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TITLE SUPERCLUSTERS AND HADRONIC MULTIPLICITY DISTRIBUTIONS

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Superclusters and Hadronic Multiplicity Distributions

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ABSTRACT

We have expressed the multiplicity distribution in terms of supercluster production in hadronic processes at high energy. This process creates unstable clusters at intermediate stages and hadrons in final stage. It includes Poisson-transform distributions (with the partially coherent distribution as a special case) and is very flexible for phenomenological analyses. The associted Koba, Nielson, and Olesen limit and the behavior of cumulant moments are analyzed in detail for finite and/or infinite cluster size and particle size per cluster. We demonstrate that in general a supercluster distribution does not need to be equivalent to a negative binomial distribution to fit experimental data well. Furthermore, the requirement of such equivalence leads to many solutions, in which the average size of the cluster is not logarithmic: e.g., it may show a power behavior instead.

We define superclustering as a two- or multi-stage process underlying observed global multiplicity distributions.[1-4] At the first stage of the production process, individual clusters are produced according to a given statistical law. For example, the clustering distribution may be described by partially coherent (or even sub-Poissonian) distribution models.[5-7] At the second stage, the clusters are considered as the sources of particle production. The corresponding distribution may then be as general as the clustering distribution just mentioned.

We shall first define the probability of having c clusters as p_c, with $\sum_c p_c = 1$. This enables us to calculate the associated moment-generating function

$$g(\lambda) \equiv \sum_c \lambda^c p_c \quad . \tag{1}$$

We shall further assume that once a cluster is created, its subsequent evolution into the experimentally-observed particles is independent of the other clusters. The probability of creating n_j particle in the j^{th} cluster is then defined to be $f_j(n_j)$ with $\sum_{n_j} f_j(n_j) = 1$. With the j cluster, we may also calculate its associated moment-generating function

$$f_j(\lambda) = \sum_{n_j} \lambda^{n_j} f_j(n_j) \quad . \tag{2}$$

However, neither p_c nor $f_j(n_j)$ are observed directly in total multiplicity measurements. What can be measured is the sum total of all the particles produced by all the clusters. It is necessary to relabel the particles in terms of an overall index N, and to re-evaluate the corresponding probability distributions P_N. With the P_N properly normalized, we get

$$P_N = \sum_c p_c \sum_{n_1, n_2, \ldots n_c} f_1(n_1) \ldots f_c(n_c)\, \delta(n_1 + n_2 + \ldots + n_c - N).$$

and the associated moment-generating function

$$G(\lambda) = \Sigma \lambda^N P_N \quad . \tag{4}$$

Clearly, the overall distribution P_N is completely determined by the distribution p_c and f_n. For the most general cases, analytical calculations are, however, rather tedious. There is no obvious analytic method for further investigation. In Ref. 1), $f_j(n_j)$ are now identical distributions. However, this procedure may tend to lose information on semi-global correlations. Alternatively, we may consider using an identical distribution $f(n_j)$ for all $f_j(n_j)$'s as a first order approximation. This would ignore, for example, possible differences in multiplicity distributions between the fragmentation and central region. The semi-global correlations are then somewhat better preserved. For simplicity, we shall from now on assume that the same distribution governs the evolution of each cluster, i.e.,

$$f_j(n_j) = f(n_j), \qquad j = 1, \ldots c \quad , \tag{5}$$

so that for all the clusters the generating functions simplify as

$$f_j(\lambda) = f(\lambda), \qquad j = 1, \ldots c \quad . \tag{6}$$

The relationships between f_n, p_c and P_N can now be expressed directly as

$$G(\lambda) = g(\mu), \qquad \mu = f(\lambda) \quad . \tag{7}$$

In terms of $G(\lambda)$, the various factorial moments ξ_L can then be evaluated as

$$\xi_L = \frac{\partial^L G}{\partial \lambda^L} \Big|_{\lambda=1} \quad . \tag{8}$$

For example, the Poisson X Poisson distribution (composition of the Poisson distributions) is given by

$$G(\lambda) = \exp \{\langle c\rangle [\exp (\langle n\rangle (\lambda-1))-1]\} \tag{9}$$

Here the Poisson distributions are characterized by the average number of cluster $\langle c\rangle$ and the average number of particles per cluster $\langle n\rangle$; the $N_B \times N_B$ distribution is given by

$$G(\lambda) = \{1 + \frac{\langle c\rangle}{k_c} [1-(1 + \frac{\langle n\rangle}{k_n} (1-\lambda))^{-k_n}]\}^{-k_c} \tag{10}$$

with the negative binomial distribution NB for the clusters characterized by $\langle c\rangle$, and a cell number k_c; the NB distribution for particles within one cluster, characterized by $\langle n\rangle$ and a different cell number k_n.

Even for these relatively simple generating functions, their associated probability functions P_N are rather complicated. In order to get a better feeling of the structures of the superclustering distributions, we shall work out explicitly several normalized cumulant moments. We get

$$<<N>> \equiv \bar{N} = <c><n> \quad . \tag{11}$$

$$\Gamma_2 \equiv <<(N-\bar{N})^2>>/\bar{N}^2 = \gamma_2(c) + \frac{\gamma_2(n)}{<c>} \tag{12}$$

$$\Gamma_3 \equiv <<(N-\bar{N})^3>>/\bar{N}^3 = \gamma_3(c) + \frac{3\gamma_2(c)}{<c>} \gamma_2(n) + \frac{1}{<c>^2} \gamma_3(n) \tag{13}$$

$$\Gamma_4 \equiv [<<(N-\bar{N})^4>> -3 << (N-\bar{N})^2 >>^2]/\bar{N}^4 = \gamma_4(c) + 6 \frac{\gamma_3(c)}{<c>} \gamma_2(n) \tag{14}$$

$$+ 3 \frac{\gamma_2(c)}{<c>^2} (\gamma_2(n))^2 + 4 \frac{\gamma_2(c)}{<c>^2} \gamma_3(n) + \frac{1}{<c>^4} \gamma_4(n) \tag{15}$$

Higher moments can be calculated in a straightforward way, and are not presented here.

In the situation with infinite number of clusters many simplifications occur, whether or not $<n>$ approach ∞. Notice that $<c> = \infty$ allow us to ignore the contribution of $\gamma_j(n)$ to Γ_j completely. Thus the Γ_j is equal to $\gamma_j(c)$

$$\Gamma_j = \gamma_j(c), \qquad <c> \equiv \infty \tag{16}$$

This is a reflection of the central limit theorem in statistics; the scaling limit is completely dictated by the scattering limit of the clusters. However, for large but finite $<c>$, both the $\gamma_j(c)$ and $\gamma_j(n)$ contribute to the scaling violation of P_N.

Recently negative binomials have been used extensively to analyze experimental data. The success of these analyses encouraged renewed interests in the origin of negative binomials.[8,9] Since both the superclustering distribution and the negative binomial distribution are important types of distributions, we shall now analyze their relationship.

In order to get a negative binomial for P_N, we may set the $G(\lambda)$ of Eq. 7 to the form associated with negative binomial distributions. This requirement alone does not uniquely determine the probability distribution of f_n. We shall first examine the special example[1)] of Giovannini and Van Hove where p_c is further assumed to be a Poissonian distribution. The form of f_n is then uniquely determined. We get

$$(1 + \frac{<<N>>}{K}(1-\lambda))^{-K} = \exp(<c>\ (f(\lambda)-1)) \tag{17}$$

leading to

$$f(\lambda) = 1 - \frac{K}{<c>} \ln\ (1 + \frac{<<N>>}{K}(1-\lambda)) \tag{18}$$

The explicity expression for f_n is now

$$f_0 = 1 - \frac{K}{<c>} \ln\ (1 + \frac{<<N>>}{K}) \quad ,$$

$$f_n = \frac{K}{<c>} (\frac{<<N>>}{K+<<N>>})^n \frac{1}{n}\ ,\ n \neq 0 \tag{19}$$

Here the value of f_n, $n > 0$ are up to a constant factor the same as those derived by Giovannini and Van Hove in Ref. 1). However, the f_0 is different. The requirement that $f_0 = 0$, is in fact a rather restrictive requirement. Notice that from the definition of P_n, P_o is bounded below by $P_{c=0}$. The requirement $P_o = P_{c=0}$ leads to $G(0)=g(f(0))$, i.e.,

$$(1 + \frac{<<N>>}{K})^{-K} = \exp(-<c>) \text{ i.e.,}$$

$$<c> = K \ln(1 + \frac{<<N>>}{K}) \quad <n> = \frac{<<N>>}{<c>} = \frac{<<N>>}{K\ln(1+\frac{<<N>>}{K})} \tag{20}$$

This condition $f_0 = 0$, in Ref.1) is a simplification for the purpose of obtaining solutions with the least number of parameters. Consider for example the situation where the clusters described by p_c may emit neutral particles. There is then a nonzero probability that any individual cluster may decay completely into neutrals without charged secondaries ($f_{n=0} \neq 0$). A more desirable restriction is for $P_{N=0} > p_{c=0}$. We then get Eq. 19 with $\langle c\rangle$ essentially a free parameter as long as $f_0 > 0$.

We may construct a large number of superclustering distributions that are equivalent to the negative binomial. This can be recognized in Eq. 7 with choices of p_c different from a Poissonian distribution. For instance, we may let p_c itself be a negative binomial. This leads to

$$[1 + \frac{\langle\langle N\rangle\rangle}{K}(1-\lambda)]^{-K} = [1 + \frac{\langle c\rangle}{k_c}(1-f(\lambda))]^{-k_c} \quad ,$$

$$f(\lambda) = 1 + \frac{K_c}{\langle c\rangle}\{1 - [1 + \frac{\langle\langle N\rangle\rangle}{K}(1-\lambda)]^{K/k_c}\} \qquad (21)$$

As far as the cluster distribution f_n is concerned, the above example is not very different from the previous example with

$$f_0 = 1 - \frac{K}{\langle c\rangle}[(1 + \frac{\langle\langle N\rangle\rangle}{k_c})^{k_c/K} - 1] \quad ,$$

$$f_n = \frac{1}{n!}\frac{k_c}{\langle c\rangle}(1 + \frac{\langle\langle N\rangle\rangle}{k_c})^{kc/K}(\frac{\langle\langle N\rangle\rangle}{\langle\langle N\rangle\rangle+K})^n (1 - \frac{k_c}{K})\ldots(n+1 - \frac{k_c}{K}), \ n\neq 0 \qquad (22)$$

In the language of Giovannini and Van Hove, both examples are partial stimulated emissions. However, they possess very different distribtuion in p_c. If we require f_n to be zero

$$\langle c\rangle = k_c [(1 + \frac{\langle\langle N\rangle\rangle}{K})^{K/k_c} - 1] \quad \langle n\rangle = \frac{\langle\langle N\rangle\rangle}{\langle c\rangle} = \frac{\langle\langle N\rangle\rangle}{k_c}[(1 + \frac{\langle\langle N\rangle\rangle}{K})^{K/k_c} - 1] \qquad (23)$$

In fact, we may construct a large class of superclustering distributions, all equivalent to the negative binomial. In particular, if p_c is a partially coherent distribution we need to solve $f(\lambda)$ through

$$G^{(NB)}(\lambda) = g^{(PC)}(\mu),\ \mu = f(\lambda) \tag{24}$$

If we take the form-invariant partially coherent distributions $f_n = p_n^{PC)}(n_o,\ k_o,\ m)$ as a generalization to replace the Poisson distribution, Eq. V.1 can then be replaced by

$$P_N = \int dX\ F^{(PC)}(X) P_N^{(PC)}\ (X\ \bar{N},\ XK, m) \tag{25}$$

where the $P_N^{(PC)}\ (\bar{N})$ is the partially coherent distribution with average $\bar{N}$, $K = c_0\ k_0$. The KNO limit is

$$\langle N\rangle P_N \to \text{const} \int dX\ F^{(PC)}(X)\ \frac{1}{X}\ \psi^{(PC)}(Z/X,\ XK,\ m),\ Z = N/\langle N\rangle \tag{26}$$

where $\psi^{(PC)}(Z,\ K,\ m)$ is the asymptotic KNO limit of the PC distribution characterized by K and m.

The above examples show explicitly that negative binomial distributions for the total multiplicity distribution can be constructed in many ways. The simple example investigated by Giovannini and Van Hove may be somewhat too restrictive. Its dependence on the logarithmic behavior of the cluster size should therefore be reexamined. For example, solutions for Eq. 19 and Eq. 22 are all negative binomial solutions without the logarithmatic behavior in $\langle n\rangle$. An additional requirement, that the probability of no charged particle per cluster be zero, forces the solution to Eq. 19 to become the Giovannini and Van Hove solution, Eq. 20, with the logrithmatic behavior in $\langle n\rangle$. However, the same requirement leads to Eq. 22 to Eq. 23 with a power law behavior in $\langle n\rangle$ instead. After all, the whole requirement of the equivalence between the superclustering distribution and negative binomial distribution may not be necessary. With a fixed hadronic multiplicity $\langle\langle N\rangle\rangle$, the requirement of an increase in the size of $\langle n\rangle$ always corresponds to a slower increase in the size of $\langle c\rangle$. The broadening of the KNO function in $\langle\langle N\rangle\rangle$ can be achieved without a real need for the logrithmatic behavior $\langle n\rangle$.

Fluctuations in the hadronic multiplicity distribution may be naturally described by quantum stochastic processes with mixed coherent and incoherent components.[6] Since superclustering representations can be very flexible in representing experimental data, total multiplicity data may easily leave a number of free parameters undetermined. This is very natural: global properties should be insensitive to a large amount of detailed information. Recent measurements on conditional multiplicities can, however, be very useful in eliminating many of the ambiguities just mentioned. We also strongly urge the measurement of correlations between conditional probability distributions. Information on global correlations may ultimately provide the best method of understanding the supercluster structure of multi-particle production processes.

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References

1. A. Giovannini and L. Van Hove, Z. Phys. C30 (1986) 391, and preprint in the proceeding of the XVII International Symposium on Multiparticle Dynamics, Seewinkel, Austria (1986) (World Scientific).
2. V. Simik and M. Sumbera preprint, Institute of Physics of CSAV, Prague, and Nucl. Phys. Institute of CSAV, Prague, (1985). F. Hayot and G. Sterman, Phys. Lett. (2) B (1983) 419.
3. W. Fiakowski, K, Phys. Letters. 169B (1986) 436.
4. A. Biaas and A. Szczerba, reprint, Jagellonian University, Krakow (1986).
5. P. Carruthers and C. C. Shih, Phys. Lett. 127B, (1983) 242 ibid. Phys. Lett. 137B, (1984) 425, P. Carruthers and C. C. Shih, preprint, to be submitted to Phys. Review (1986).
   P. C. Carruthers and C. C. Shih, J. Modern Phys. A to be published (1986).
6. C. C. Shih, Phys. Rev. D33 (1986) 3391; Univ. of Tenn. preprint, to appear in Proceeding of the 2nd International Workshop on Local Equilibrium in Strong Interaction Physics; Santa Fe, N.M. April 1986 (World Scientific).

7. G. N. Fowler and E. M. Friedlander, R. M. Weiner and G. Wilk, Phys. Rev. Lett. 56, 14 (1986).
8. G. J. Alper, et. al. (UA5 Collaboration) Phys. Lett. 160B, (1985) 199.