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TITLE: EXPLOSIVE VAPORIZATION OF SMALL DROPLETS

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EXPLOSIVE VAPORIZATION OF SMALL DROPLETS

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A) "Selfsimilarity in Electrohydrodynamics", S.M. Chitanvis, *Physica A* (1986), in print.

B)"High energy laser interactions with water droplets", S.M. Chitanvis, *App. Opt.* **24**,3552,(1985)

C) "Explosion of water droplets", S.M. Chitanvis. *App. Opt.* **25**,1637 (1986)

ABSTRACT.

We have created a model of the explosive vaporization of small droplets by the absorption of energy from a high energy laser beam. The model consists of a polarizable drop of fluid interacting with laser radiation. A criterion for the explosion of the droplet has been introduced. Selfsimilarity is invoked to reduce the spherically symmetric problem involving hydrodynamics and Maxwell's equations to simple quadrature. We point out that there is experimental evidence in favor of our model.

I. INTRODUCTION.

In an earlier paper[1] (hereafter referred to as I) we reported the serendipitous event that the combined equations of Maxwell and those of hydrodynamics admit a selfsimilar set of solutions. The main reason selfsimilarity/scaling works is that Maxwell's equations (source-free) as well as the hydrodynamic equations (source-free) have no scale associated with them per se. It is the particular physical problem being solved that sets its own scale. It is therefore no surprise that when electromagnetic waves interact with matter fields in the hydrodynamic regime, selfsimilarity survives as well.

We realized that some details in our earlier model[1] had been left unclear, especially in the way some of the parameters could be related to measurable quantities. We therefore made changes in the source term to be used in the hydrodynamic equations. This led naturally to an elegant model which turned out to be more realistic than our previous one. We can now make specific predictions of how a high energy laser will vaporize a small droplet (the initial radius a_0 of the droplet is $<<$ incident wavelength λ).

We have introduced a simple criterion for the explosion of droplets viz., the absorbed energy is greater than the energy due to surface tension.

We now have a better physical picture of the explosion process, coupled to a better knowledge of the limitations of our approach to this problem. In particular, our model is rigorously valid for short pulses of electromagnetic radiation. On the other hand, it may turn out that our assumption that the radius of the drop is increasing linearly in time may give an adequate average description of the explosion process for longer pulses as well.

II. THE PHYSICAL PROBLEM.

We have taken a simple criterion for the explosive vaporization of the droplet. We shall suppose that the incident laser radiation dumps energy into the droplet at a high rate so that the droplet becomes superheated[2] i.e., enters a metastable liquid state in which the temperature rises above the usual boiling temperature. Following a conventional picture of explosive vaporization,[2] the metastable liquid goes into a vapor phase "instantaneously" (via the formation of miniscule bubbles) at a temperature $\sim 0.8T_C$, T_C being the critical temperature of the liquid. We then say the vapor in the drop will explode if the energy absorbed per unit volume during the duration of the pulse is greater than the pressure due to the surface tension of the liquid surrounding the vapor bubbles. To get an order of magnitude estimate, we write the following inequality:

$$\alpha' I \tau_p >> 2\sigma/a_0 \tag{2.1}$$

where $\alpha' = \alpha/a_0$, α being the dimensionless Mie absorption efficiency, I is the intensity of the incident laser radiation (power/unit area), τ_p is pulse length, σ is the surface tension (72.0e-3 N/m), and a_0 is the radius of the drop. It turns out that for hygroscopic aerosols with $a \sim 10.0$ µm, $\alpha' \sim 10^4$/m, with $I \sim 10^6$ W/cm^2 , $\tau_p \sim 1.0$µsec., $[\ \alpha' I \tau_p\ ] \sim 10^3 \cdot [\ 2\sigma/a_0\ ]$. Thus the inequality in Eqn. (2.1) is eminently satisfied and the droplet will explode. These numbers are relevant to the Kafalas and Hermann[3] experiment. Since Kafalas and Hermann were able to explode their droplets, we claim that our criterion for the explosion of a droplet is consistent with at least one set of experiments.

If the wavelength of the laser radiation is longer than the size of the spherical droplet, it is natural to assume that we shall have spherical symmetry in the problem. Since the radius of the droplet is small compared to the wavelength, we shall also assume the electromagnetic and hydrodynamic variables inside the droplet vary negligibly, and will be taken to be a constant. Since our model is expected to be true for a short time after the explosion begins, we shall assume that the density within is the unperturbed density. The temperature will be taken to be the superheated temperature[2] $T_h \sim 0.8\ T_c$ (T_c is the critical temperature), and the velocity to be zero. The electric field inside is related to the intensity I (power/area) by:

$$E_{in} = \sqrt{(4\pi/c)I} \qquad (2.2)$$

Mass, momentum, energy will be conserved at the explosion interface. If the interface speed (related to the energy absorbed) is sufficiently large, we will have a shock wave at the interface, and Hugoniot conditions apply. Otherwise we shall take the hydrodynamic variables to be continuous at the surface.

Since there are no free charges in the problem (we assume that the laser beam causes no ionization) $\nabla \cdot \mathbf{E} = 0$, where $\mathbf{E}$ is the electric field.

III. SELFSIMILARITY (SCALING).

Using spherical symmetry, the hydrodynamical equations for mass and momentum conservation are:

$$\partial \rho(r,t)/\partial t = - (\partial/\partial r + 2/r) (v(r,t)\ \rho(r,t)) \qquad (3.1)$$

$$\rho(r,t)\,(\partial v(r,t)/\partial t + v(r,t)\partial v(r,t)/\partial r\,) = -\partial P(r,t)/\partial r \qquad (3.2)$$

The equation for the conservation of energy is

$$\rho(r,t)\,C_V\,(\,\partial/\partial t + v(r,t)\,\partial/\partial r\,)T(r,t) =$$
$$- P(r,t)(\partial/\partial r + 2/r)v(r,t) + (\alpha/8\pi)\,\partial/\partial t\,|E(r,t)|^2 \qquad (3.3)$$

where C_V is the specific heat at constant volume, T is the temperature. $E(r,t)$ is the electric field, and α is the dimensionless Mie absorption coefficient. The first term on the right hand side of Eq.(3.3) denotes the "usual" cooling due to expansion. The second term represents the temporal rate of absorption of the electromagnetic field energy. In our previous paper we had taken this term to be proportional to E^2, so that it was not clear how the proportionality constant was to be determined. This term allows a more elegant set of selfsimilarity transformations.

In the Lorentz gauge, Maxwell's equations for a deformable, polarizable medium with no free charges can be written in terms of the electric field alone:[4]

$$(1/c_0^2\,\partial^2/\partial t^2 - \nabla^2\,)\,\mathbf{E}(r,t) = b\,\nabla\,(\,\nabla\cdot(\rho(r,t)\,\mathbf{E}(r,t)))$$
$$+ \upsilon\,\partial^2(\rho(r,t)\,\mathbf{E}(r,t))/\partial t^2 \qquad (3.4)$$

$$\nabla\cdot\mathbf{E}(r,t) = 0 \qquad (3.4a)$$

where

$$b = -\chi_0/\rho_0\epsilon_0 \qquad (3.5)$$

$$\upsilon = -\mu_0\chi_0/\rho_0 c_0 \qquad (3.6)$$

Here, c_0 is the speed of light. Equation (3.4) derives its form from the fact that we took the polarizability $\chi(r,t)$ to be:

$$\chi(r,t) = (\chi_0/\rho_0)\, \rho(r,t) \tag{3.7}$$

In other words, an external beam dumps energy in the medium, changing the density, velocity, temperature of the medium; a change in the density of the medium must necessarily affect the refractive index of the medium, which in turn must change the electromagnetic fields themselves. Equations (3.4)-(3.7) represent a model for the interaction of polarizable media with electromagnetic fields. μ_0, ϵ_0, ρ_0 refer to the undisturbed medium.

Thus, we have the hydrodynamic variables and the electromagnetic fields "driving" each other.

In order to satisfy Eqn(3.4a), we assume **circular** polarization:

$$\mathbf{E}(r,t) = \boldsymbol{\phi}\, E(r,t) \tag{3.8}$$

where $\boldsymbol{\phi}$ is the unit vector in the azimuthal direction in spherical co-ordinates. Since $\boldsymbol{\phi}\cdot\mathbf{r} = 0$, Eqn.(3.4a) is automatically satisfied. Note that we have imposed radial symmetry on the electromagnetic fields, just as we did on the hydrodynamic fields. In our previous paper,[1] we had assumed a radial polarization. Equation (3.4) then takes the form:

$$[\, 1/c_0^2\, \partial^2/\partial t^2 - (\, \partial^2/\partial r^2 + (2/r)\, \partial/\partial r\,)\,]\, E(r,t) = \upsilon \partial^2(\rho E)/\partial t^2 \tag{3.9}$$

The terms involving the space derivative of $\rho(r,t)$ have dropped out. The reason is that the electric fields are assumed to be polarized in the $\boldsymbol{\phi}$ direction and hence are "transverse". These transverse fields then cannot excite the density along the radial/"longitudinal" direction.

Since we are considering an exploding drop, there are two relevant dimensional parameters, viz., the energy density ϵ inside the droplet (we include in ϵ the pressure and the electromagnetic field energy density) which supplies the explosive energy, and the density $\rho_{(0)}$ of the aerosol at the time of the explosion. The simplest way to get a length parameter out of these is:

$$R \sim \sqrt{(\epsilon/\rho_{(0)})}\ t \tag{3.10}$$

where t is the time variable. We shall therefore assume that the surface of the aerosol is expanding at a uniform speed. Let this speed be c. Thus, the radius of the aerosol is:

$$R_S(t) = c\ (t+t_0) \tag{3.11}$$

with

$$c \approx \sqrt{(\alpha I \tau_p/(a_0 \rho_{(0)}))} \tag{3.11a}$$

where t_0 is some initial time, α is the dimensionless Mie absorption coefficient, I is the intensity of the laser (power/area), a_0 is the initial radius of the droplet. α is given in the long wavelength limit in Mie theory by:[5]

$$\alpha = 4\ (2\pi a_0/\lambda)\ \mathrm{Im}\ \{\ (n^2(\lambda) - 1)/(n^2(\lambda) + 1)\ \} \tag{3.11b}$$

In Eq. (3.11b), a_0 is the initial radius of the droplet, λ is the incident wavelength, and $n(\lambda)$ is the complex refractive index.

These considerations hold for a short time after the explosion starts.[6] After a long time,[6] $R_S(t) \sim t^{1/2}$, which is indicative of diffusive behaviour.

If we now make the following ansatze:

$$\rho(r,t) = \rho'(\xi) \tag{3.12}$$

$$v(r,t) = v'(\xi) \tag{3.13}$$

$$T(r,t) = T'(\xi) \tag{3.14}$$

$$E(r,t) = F'(\xi) \tag{3.15}$$

where

$$\xi = r/(c(t+t_0)) \equiv r/R_s(t) \tag{3.16}$$

We assume a perfect gas law for ease of computations:

$$P(\xi)\rho^{-1}(\xi) = R_g T(\xi) \tag{3.17}$$

we get the following set of coupled ordinary differential equations (after straightforward but somewhat lengthy algebra)

$$dv(\xi)/d\xi = 2\kappa_2[(v(\xi)-\xi)^2 - \kappa_2(\kappa_3-1)T(\xi)]^{-1}$$
$$[\,(\kappa_3+1)v(\xi)T(\xi)/\xi + \kappa_1\,\xi\,(d/d\xi|F(\xi)|^2)\,/(2\rho(\xi))\,] \tag{3.18}$$

$$dT(\xi)/d\xi = -\kappa_3 T(\xi)(dv(\xi)/d\xi+2v(\xi)/\xi)/(v(\xi)-\xi)$$
$$-\kappa_1\,\xi(d/d\xi|F(\xi)|^2)\,/(\rho(\xi)\,(v(\xi)-\xi)) \tag{3.19}$$

$$d\rho(\xi)/d\xi = -\,\rho(\xi)(dv(\xi)/d\xi+2v(\xi)/\xi)/\,(v(\xi)-\xi) \tag{3.20}$$

$$d^2F(\xi)/d^2\xi^2 = -2F(\xi)/\xi \tag{3.21}$$

where ρ,v,T,F are normalized functions as defined below

$$\rho(\xi) = \rho'(\xi)\,/\rho_{(0)} \tag{3.22}$$

$$T(\xi) = T'(\xi)/T_{(0)} \tag{3.23}$$

$$v(\xi) = v'(\xi)/c \tag{3.24}$$

$$F(\xi) = F'(\xi)/F'(\xi=1) \tag{3.25}$$

where $\rho_{(0)}$ is the density of the droplet at the surface, $T_{(0)}$ is the temperature of the drop at the surface, and c is the speed of the surface.

Also

$$x_1 = \alpha \, |F'(\xi=1)|^2/(8\pi C_V T_{(0)} \rho_{(0)} c) \tag{3.26}$$

$$x_2 = R_g T_{(0)}/c^2 \tag{3.27}$$

$$x_3 = R_g/C_V \tag{3.28}$$

It is important to point out that the speed of light c_0 is much greater than c, the speed of the expanding surface. And in Eq. (3.21) for $F(\xi)$, we have neglected terms of the order of c/c_0. In other words, we are not looking for the transient response of the electric fields, but rather the "long-term" effects of the hydrodynamic fields on the electric fields. As a consequence of this and the fact that we assumed a **circular** polarization, we find that there is no longer an explicit dependence on any of the hydrodynamic variables. Nevertheless, the invocation of self-similarity to describe the combined dynamics of the electric fields and the hydrodynamic fields presupposes an implicit connection between the two types of fields.

In refcrence 1, we had to scale $E(r,t)$ by $\sqrt{t}$ to obtain selfsimilarity. In the present model, this undesirable feature has been eliminated.

For sufficiently large c, the boundary conditions on the matter variables are given by the Hugoniot conditions because at the explosion interface between the droplet and the air around it, it would be safe to assume that that mass, momentum and energy are conserved. Thus:

$$(R_g/c^2)T(1)\,\rho(1) = 2/(T_{(0)}(\gamma+1)) - (\gamma-1)/((\gamma+1))\,(R_g/T_{(0)}c^2) \tag{3.29}$$

$$\rho(1) = [(\gamma-1)/(\gamma+1) + T(1)][1+(\gamma-1)/(\gamma+1)\;T(1)\;]^{-1} \tag{3.30}$$

$$v(1) = 2\,(\,T(1) - 1\,)/(\gamma-1 + (\gamma+1)\,T(1)) \tag{3.31}$$

These boundary conditions obviously preserve the self-similarity of the hydrodynamic variables.

If c is not large enough to obtain physically meaningful solutions to eqns.(3.29)-(3.31) the matter variables are assumed to be continuous across the surface.

We obtain $F'(1)$ by considering a temporally flat laser pulse, so that:

$$|F'(1)|^2 = (4\pi/c)\, I \qquad (3.32)$$

In addition we have $F'(\xi \to \infty) \sim 0$, so that

$$F'(\xi) = F'(1)/\xi \qquad (3.33)$$

The first-order ordinary hydrodynamical equations are solved by the Euler method with a first order predictor-corrector correction. We start at the surface $\xi = 1$ and propagate the solution outwards. The code was tested in the absence of any electric field ($F'(\xi) \equiv 0$), in the linearized regime where analytic solutions can be easily found.

IV. THE SOLUTION.

We chose to work at $\lambda = 10.6\ \mu m$, $I = 10^7\ W/cm^2$, $\tau_p = 1.0\ \mu sec.$, $a_0 = 10.0\ \mu m$ and we took the super-heated temperature of the vapor within to be ~ 380°C in order to approximate the physical situation in the experiment of Kafalas and Hermann.[3] To be accurate, Kafalas and Hermann allowed the pulse to go by and then observed the subsequent explosion. We attempted to describe the post-irradiation hydrodynamics in an earlier paper.[7] We now wish to model the dynamics while the laser pulse is still on.

Figure 1 clearly demonstrates a shock tube type of behaviour in which a layer of vapor is formed, with a shock front at the head. Figure 2 demonstrates the existence of a shock wave in the velocity profile. However, the $v(\xi)$ is now increasing at the shock front, in contrast to the density profile, as if to conserve $1/2\ \rho v^2$. Figure 3 demonstrates a cooling curve in the temperature. The droplet in this case acts as a source of heat and the temperature decreases away from it. Figure 4 shows the electric field. In all four figures, the variables eventually decrease to their ambient levels.

It is unfortunate that we cannot do more to compare our results with experimental observations.[3] This is because the schlieren photographs in reference 3 are not clear enough to provide even a qualitative profile of the layer of vapor depicted in Fig. 1. We are not aware of a similar experiment by any other group depicting spherical symmetry and providing details of the density profile, etc.

The shortcoming of our model is that it strictly applies only for a short time after the explosion begins. This problem may be overcome by linearizing a more complicated temporal behaviour of the surface in small time slices, and then applying our model in each time slice. In addition, we have ignored the ambient medium. This means that we cannot study the shock wave that will propagate in the air surrounding the droplet.

We would like to conclude that the process we have modeled here is more violent in nature than the phenomenon of convective vaporization studied by R.L. Armstrong et al.[8] Convective vaporization presumably takes place when the inequality in Eqn. (2.1) is not satisfied.

V. ACKOWLEDGMENTS.

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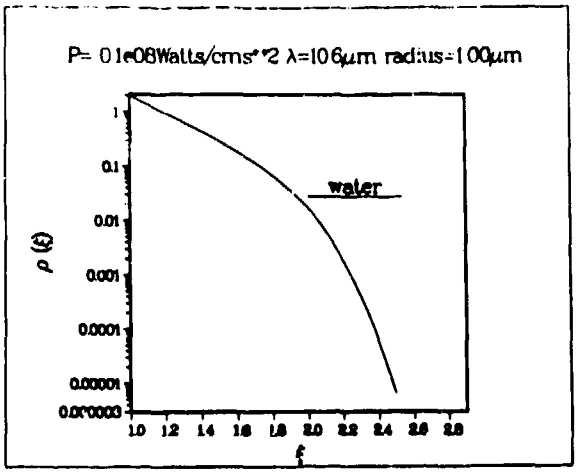

Fig. 1: Normalized profile for the density desplaying shock tube type of behaviour.

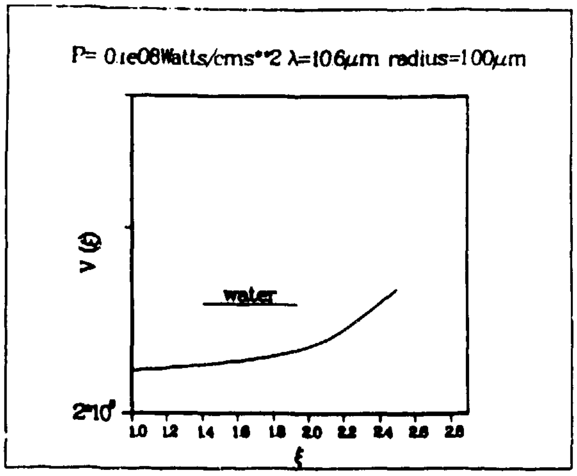

Fig. 2: Normalized velocity displaying a shock wave.

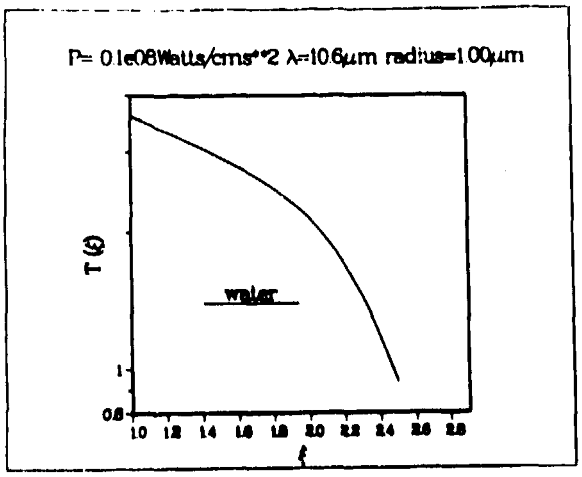

Fig. 3: Normalized temperature showing a cooling curve.

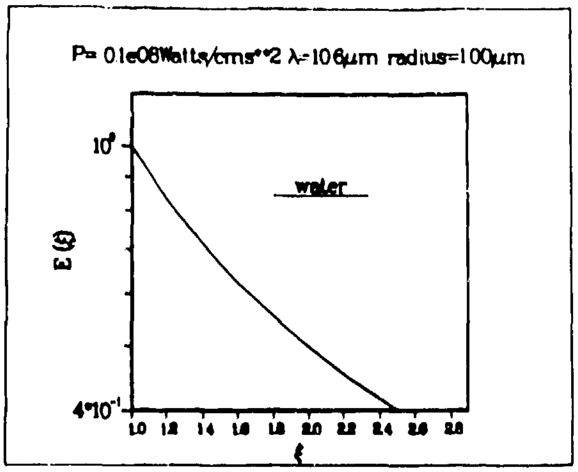

Fig. 4: Normalized electric field.