8-T, 1.9-K magnets with their smaller stability margin and increased cryogenic requirements, or the higher breakdown reliability requirements necessary for the lower field magnets to avoid excessive down-time of the accelerator. The given cost brackets are not sufficiently narrow to favor any particular field; although, they seem to indicate that the cost optimum will be found in the middle rather than at the high or low extremity of the field range.

We may assume that the cost of tunnel + bending magnets represents about three-fourths of the total accelerator cost. Thus, a change of 20% in r_i starting from $r_i = 2.5$ (according to Fig. 2, and with the use of above cost brackets and assumptions) results in a total accelerator cost change almost independent of the bending field. The change is 4-10% for 10 T; 2.5-11% for 8 T; 3-10% for 5 T and 2-5% for 2 T (having, in this latter case, assumed a change with aperture in magnet cost of 80 M\$/cm).

Conclusion

In conclusion one can say that in the range of interest, a change in aperture diameter by a certain percentage will change the total cost of the accelerator by between a tenth and half that percentage only. The pressure to choose an uncomfortably small aperture is, therefore, not as strong as might have been assumed.

References

1. M. N. Wilson, Superconducting Magnets, Ch. 3.2, (Clarendon Press, Oxford, 1983).
2. Physics Vademecum, ed. H. L. Anderson, Sec. 7.02, (Am. Inst. Physics, NY 1981).
3. D. C. Larbalestier, "Superconducting Materials for Particle Accelerator Magnets," IEEE Trans. Nucl. Sci., NS-30, 3299-3303 (1983).
4. W. Hassenzahl, "A Comparison of the Costs of Superconducting Dipoles using NbTi, Nb_3Sn and NbTiTa," IEEE Trans. Nucl. Sci., NS-28, 3277-79 (1981).
5. W. Hassenzahl, "Cost of High-Field Nb_3Sn and NbTi Accelerator Dipole Magnets," paper LH-14, 1982 Applied Superconductivity Conference, Knoxville, Tennessee.