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OFF-AXIS SCATTERING OF LASER BEAMS USING SINGLE AND MULTI-GAUSSIAN PHASE FUNCTION APPROXIMATION

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RECENT PUBLICATIONS, SUBMITTALS FOR PUBLICATION AND PRESENTATIONS:

A) A. Zardecki, S. A. W. Gerstl, and R. E. DeKinder, Jr., "Diffusion Approximation for Modeling of 3-D Radiation Distributions," Proceedings of the 1985 CRDC Scientific Conference on Obscuration and Aerosol Research, R. H. Kohl, Ed., Chem. Res. and Devel. Center report. In preparation.

B) R. L. Armstrong, S. A. W. Gerstl, and A. Zardecki, Nonlinear Pulse Propagation in the Presence of Evaporating Aerosols," J. Opt. Soc. Am. A2, 1739-1746 (1985).

C) A. Zardecki, S. A. W. Gerstl, W. G. Tam, and J. F. Embury, Image Quality Degradation in a Turbid Medium under Partially Coherent Illumination," J. Opt. Soc. Am. A3, 393-400 (1986).

D) S. A. W. Gerstl, A. Zardecki, and H. L. Wiser, "A New UV-B Handbook, Vol. 1," Stratospheric Ozone Reduction, Solar Ultraviolet Radiation and Plant Life, Edited by R. C. Worrest and M. M. Caldwell (Springer-Verlag, Berlin Heidelberg, 1986) pp. 63-74.

E) S. A. W. Gerstl, S. M. Chitanvis, and A. Zardecki, "The Scattered Radiation Field of a Laser Beam Propagating through Turbid Media," Proceedings of Smoke/Obscurants Symposium X at Harry Diamond Laboratories, Adelphi, Maryland, April 22-24, 1986.

F) R. L. Armstrong, A. Zardecki, and S. A. W. Gerstl, "Hydrodynamics of Evaporating Aerosols Irradiated by Intense Laser Beams," International Conference on Lasers '85, Las Vegas, Nevada, December 2-6, 1985.

G) A. Zardecki and S. A. W. Gerstl, "Significance of Multiple Scattering in Imaging through Turbid Media, SPIE Annual Meeting, Orlando, Florida, March 31 - April 4, 1986.

H) A. Zardecki and S. A. W. Gerstl, "Multiple Scattering Effects in the Off-Axis Propagation of Laser Radiation," International Conference on Optical and Millimeter Wave Propagation and Scattering in the Atmosphere, Florence, Italy, May 27-30, 1986.

I) A. Zardecki, S. A. W. Gerstl, W. P. Unruh, G. H. Stokes, D. M. Stupin, N. E. Elliot, and J. A. Weinman, Multiple Scattering of Laser Beams in Dense Hydrosols," International Laser Radar Conference, Toronto, Canada, August 11-15, 1986.

J) R. L. Armstrong, P. J. O'Rourke, and A. Zardecki, "Vaporization of Irradiated Droplets," submitted to Phys. Fluids, March, 1986.

ABSTRACT

Within the small-angle approximation, a multi-Gaussian phase function model provides a new description of the multiple scattering process and leads to improved agreement with experimental data. Future work will be concerned with the combination of small-angle and diffusion approximations in the radiative transfer. This should lead to an improved predictive model of the performance of electrooptical systems in aerosol-loaded atmospheres.

INTRODUCTION

Recent results of Battistelli et al.[1,2] demonstrate the significance of multiple scattering in the on-axis laser beam propagation through turbid media. When the mean size of particles is larger than or comparable to the wavelength of radiation, the small-angle approximation provides an adequate interpretation of on-axis experimental data. For the off-axis configuration the small-angle approximation remains valid when the scattering

angles are of the order of the angular spread of the scattering phase function. When the small-angle and diffusion approximations are combined, the range of validity of the theory can be extended considerably.[3,4]

In this paper, we use an earlier idea due to Weinman et al.[5] to represent the scattering phase function as a linear combination of Gaussian components. We derive an expression for the irradiance distribution and demonstrate the usefulness of our solution in interpreting experimental data.

RADIATIVE TRANSFER CALCULATIONS

Under the small-angle approximation and with the z axis chosen along the direction of the incident beam, the equation of radiative transfer[6,7] reads:

$$\left(\vec{\phi}\cdot\frac{\partial}{\partial\vec{r}} + \frac{\partial}{\partial z} + \sigma\right) I(\vec{\phi},\vec{r},z) = \sigma_s \int P(\vec{\phi}-\vec{\phi}')I(\vec{\phi}',\vec{r},z)d\vec{\phi}' \quad , \tag{1}$$

where $I(\vec{\phi},\vec{r},z)$ is the spectral radiance function of a monochromatic beam wave at a point with coordinates $(\vec{r},z)$ along a direction whose unit directional vector $\vec{\Omega}$ has a projection $\vec{\phi}$ transverse to the beam axis. The volume extinction and scattering coefficients are denoted σ and σ_s, respectively. The normalization condition for the scattering phase function $P(\vec{\phi})$ is

$$\int P(\vec{\phi})d\vec{\phi} = 1 \quad . \tag{2}$$

We assume that the incident laser beam can be represented by a Gaussian function in both spatial distribution and angular divergence, i.e.,

$$I_0(\vec{\phi},\vec{r}) = \frac{\beta^2\gamma^2}{\pi^2}\exp(-\beta^2\vec{\phi}^2 - \gamma^2\vec{r}^2) \quad , \tag{3}$$

and that the phase function is the sum of Gaussian functions, i.e.,

$$P(\vec{\phi}) = \frac{1}{\pi}\sum_{m=1}^{M} K_m\alpha_m^2\exp(-\alpha_m^2\vec{\phi}^2) \quad . \tag{4}$$

Equation (2) implies that

$$\sum_{m=1}^{M} \kappa_m = 1 \quad . \tag{5}$$

It is not difficult, by going to the Fourier space, to derive the general solution of Eq. (1) valid for an arbitrary value of M. Following Ref. 6, we give here the formula for the irradiance N(r,z), which in the small angle approximation is defined as

$$N(\vec{r},z) = \int I(\vec{\phi},\vec{r},z)\, d^2\phi \quad . \tag{6}$$

For brevity, we write the irradiance for the special case, M = 2; the general case essentially involves the multinomial formula instead of the binomial formula used here. Thus, for M = 2, the result is

$$N(\vec{r},z) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} N_{k_1k_2}(\vec{r},z) \quad , \tag{7}$$

where

$$N_{k_1k_2}(\vec{r},z) = \frac{1}{\pi} e^{-\sigma z} \frac{(\kappa_1\sigma_s z)^{k_1}}{k_1!} \frac{(\kappa_2\sigma_s z)^{k_2}}{\kappa_2!}$$

$$\times \int_0^1 dx_1 \cdots dx_{k_1} \int_0^1 dy_1 \cdots dy_{k_2} \frac{1}{\Lambda_{k_1k_2}} \exp\left[-\frac{\vec{r}^2}{z^2\Lambda_{k_1k_2}}\right] \quad . \tag{8}$$

In Eq. (8),

$$\Lambda_{k_1k_2} = \frac{1}{\alpha_1^2} \sum_{j=0}^{k_1} x_j^2 + \frac{1}{\alpha_2^2} \sum_{j=0}^{k_2} y_j^2 + \frac{1}{\beta^2} + \frac{\sigma^2}{\tau^2\gamma^2} \quad . \tag{9}$$

The double-infinite series in Eqs. (7)-(9) generalizes the multiple scattering series of Ref. 3 for the case of a two-component Gaussian phase function. Each term of the series, labeled (k_1,k_2), represents the scattering event of order k_1 and k_2 due to the first and second component of the phase function, respectively. We stress that this interpretation becomes transparent only when the solution to the radiative transfer equation is cast into the form of a multiple scattering series. A crucial point of the derivation is the Gaussian model of the phase function.

COMPARISON OF COMPUTED WITH EXPERIMENTAL RESULTS

In principle, the parameters κ_1 and κ_2 in Eq. (4) can be determined from the normalization condition (5) combined with condition at $\theta = |\vec{\phi}| = 0$:

$$\kappa_1 \frac{\alpha^2}{\pi} + \kappa_2 \frac{\alpha^2}{\pi} = P(0) \quad . \tag{10}$$

In practice, however, it is more effective to determine the κ's by fitting Eqs. (7)-(9) to experimental data. Weinman et al.[5] sought the best approximation to the scattering phase function using an expansion similar to that given by Eq. (4); in a different context, de Wolf and Pack[8] discretized the Wigner distribution function into a sum of Gaussians.

To determine how well our model describes laser beam scattering, we compare the predictions of the formula for irradiance with the measurements of optical scattering by high-density hydrosols. The laboratory system, which is described in detail elsewhere,[9,10] monitors the off-axis irradiance profile as a function of radial distance with the aid of a wide-angle detector (field of view half angle of 90 deg). The hydrosol consists of a colloidal suspension of 2.26 ± 0.07 μm latex spheres in deionized water. The path length through the hydrosol is 10.2 cm; the light source is generated by a 1 mW He Ne laser with a wavelength of 0.6328 μm (0.4754 μm in water) and an unscattered beam spot size of 0.4 cm.

Figure 1 shows the fit to the Mie phase function, in which the two Gaussians with $\kappa_1 = 0.9$ and $\kappa_2 = 0.5$ are employed; the corresponding spread parameters of the Gausssians are $\alpha_1 = 8.46$ rad^{-1} and $\alpha_2 = 2.29$ rad^{-1}. Figures 2 and 3 compare the measured and calculated irradiance distributions for two optical depths of 5.02 and 10.16, respectively. Table I lists relative contributions of different scattering events.

TABLE I. RELATIVE CONTRIBUTIONS OF SCATTERING EVENTS (%).

	τ=5.02	τ=10.16
N_{00}	<0.01	<0.01
N_{10}	6.97	0.05
N_{01}	1.23	<0.01
$\sum_{k=2}^{\infty} N_{k0}$	34.02	5.40

$\sum_{m=2}^{\infty} N_{0m}$	0.58	0.02
N_{11}	3.24	0.05
$\sum_{k=2}^{\infty} N_{k1}$	24.30	11.27
$\sum_{m=2}^{\infty} N_{1m}$	2.11	0.14
$\sum_{k=2}^{\infty} \sum_{m=2}^{\infty} N_{km}$	27.55	83.07

In conformity with earlier findings,[6] higher-order scattering events play a dominant role as the optical depth increases.

CONCLUSIONS

For an optically dense medium of Mie scatterrers, where the radius of the particles is larger than the wavelength of the laser radiation, we analyzed the off-axis scattering experiment using the small-angle approximation in which the scattering phase function was modeled in terms of a linear combination of Gaussian components. This approach has definite advantages over the conventional small-angle approximation, which often uses a single Gaussian. When the weight factors K_1 and K_2 are regarded as adjustable parameters--the strategy used here--we actually can extend the range of validity of the theory for larger radial distances than predicted by a combination of a single Gaussian and diffusion.[10]

Still, because the energy conservation condition, Eq. (4), is violated, we need a better theory to account for the large-angle scattering. A combination of the approach advocated in this paper with the diffusion theory should lead to the desired goal.

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REFERENCES

1. E. Battistelli, P. Bruscaglioni, A. Ismaelli, and G. Zaccanti, "Use of Two Scaling Relations in the Study of Multiple-Scattering Effects on the Transmittance of Light Beams Through a Turbid Atmosphere," J. Opt. Soc. Am. A2, 903-912 (1985).
2. E. Battistelli, P. Bruscaglioni, A. Ismaelli, and G. Zaccanti, "The Effect of a Strongly Inhomogeneous Medium on the Propagation of Light Beams under Multiple Scattering Conditions," Opt. Acta 32, 717-730 (1985).
3. W. G. Tam and A. Zardecki, "Off-Axis Propagation of a Laser Beam in Low Visibility Weather Conditions," Appl. Opt. 19, 2822-2827 (1980).
4. A. Zardecki and S. A. W. Gerstl, "Multiple Scattering Effects in the Off-Axis Propagation of Laser Radiation," International Conference on Optical and Millimeter Wave Propagation and Scattering in the Atmosphere, Florence, Italy, May 27-30, 1986.
5. J. A. Weinman, J. T. Twitty, S. R. Browning, and B. M. Herman, "Derivation of Phase Functions from Multiply Scattered Sunlight Transmitted Through a Hazy Atmosphere," J. Atmos. Sc. 32, 577-583 (1975).
6. W. G. Tam and A. Zardecki, "Multiple Scattering Corrections to the Beer-Lambert Law. 1: Open Detector," Appl. Opt. 21, 2405 (1982).
7. A. Zardecki and W. G. Tam, "Multiple Scattering Corrections to The Beer-Lambert Law. 2: Detector with a Variable Field of View," Appl. Opt. 21, 2413 (1982).
8. D. A. de Wolf and J. K. Pack, "Wave Kinetic Numerical Approach to Propagation of Optical Beams," J. Opt. Soc. Am. A3, 532-535 (1986).
9. A. Zardecki, S. A. W. Gerstl, W. P. Unruh, G. H. Stokes, D. A. Stupin, N. E. Elliott, and J. A. Weinman, "Multiple Scattering of Laser Beams in Dense Hydrosols," International Laser Radar Conference, Toronto, Canada, August 11-15, 1986.
10. S. A. W. Gerstl, A. Zardecki, W. P. Unruh, D. M. Stupin, G. H. Stokes, and N.E. Elliott, "Off-Axis Multiple Scattering of a Laser Beam in Turbid Media; Comparison of Theory and Experiment," Submitted for publication to Applied Optics, July 1986.

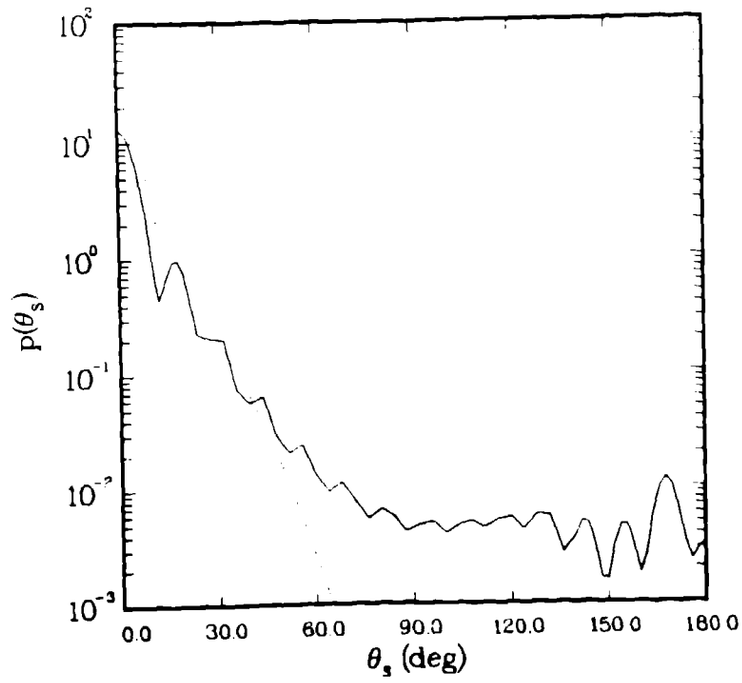

FIGURE 1. Comparison between the Mie theory phase function (solid line) for 2.26 μm polystyrene spheres and the two-component Gaussian phase function. $\kappa_1 = 0.9$, $\kappa_2 = 0.5$, $\alpha_1 = 8.46$ rad^{-1}, $\alpha_2 = 2.29$ rad^{-1}.

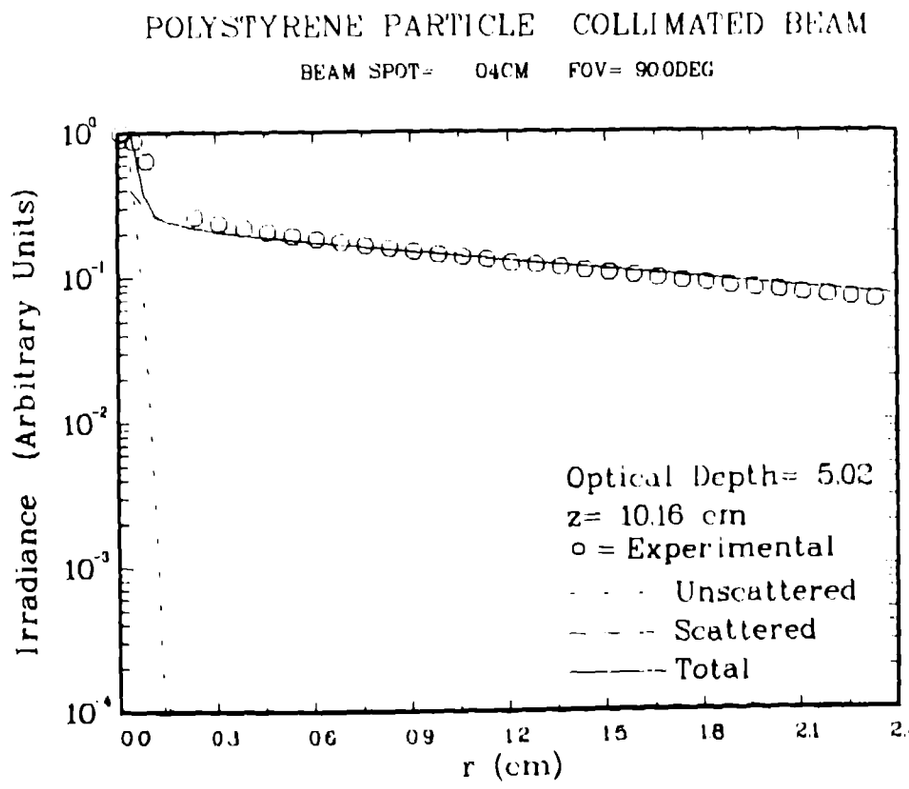

FIGURE 2. Measured and calculated intensities (irradiances) for the optical depth of 5.02.

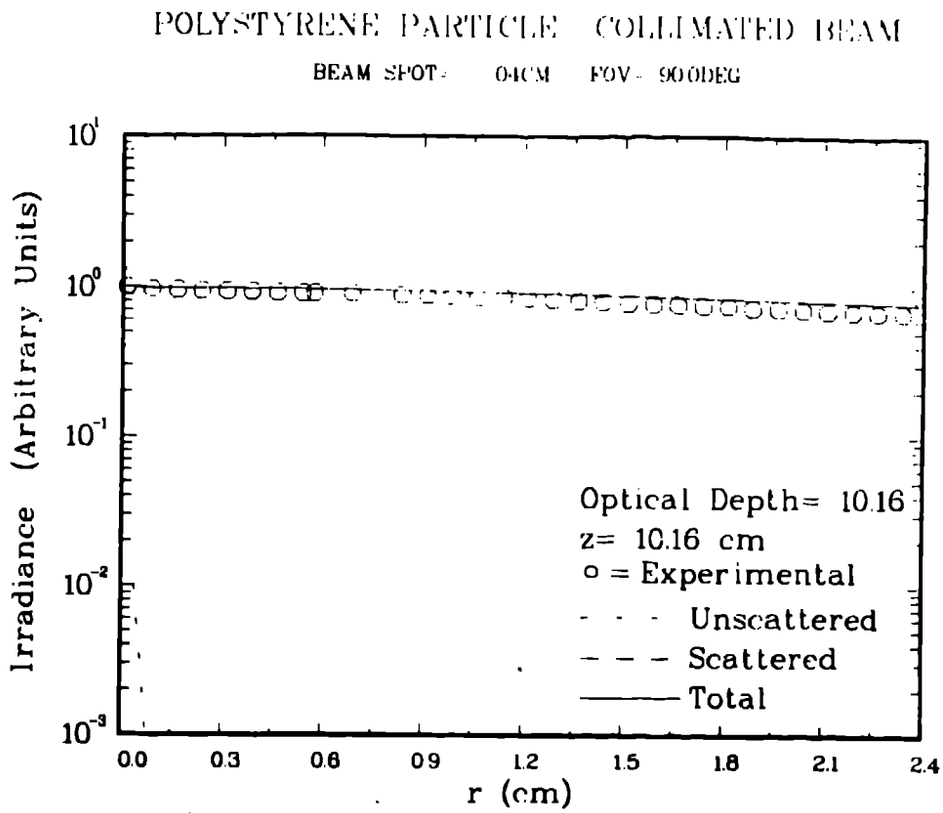

FIGURE 3. Measured and calculated intensities (irradinces) for the optical depth of 10.16.