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IS LOCAL EQUILIBRIUM A USEFUL CONCEPT IN HADRONIC INTERACTIONS?*

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ABSTRACT

Aspects of multiparticle production phenomena are reviewed, which bear on the existence of local equilibrium in all or part of a collision event. Several universal features of purely hadronic events, such as the $p_\perp$ distribution of secondaries, the independence of multiplicities and multiplicity distributions on the quantum numbers of the colliding particles are easily interpreted by postulating the existence of local thermodynamic equilibrium for the dominant non-diffractive events. Except in the case of the multiplicity distribution, other interpretations often do not exist. Equilibration mechanisms which might establish local equilibrium are examined. We point out that several mechanisms besides the usual kinetic relaxation have not been seriously studied. These include collective instabilities, turbulence and chaos, which could be more effective in establishing equilibrium. Developments in the use of the hydrodynamic model are reviewed, with particular attention to the initial conditions appropriate to hadronic and nuclear collisions. We conclude that local equilibrium is indeed a useful concept but that much effort is needed to assess its accuracy and domain of applicability.

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1. INTRODUCTION

Old (and mostly unanswered) questions in classical strong interaction physics have been sharpened by new results from the CERN $p\bar{p}$ collider. Fascinating trends in total, elastic, diffractive and differential cross sections demand explanation. Old (non)truths about scaling and constant rapidity plateaus already suspect to some in 1973 are being quickly forgotten. Although most theorists believe QCD to be the theory of strong interactions, most of the evidence comes from electroweak probes of hadrons, together with a lot of circumstantial evidence. Only the successful quantitative theoretical calculation of hard jet production rates at 540 GeV stands as solid support for QCD in purely hadronic events (to within a factor of two, say). In the meantime, the remaining 99.9% of events are outside the reach of QCD technology of 1984. For this reason it seems appropriate to adopt a somewhat more phenomenological approach, hoping that nature will provide hints not easily seen from a "first principles" calculation. We shall suggest that the flexible and powerful techniques used to describe stochastic behavior in statistical physics can be adapted to a new phenomenology of strong interaction, such that the structure of these equations and results obtained guide the way in which the QCD (or other?) equations should be cast.

Particle production events are quite dramatic. When two high energy hadrons collide at high energy, much of the initial kinetic energy is converted into many (say dozens) of final hadrons. All of this takes place in a very small region of space-time. This circumstance presents problems for each of the extremist techniques of collision theory: (1) the perturbative world view and (2) the thermodynamic world view. The required program should extract insights from these views without getting trapped by their known limitations.

QCD indicates the presence of many more field degrees of freedom than would have been contemplated twenty years ago. In addition the zero mass of the gluon field suggests the possibility of collective, possibly chaotic behavior difficult to intuit from a collisional, perturbative approach. The framework in which equilibration can occur is much improved when one studies this aspect of QCD.

It is impossible here to outline the history of applications of statistical mechanics and hydrodynamics to particle production.[1] Beginning with Heisenberg, Fermi, Pomeranchuk and Landau, we find important contributions from the Russian school (Khalatnikov, Feinberg, Bilenkii, Emelyanov, Milekhin, ...) and Japanese schools. Beginning in the '60's the main defender of the faith was Hagedorn and his influential school. The reader should consult Hagedorn's fine history[2] of the statistical bootstrap model presented at Helsinki to understand these developments.

It has always been interesting to study the properties of bulk hadronic matter, beginning in modern times with the Bethe-Breuckner theory of nuclear matter and continuing to contemporary speculations on early universe dynamics. Recent attempts to calculate the thermodynamic properties of extended QCD matter are hoped to apply to ultrarelativistic heavy ions.

In the next section we shall be more daring, however. We ask which features of hadron-hadron events can be easily understood <u>if</u> substantial thermodynamic equilibrium were established. Then we look for less restrictive explanations, when such exist. Section 3 discusses equilibration mechanisms. In Sec. 4 developments in the hydrodynamical model are reviewed. Throughout we emphasize the tentative nature of many interpretations. An attempt is made to identify promising theoretical approaches which will improve our understanding of multiparticle production.

2. FEATURES OF HADRONIC COLLISIONS WHICH CAN BE INTERPRETED BY LOCAL EQUILIBRIUM CONCEPTS

Certain features of hadronic collisions point to global, model independent explanations for which statistical interpretations are natural. In all such applications it must be understood that only a certain fraction of the incident energy is available for thermalization, since the "leading particles" carry away roughly one-half of the initial energy on the average. Some physicists persist in believing that this situation, contradicting the original Fermi-Landau assumption of total stopping and instant thermalization, discredits the picture altogether. Others imagine this effect to indicate a transparency of the proton incompatible with equilibration. We have, of course, learned that the hadron is a structured object, whose components behave differently when colliding with another object. Elsewhere we have discussed[3] how QCD provides support to the Pokorski-van Hove model,[4] wherein to first approximation the valence quarks pass through while the confining glue clouds interact more strongly, leading to hadronization of about one-half the kinetic energy of collision. Apart from modelistic scenarios, however, there are impressive phenomenological facts whose behavior looks very much like that expected of thermalized systems.

2.1 Universality of Hadronic Multiplicities.

In contrast to cross sections, which vary according to the nature of the projectile and target (e.g., πp, $\bar{p}p$, Kp, ...) the multiplicity of detected (usually charged, for practical reasons) particles depends only on the c.m. energy of the collision and not on the species. Or if you like, the valence quark composition of projectile and target are not relevant. Curiously enough, once the leading particle energies and preexisting charges in hadron-hadron collisions are removed, the mean charged multiplicity $\bar{n}(W_{had})$ expressed as a function of the energy available for nondiffractive hadronization coincides[5] with that seen in e^+e^- annihilation to hadrons. Failure to account for this effect continues to slow progress even in the study of bulk multiplicities. One has to admit also the presence of conceptual and practical ambiguities in attempting to separate "leading" particles, diffractive events, and so forth.

The traditional way of interpreting universal behavior as a function of the total accessible energy is to assume that the system in question has reached thermodynamic equilibrium. The Fermi-Landau statistical-hydrodynamical model (SHM) gives an elementary and almost too successful prediction of the magnitude and energy dependence of the multiplicity. Recently we reexamined[3] this calculation using a

QCD interpretation of the Pokorski-van Hove model. We envisioned a fraction of the initial energy (actually the gluon energy) to be available for nondiffractive hadronization. From Landau's ingenious entropy argument, the final multiplicity can be related to the initial entropy (subject to corrections due to viscosity and possible latent heats at first-order phase transitions). The entropy of the initial system is

$$S_i = \frac{2\pi^2}{45}\left(\frac{30}{\pi^2}\right)^{3/4} (N_d V_i)^{1/4}\, W_{had}^{3/4} \tag{1}$$

where N_d is the effective number of degrees of freedom and V_i the volume at which initial thermalization occurs. Using free particle thermodynamics, one has $N_d = 37$ for two active flavors, 47.5 for three.

The original SHM assumed rapid equilibration in the Lorentz contracted Fermi-Landau volume $V_i = (4\pi/3 \cdot 1/m_\pi^3)/\gamma$ with $\gamma = W/2M_p$ for pp collisions. Computing the charged multiplicity in a pion-dominated model gives

$$N_{ch} \simeq \frac{2}{3} \times \frac{1}{4} S_i = 1.23(N_d f)^{1/4}\, W_{had}^{1/2} \quad . \tag{2}$$

Taking $f = W/W_{had}$ as in the original version gives an f dependence seemingly ruled out by the data of Basile <u>et al</u>.[6] However, if f really refers to the gluon component a constant f (not yet predictable from theory) representing the initial geometry of the gluon cloud, the agreement is surprisingly good in the absence of adjustable parameters, a point made to me by R. Weiner.[7] For $f \simeq 0.4$ appropriate to ISR data, we find

$$N_{ch} \simeq 2.41\, W_{had}^{1/2} \tag{3}$$

to be compared with our earlier X^2 fit[8] $N_{ch} \simeq 2.23\, W_{had}^{0.46}$.

Why has this seeming triumph been ignored so totally? First, as mentioned, the confusion over the proper energy to use has led to a casual dismissal of what is known as the "$s^{1/4}$ law." Secondly, there are legitimate questions about the extremity of the "instant-equilibration" initial condition. Finally, there is a sociological trend to prefer those latest calculations whose shortcomings may not yet be visible. As an example, we mention the jet calculus multiplicity formula $n(Q^2) \sim A \exp(B\sqrt{\ell n Q^2/\Lambda^2})$. As we showed earlier,[8] this expression closely tracks the F-L formula (3) over all accessible energies. To keep it from contradicting data (assuming it to be applicable to the hadronic case), one has to make the same energy deduction as used above. Yet no one criticizes the QCD formula!

In the foregoing we assumed that the energy $(1-f)W$ kept by the leading or diffractively excited particles was constant. Such is not the case at all, nor is the average $\langle f\rangle$ necessarily constant as a function of energy. This point was dealt with in the talk by Plümer[9] (see also refs. 10 and 11).

2.2 Universality of Hadronic Multiplicity Distributions in Hadron-Hadron Collisions

Large fluctuations are observed in the number of produced hadrons when observed event by event. As in other fields of science, the probability (or frequency) of a given number is indicative of the statistics of the emitting system. In this case, too, the shape of the probability distribution P_n is believed to be informative with regard to the statistical nature of the hadronization process.

Some time ago Koba, Nielsen and Olesen[12] (referred to as KNO in the sequel) proposed a simple way to remove the energy dependence of the probability distribution $P_n = \sigma_n/\sigma_{inel}$, where σ_n is the n-prong charged inelastic cross section. Motivated by the then-principle of Feynman scaling, they suggested that $\bar{n}P_n$, plotted as a function of the scaled multiplicity $n/\bar{n}$, should be independent of energy. This idea, subject to the revisionist tendencies of experts, has proved quite successful, indeed surviving the demise of its derivation. Nowadays apparent deviations from this principle observed at 540 GeV attract considerable attention.

To date, the shape of the KNO "plot" of $\bar{n}P_n$ vs. $n/\bar{n}$ seems to be independent of target and projectile, be these hadrons. (To be sure, the corresponding distribution for e^+e^- and eh $\rightarrow$ hadrons is different, as we have discussed elsewhere.)[13] This fact is compatible with the existence of a common glue fraction for all hadrons.

This phenomenon of scaling of the appropriately normalized counting distributions had been discovered by Mandel[14] in 1959 in his analysis of photocount distributions from "thermal" sources of the radiation field. For such sources (of equal strength for simplicity) the semiclassical theory of photocounts leads to a generalized Bose-Einstein distribution to good approximation:

$$P_n^k = \frac{(n + k - 1)!}{n!(k - 1)!} \frac{(\bar{n}/k)^n}{(1 + \bar{n}/k)^{n+k}} \tag{4}$$

where each emitting "cell" has equal average occupancy $\bar{n}/k$. It should be noted that (4) does not necessarily depend on quantum theory. k does not have to be an elementary cell in phase spaces nor does $\bar{n}$ have to be given by the elementary Bose-Einstein formula depending on μ, T, etc. Indeed there are many ways to arrive at Eq. (4), which has many names, among which we shall use the "negative binomial distribution" much used in mathematical statistics. It was observed very early by Giovannini,[15] Knox[16] and Suzuki[17] that Eq. (4) gives an excellent account of hadron-hadron multiplicity data. Giovannini in particular emphasized the connection with quantum optics and the gaussian random variable nature of the hadronization process.

KNO scaling, in the particular form of Eq. (4) is easily shown to take the form

$$\bar{n}P_n^k \sim \psi_k(n/\bar{n}); \qquad \psi_k(z) = \frac{k^k}{(k - 1)!} z^{k-1} e^{-kz} \tag{5}$$

In recent years new data have led various groups[18-20] to rediscover the utility of Eqs. (4) and (5) for the accurate description of data. Later we want to emphasize quite general contexts which can lead to

Eqs. (4-5). Here, however, we note the simplest possible interpretation of Eq. (4).

If the hadronizing system is a set of k (on the average) fireballs, superclusters or whatever, wherein the asymptotic hadron fields are represented as a gaussian field ensemble (simplest example, with a temperature), then the probability distribution is just Eq. (4).

Other interpretations may be possible, even preferable, but the structure of (5) is entirely compatible with the existence of thermalization within the set of emitting sources. Indeed the multiplicity distribution (4) is a natural (thermodynamic cluster) extension of the Fermi-Landau formula for the average multiplicity. (Consult the early work of Cooper and Frye,[21] which is equivalent to the foregoing picture with one cell.)

We remark briefly on recently observed deviations from KNO scaling reported by the UA5 group.[12] As discussed elsewhere,[22] the increased "tail" of the multiplicity distribution can be fit by allowing the effective cell number k to decrease with increasing energy, to abort 3 at collider energy. As Meng[23] has discussed in his lecture, a suggestive physical explanation is that the central "fireball," in his language, increases in relative strength to that of the diffractive fireballs. Since the "cell" is clearly to be identified with the "fireball" of the Berlin group, one sees that an energy dependent modification of the "equal strength" assumption facit in Eq. (5) provides a natural way to break KNO scaling.

Meng et al arrive at Eq. (6) by a simple statistical argument. Physically there is little difference between the Berlin and Los Alamos models. To the eye the discrete form (5) is well approximated by the asymptotic approximation (6) except for large n. Elsewhere we have argued that on the basis of moments, the discrete form is to be preferred. Due to the difficulties in both theory and experiment, this conclusion must be regarded as tentative.

Another treatment of KNO scaling, which takes into account the role of the impact parameter and coordinates pp and e^+e^- results, has been presented by Barshay.[24] In any geometrical model it is intuitiously clear that the multiplicity should depend on the impact parameter of the collision. Moreover we expect larger multiplicities for smaller impact parameters. In particular there is a probability $P_n(b,s)$ for a given multiplicity for a given b and energy s. The next step is to use an eikonal method to evaluate σ_{inel} (b,s) and therefore the total n prong cross section. This approach is physically well founded but hard to make truly quantitative because of uncertainties in evaluation of the eikonal. Nevertheless the fits are rather good except for very high multiplicities, where the predicted gaussian KNO function (see Table I) falls off too rapidly. In addition the different shapes of the KNO function in e^+e^- annihilation and hadron-hadron scattering has a natural explanation.

2.3. Universality of Transverse Momentum Distributions

Recall that in Fermi's original picture, the collision energy was converted to heat energy, with the final hadrons (pions) leaping into phase space without further interaction. Such a picture gives rather high temperatures even at Fermilab energies, so that, for example, there would be 4/3 as many kaons as pions when $T>m_K$. Soon Pomeranchuk pointed out a significant flaw in Fermi's argument, namely that the expanding hadronic matter will not break up until the temperature drops to the mass of the lightest particle, here the pion. When $T<m_\pi$ the pion density drops exponentially to zero so that no further interactions are possible. In this way the hypersurface $T(\underline{x},t) = m_\pi$ becomes identified with the hadronization surface. Note that as the system cools, the heavier particles are suppressed by ordinary rate processes provided the conversion for $K\bar{K}\to\pi\pi$ is fast enough. Note that (see Shuryak[25]) because of strangeness conservation, the relative population of K's is <u>not</u> given by the Boltzmann factor. The agreement of the K/π ratio with Shuryak's calculation is an interesting piece of evidence in favor of the existence of a thermodynamic phase in the hadronization process.

Experimentally one observes a strong difference between longitudinal (defined by collision axis) and transverse directions. Landau's original hydrodynamic model predicted a secondary distribution which is gaussian in pseudorapidity of ($\eta = -\ell n \tan \theta/2$) with a width depending on the proton size. Milekhin's extension[26] of Landau's calculation led to an approximately factorized form for the secondary distribution

$$E \frac{d^3N}{d^3p} \propto \exp(-p_\perp/m_\pi) \exp(-y^2/2L) \tag{6}$$

with $L = \frac{1}{2}\ln(s/4m_p^2)$. Here y is the true rapidity $y = \frac{1}{2} \ln(E + P_\parallel)/E - P_\parallel)$. This formula, which violates Feynman scaling for small x, has been found to give an excellent phenomenological description of hadronic data (mostly $pp \to \pi$ + anything) up to and including ISR energies. As before, (6) is meant to apply to nondiffractive events.

For some time the transverse momentum distributions ~exp $(-6p_\perp)$, with $p_\perp$ in GeV/c, has been known to give a good description of data for $p_\perp < 1$ GeV/c. In these units we see that 1/6 GeV $\simeq$ 166 MeV $\simeq m_\pi$, in plausible agreement with Milekhin's picture that the transverse distribution is mainly due to thermal fluctuations. However, as Hama[27] has shown here, a more careful calculation of the transverse hydrodynamic expansion increases $\langle p_\perp \rangle$ beyond acceptable values. As discussed subsequently, this and other puzzles may require modification of the Fermi-Landau boundary condition.

Eq. (6) predicts an average transverse momentum of $\langle p_\perp \rangle = 2m_\pi$, as compared with the empirical result $\langle p_\perp \rangle$ = 350 MeV/c (i.e., 2 × 1/6 = 333 GeV/c). Since the collider now indicates $\langle p_\perp \rangle$ ~ 500 GeV/c, we see that the simpler rule (6) must break down at sufficiently high energy. The situation is complicated by the increasingly non-negligible fraction of heavy particles. Advocates of LTE will say that the effective hadronization temperature has increased by 50% as the energy is increased by an order of magnitude.

In bubble chamber experiments, one can investigate small secondary momentum $p_\perp$ < 100 MeV/c. Here it is found that the <u>same</u> temperature (~120 MeV) fits $\exp(-E_\perp/T)$, with $E_\perp = (p_\perp^2 + m^2)^{1/2}$, for a large variety of secondary masses m.

The collider η distributions are much narrower than expected from phase space considerations. In fact the width is not far from that predicted in the Landau model of 30 years ago. Wehrberger has shown in his talk[28] that the shapes $dN/d\eta$ are in fact compatible with the hydrodynamical model. This does not prove anything, of course, since the Berlin group has given a good account of the same data in their (non-hydrodynamic) fireball model.

2.4 Possible Indirect Evidence: Nucleon Structure Functions and Bose-Einstein Correlations

Nucleon structure functions look very much like thermal Bose-Einstein distributions and have been thus interpreted.[29] This is suggestive though not compelling to a skeptic. In addition one might object in principle to the idea of attributing a temperature to an energy eigenstate, the physical proton. However, the nucleon is a complex object with fluctuating components, so that the contradiction is only apparent. We have noted elsewhere[30] that it is plausible to regard the valence quarks as immersed in a chaotic gluon cloud, so that coarse graining can result in an effective heat bath for the valence quarks (whose distribution is probed by the electroweak probes.)

2.5 Bose-Einstein Correlations

Correlations between like particles at small momentum transfer provide evidence on the coherence of the hadronization process.[31] The ratio $R = N(--)/N(+-)$ for zero momentum transfer for a single source is two for incoherent emission and one for coherent emission. In hadronic collisions the ratio appears to be between 1.5 and 2, pointing to a strong incoherent signal. This is in accord with the statistical interpretation of the negative binomial distribution in subsection 2. At this meeting Mattig has presented[32] new results from the Goldhaber group (for $e^+e^- \to J/\psi \to$ hadrons) showing an intercept close to 1.0. Again, predominant coherence in the e^+e^- hadronization process is in accord with the (narrow) shape of the KNO plot for $e^+e^- \to$ hadrons.

2.6 Universal Hagedorn Temperature

It is a striking fact that the hadronic mass spectrum fits a universal exponential curve

$$\rho(m) \propto \exp(m/T) \qquad (7)$$

where T is 160 MeV. This form is exactly that required by Hagedorn's self-consistent bootstrap.[2] As far as I know these results have not yet been properly integrated into the world view of QCD.

It must be stressed that most of the results of this section are not only obtained easily but are quantitatively successful. The same results are often outside the reach of other more fashionable theoretical frameworks.

3. EQUILIBRATION MECHANISMS

A system composed of many components, subject to fixed boundary conditions, typically relaxes to thermodynamic and chemical equilibrium if given sufficient time. Different time constants will characterize the differing kinds of relaxation of the various components. Approximate methods for the description of the time evolution include kinetic theory [exemplified by the Boltzmann equation for the single particle distribution functions $f_i(p,R,t)$] and rate equations for the number populations. Usual intuitions are best for dilute systems near equilibrium. Hence applications of traditional methods to collision problems, which involve transient effects in space time and frequently unknown boundary conditions, are expected to be very crude. Nevertheless it is useful to have estimates of equilibration times and transport coefficients in uniform extended media, in order to assess the likelihood of equilibrium being attained during the lifetime of the system.

3.1 Binary Collision Scenario

The most primitive estimate (freshman level kinetic theory) gives a collision time

$$t \underset{\sim}{<} 1/(\sigma n v) \tag{8}$$

where σ is the cross section, n the number density of targets and v the projectile velocity. A slightly more sophisticated procedure (written here for spatially uniform systems) requires the solution of rate equations for the number populations

$$\frac{dN_i(\vec{p})}{dt} = \sum_{\{p'\}} \rho\, F(\{N_i\}) \tag{9}$$

where ρ is a phase space factor and F a nonlinear function of the occupation numbers. Eqs. (8) and (9) tacitly assume that the particles are on the mass shell, which is false when the system is dense or strongly interacting. In this case the formalism of covariant transport theory[33] (or some equivalent) is required, bringing many complications.

Several authors[34-36] have estimated relaxation times in the QCD plasma using standard kinetic theory. Details and references will be found in McLerran's contribution to this workshop.[37] Salient points are: (1) gluon relaxation is more efficient than that of quarks, due to the color phase space factors; (2) the mean free paths are typically a fraction of a fermi. This latter result suggests that equilibration is quite likely in heavy ion collisions but leaves open the situation in hadron-hadron collisions. Finally, the entire calculation is based on perturbative matrix elements. Clearly more work needs to be done.

3.2 Fokker-Planck Relaxation

The "binary collision" scenario of the preceding section is most appropriate for collisions due to short-range forces in dilute systems. For long-range forces, the net scattering is instead the cumulative effect of many small scatterings. As described in plasma textbooks,[38] the appropriate evolution equation for this stochastic-diffusive process is the Fokker-Planck equation. Clearly the collision of two hadrons can be regarded as the flow of two parton currents in opposite directions, with long-range QCD forces acting to decelerate the system. Hwa has adapted[39] the Fokker-Planck technique to the QCD description of heavy ion collisions with interesting results. He obtains a somewhat smaller mean free path (~0.1 f) than the binary collision approach. Color and spin corrections should be made to see whether this diffusive relaxation is indeed more efficient.

3.3 Collective Instabilities

Both preceding mechanisms were stochastic in the sense that successive collisions were assumed to occur randomly. This neglects the well-known phenomenon of collective motions in systems whose constituents interact with long-range forces. In addition to providing collective excitations (such as plasmons in the coulomb gas), there are the many instabilities notorious in the field of magnetic confinement fusion.[38] In typical plasmas the collective instabilities cause the system to relax faster than do binary collisions. Hence what is bad in the fusion program may be good for the establishment of local equilibrium in particle physics. As far as I know, this mechanism has not yet been considered in strong interaction physics. Some crude estimates (including the color variable) would be of great interest.

3.4 Turbulent and Chaotic Relaxation

Many years ago Heisenberg speculated that the excited hadronic matter produced in hadronic collision could exhibit turbulent flow. Even now it is not clear to what extent such matter behaves as a fluid, but the idea that the flow of the non-linearly coupled fields should be turbulent in some sense is quite plausible. In such a case transport mechanisms are much enhanced over the kinetic-diffusive mechanism and, in particular, equilibration times could be much shorter than current estimates.

Another possible behavior of highly excited QCD matter is the manifestation of chaos. In the classical gluon field inhabiting a spatially uniform world, the time dependence of the field has been found to be intrinsically chaotic.[40-41] The extended system of excited matter created in a heavy ion collision should be a good candidate for this limit. As in the case of convective turbulent transport, the system will mix in phase space much more efficiently than in kinetic estimates.

To summarize, traditional kinetic theory gives crude estimates of relaxation times which suggest that local equilibrium is plausible, though not compelling. We have drawn attention to other possible, more efficient relaxation mechanisms for which quantitative estimates should be made.

3.5 Stochastic Number Evolutions

The universality of hadron-hadron KNO plots, and the success of the negative binomial distribution in fitting these, led us to conjecture[42] the existence of a generic statistical framework for hadronization. The statistical behavior of the hadronization process itself is described following Eq. (4). What about the evolution of particle numbers before hadronization? (Note that in the Landau model "before" means within the surface $T(\underline{x},t) = m_\pi$ while in the inside-outside cascade the hadronization time depends on the particle momentum.) Rate equations such as (9) provide a traditional context for number evolutions. Indeed jet calculus equations for the branching processes involving quarks and glue are of such form with $t \rightarrow \ell n\, Q^2$. Giovannini[43] has found the negative binomial as a solution to (approximate) evolutions involving pure glue.

As pointed out elsewhere,[11] the effect of mode coupling in (9) is frequently represented as noise acting on a chosen mode α of given momentum. The evolution is then given by a Langevin equation

$$\frac{dN_\alpha}{dt} = F(N_\alpha) + G(N_\alpha)\, f(t)$$

Here F and G are functions of N_α, and f(t) represents the noise. We assume that f(t) can be described by gaussian white noise. F is typically nonlinear; when G is independent of N, one speaks of additive noise as in Brownian motion. If G depends on the value of N_α, one has multiplicative noise.

A good class of models which includes many physical systems and is analytical solvable[44] is described by the Langevin equation for multiplicative noise

$$\frac{dx}{dt} = dx - bx^{1+\gamma} + xf(t) \tag{10}$$

with the associated Fokker-Planck equation

$$\frac{\partial f(x,t)}{\partial t} = \frac{1}{2} Q \frac{\partial^2}{\partial x^2} [x^2 f(x,t)] - \frac{\partial}{\partial x} [(d + \frac{1}{2} Q) x - bx^{1+\gamma}] f(x,t) \tag{11}$$

where the noise strength Q is defined by the ensemble average

$$\langle f(t) f(t') = Q\delta(t - t') \tag{12}$$

As mentioned in ref. 11, $f(x,t)$ is to be identified with the kernel of the Poisson transform which gives the discrete distribution P_n. The parameters (d,b,γ) can be identified with parameters in the fundamental Hamiltonian of the system. In the case $\gamma = 2$ one obtains a well-known laser model.[45] We shall see that $\gamma=1$ leads to Eq. (5) and thence to (4) via the Poisson transform.

The stationary (long-time) solution to (11) is

$$f_o(x) = Nx^{-1+d/Q} \exp(-2bx^{\gamma}/\gamma Q) \tag{13}$$

For another instuctive example, consult the paper by Biyajima.[46]

We obtain Eq. (5) for the cases $\gamma=1$, $b=d$, $k=2d/Q$. (Actually $\gamma=1$ arises naturally in ϕ^4 field theory, as mentioned elsewhere.[11]) Although physical considerations do not always permit the arbitrary adjustment of parameters, it is amusing (Table I) that most of the analytic KNO functions are described by the solution (13).

TABLE I. Asymptotic KNO Functions

	Distribution	γ	2b/Q	2d/Q
(a)	$xe^{-\pi x^2/4}$	2	$\pi/2$	2
(b)	$x^3e^{-9\pi x^2/16}$	2	$9\pi/8$	4
(c)	$x^2e^{-\alpha x^3}$	3	3α	3
(d)	$x^{k-1}e^{-kx}$	1	k	k

Some common (asymptotic) KNO functions are identified with the stationary solutions [Eq. (13)] for a suitable set of parameters. From top to bottom we find: in (a) and (b) Barshay's formulas for $\bar{p}p$ and e^+e^- collisions; in (c) the proposal of the Berlin group for $e^+e^- \to$ hadrons (the parameter α is $\Gamma(4/3)$]; in (d) the gamma distribution used by the Berlin and Los Alamos groups, among others. In this case k is 2d/Q as discussed in the text.

The creation of hadrons in a collision is a dissipative process, driven by the initial kinetic energy. It is not clear that one will reach the stationary state before hadronization occurs. Here we only wish to note that (11) has the general form of the time dependent Ginzburg-Landau theory, which suggests the possibility of an out-of-equilibrium phase transition. If this occurs, one may have scaling and universality dictating better answers than the approximate input. In this case one could construct a respectable theory of hadronization.

4. STATUS OF THE HYDRODYNAMICAL MODEL

Presently there is a working consensus that classical relativistic fluid mechanics (without dissipation) gives a useful first approximation to energy momentum flows of matter excited in relativistic heavy ion collisions. No such agreement exists for hadron-hadron collisions, the original problem for which the Landau model was invented. Disregarding this we can note some developments over the past decade or so. For more information see the lecture by Feinberg.[47]

During the sixties little attention was paid to the statistical-hydrodynamic framework for particle production. Experimental investigations of inclusive cross sections set the stage for useful applications of the Landau theory. In 1972-73 the Landau model was modernized by Shuryak and Carruthers and Duong-Van. Excellent quantitative results were obtained for multiplicities, rapidity distributions, scale violating (Feynman) distributions as well as the exponential transverse momentum distributions.[1] However, the community was not receptive, to say the least, so that these ideas found little acceptance. Even at this time it was realized that the model should not apply to the leading particles, so that the latter became something outside the model.

The physical picture provided by the Pokorski-Van Hove model, as fleshed out[3] by QCD, made possible a unified (but still qualitative) picture of these two components. The prediction of the multiplicity normalization improved when the proper number of QCD degrees of freedom were taken into account, for example. During the seventies various authors also studied the sensitivity of predictions to the equation of state.

The eighties brought a keen interest in relativistic heavy ion collisions, especially insofar as these provided a vehicle for creation of the fabled QCD "plasma" phase. Practical people soon realized that the only sensible first step was to see what hydrodynamics predicted for such processes. Another sociological influence was that nuclear physicists are traditionally more at home with concepts of statistical physics than are high energy physicists. By the time of the "Quark Matter '83" meeting held at Brookhaven, one could see workers in both fields seriously considering the same problems.

During this time Bjorken published an influential paper[48] in which an alternative to the Fermi-Landau boundary condition was proposed. His picture of the inside-outside cascade mechanism was quickly and almost universally accepted so we shall refer to it as "The New Orthodoxy." The Fermi-Landau "pancake" is replaced by a larger region of space time in which equilibration occurs. After this time hydrodynamics proceeds as usual. (It might seem that this change would ruin the energy dependence of the multiplicity, but we have shown elsewhere[30] that this is not true.) In the preceding section we suggested that what happens in the prehydrodynamic phase is the counter streaming of the constituent partons. As mentioned there, the operative damping mechanism is not yet adequately known.

In the "classic" Landau model there has been recent progress, too. We have mentioned the work of Hama on the transverse expansion.[27] Wehrberger[28] has also shown that the single particle rapidity distributions observed at 540 GeV c.m. energy can in fact be understood in terms of the "classic" framework.

For some time we have been interested in understanding to what extent hydrodynamic flow structures exist in quantum field theories even in the absence of local equilibrium. These problems are conveniently studied[49] using the covariant Wigner phase-space distribution. The most difficult problems, currently under investigation, concern the closure of the equations of motion for off-shell, nonequilibrium field configurations.

5. CONCLUSIONS

In recent years we have witnessed increasing interest in and application of the concepts of local equilibrium to the hadronic many-body problem. In large part this attention is due to excitement over the possible existence of the QCD plasma phase of hadronic matter, perhaps to be created in relativistic nucleus-nucleus collisions. In this case ideas of local equilibrium, with dynamics implemented via relativistic fluid mechanics, provide the only efficient approach to the complex physical situation. The second source of renewed interest in multiparticle production has been provided by new results at 540 GeV from the CERN $p\bar{p}$ collider. New effects, such as increasing $\langle p_\perp \rangle$ and violation of KNO scaling have attracted much attention.[50-51] The explanation of such results may require a stochastic framework more general than complete local equilibrium, as discussed before.

Earlier we stressed the fact that several prominent features of multiparticle production data which have a natural explanation in the framework of local equilibrium are beyond the reach of other approaches. One should always keep in mind the complementary nature of the equilibrium regime and the perturbative regime. Differing approximations are likely to be appropriate for different space-time regions in the history of a reaction. Conceptually all these limits, approximations, etc., can be regarded as embedded in a more general framework, namely that of relativistic transport theory as expressed in terms of covariant phase space distribution functions. Unfortunately this formalism remains inadequately developed for practical applications. In the near future it is more likely that a more phenomenological approach will be fruitful. Luckily there are many interesting problems of experimental consequence to study at this level.

In summary, the answer to the question posed in the title to this paper is: yes! But much work must be done to explain why.

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REFERENCES

1. Carruthers, P., proceedings of the New York Academy of Sciences, 229, 91 (1974); see also Feinberg, E. L., Usp. Fiz. Nauk (SSSR) 139 (1983)/Trans. Sov. Phys. Uspekhi 26 (1983).

2. Hagedorn, R., to be published in "Quark Matter '84".

3. Carruthers, P. and Duong-Van, Minh, Phys. Rev. D28, 130 (1983).

4. Pokorski, S. and Van Hove, L., Nucl. Phys. B86, 243 (1975).

5. Basile, M. et al, Nuovo Cimento 65A, 400 (1981).

6. Basile, M. et al, Nuovo Cimento 47A, 244 (1982).

7. Weiner, R. (private communication).

8. Carruthers, P. and Duong-Van, Minh, Phys. Lett 114B, 169 (1982); Carruthers, P., Los Alamos preprint LA-UR-84-1084, to be published in "Proceedings of the Conference on the Physics of the 21st Century".

9. Plümer, M., contribution to this workshop.

10. Friedlander, E. and Weiner, R., Phys. Rev. D28, 2903 (1983).

11. Carruthers, P., Los Alamos preprint LA-UR-84-2733 (to be published in the proceedings of "Quark Matter '84").

12. Koba, Z., Nielsen, H. B. and Olesen, P., Nucl. Phys. B40, 317 (1972); see proceedings of the XV Multiparticle Dynamics Symposium (Lund, Sweden, 1984) to be published and contributions by Meng and Stöcker to this workshop.

13. Carruthers, P. and Shih, C. C., Phys. Lett. 127B, 242 (1983). See Fowler, G., contribution to this workshop.

14. Mandel, L., Proc. Phys. Soc. 74, 233 (1959).

15. Giovannini, A., Nuovo Cimento 15A, 543 (1973); ibid 24A, 421 (1974).

16. Knox, R., Phys. Rev. D10, 65 (1974).

17. Suzuki, N., Prog. Theor. Phys. 51, 51 (1974).

18. Biyajima, M., Prog. Theor. Phys. 69, 966 (1983).

19. Carruthers, P. and Shih, C. C., Phys. Lett. 137B, 425 (1984).

20. Pancheri, G, review talk at Lund Symposium (see ref. 12) and private communication.

21. Cooper, F. and Frye, G., Phys. Rev. D8, 334 (1973).

22. Carruthers, P., Los Alamos preprint LA-UR-84-2505, to be published in proceedings of the Lund Symposium.

23. Ta-Chung, Meng, contribution to this workshop.

24. Barshay, S., contribution to this workshop.

25. Shuryak, E., Phys. Repts. 61, 71 (1980).

26. Milekhin, G. A., J. Expt'l Theoret. Phys. (SSSR) 35, 1185 (1958) [Sov. Phys. JETP 35, 829 (1959)].

27. Hama, Y., contribution to this workshop.

28. Wehrberger, contribution to this workshop, see also Wehrberger, K. and Weiner, R., Phys. Rev. D31, 222 (1985).

29. Angelini, C. A. and Pazzi, R., Phys. Lett. 113B, 343 (1982).

30. Carruthers, P., Nucl. Phys. A418, 501c (1984).

31. Goldhaber, G., Goldhaber, S., Lee, W. and Pais, A., Phys. Rev. 120, 300 (1960).

32. Mattig, P., contribution to this conference.

33. Carruthers, P. and Zachariasen, F., Phys. Rev. D13, 950 (1970); Rev. Mod. Phys. 55, 245 (1983).

34. Rafelski, J. and Müller, B., Phys. Rev. Lett 48, 1066 (1982).

35. Hosoya, A. and Kajantie, K., Helsinki U. preprint HU-TFT-83-62 (to be published).

36. Baym, G., U. Illinois preprint D183/11/150 (to be published).

37. McLerran, L., Fermilab preprint 84/128-T (contribution to this workshop).

38. Krall, N. and Trivelpiece, A., "Principles of Plasma Physics" (McGraw-Hill, N.Y., 1973), p. 256.

39. Hwa, R., U. of Oregon preprint OITS 263 (1984) (to be published).

40. Savvidy, K., Yerevan Physics Institute preprints 626 (16)-83 and 613 (3)-83.

41. Matinyan, S. G., Yerevan Physics Institute preprints 668 (58)-83.

42. Carruthers, P., p. 825 in "XIV International Symposium on Multiparticle Dynamics 1983", ed. Yager, P., and Gunion, J. F. (World Scientifie, Singapore, 1983).

43. Giovannini, A., Nucl. Phys. B161, 429 (1979); see also Garetto, M., Lett. al Nuovo Cimenta 28, 593 (1980).

44. Schenzle, A., and Brand, H., Phys. Rev. A20, 1628 (1979).

45. DiGiorgio, V. and Scully, M., Phys. Rev. 147, 214 (166).

46. Biyajima, M., Phys. Lett. 137B, 225 (1984); ibid 139B, 93 (1984).

47. Feinberg, E., contribution to this workshop.

48. Bjorken, J. D., Phys. Rev. D27, 140 (1983).

49. Carruthers, P. and Zachariasen, F., p. 698 in proceedings of "Intersections Between Particle and Nuclear Physics", AIP Conf. Proc. No. 123, ed. Mischke, R. (American Institute of Physics, N.Y. 1984).

50. Barshay, S., Phys. Lett. 127B, 129 (1983).

51. Van Hove, L., Phys. Lett. 118B, 138 (1982).