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PROTON DECAY IN THE SUPER-WORLD

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INTRODUCTION

One of the most dramatic predictions of any grand unified theory (GUT) is for the decay of the nucleon.[1] The experimental observation of this decay could illuminate the physical principles operating on scales as small as 10^{-30}cm. In this talk we shall elaborate on the predictions for nucleon decay in supersymmetric (SUSY) GUTs.

General Analysis

The experimental limits on nucleon decay are sensitive to the particular decay modes. It is clear however that in general

$$\tau_N \gtrsim 10^{31} \text{ years} \quad ,$$

$$= .46 \times 10^{63} \text{ GeV}^{-1} \quad .$$

Assuming that the baryon number violating processes originate from a fundamental theory at a scale $M \gg m_W$ (i.e. much greater than the weak scale) and that the only particles with mass less than M are at m_W or below, we can, by integrating out all states with mass M or greater, obtain an effective action at m_W of the form

$$\mathcal{L}_{eff} \sim \frac{b}{M^{d-4}} \; O^{(d)} \quad .$$

The baryon violating operator of mass dimension d, $O^{(d)}$ is a product of local operators including particles with mass $\lesssim O(m_W)$. $O^{(d)}$ is invariant under the low energy gauge group $SU_3 \times SU_2 \times U_1$. The nucleon lifetime is then given by

$$\tau_N = \frac{c^{-2}}{m_N}\left(\frac{M}{m_N}\right)^{2(d-4)}$$

where c is proportional to b and the matrix element of $O^{(d)}$ between the nucleon and particular final states.

For d = 6 and c ~ 1 we find that $M \gtrsim 4 \times 10^{15}$ Gev. Note that in the standard model with the minimal particle content, dimension 6 four fermi operators are the lowest dimension baryon violating operators consistent with $SU_3 \times SU_2 \times U_1$.[1] All such operators conserve the quantum number B-L (Baryon # - Lepton #). Dimension 6 operators require a new scale $M \sim 10^{15}$ Gev in order to give reasonable amounts of nucleon decay. In GUT's this scale is naturally associated with the grand unification scale, M_{GUT}.

Consider now d = 5 (4). If we assume $M \sim 4 \times 10^{15}$ Gev we find

$$c \lesssim 10^{-16}\ (10^{-32})$$

in order to be consistent with observation.[F1]

It was realized that Baryon violating SUSY operators consistent with $SU_3 \times SU_2 \times U_1 \times$ SUSY can have d = 4,5 or 6.[2] Clearly for d = 4 or 5 either one must forbid these operators by introducing new symmetries or there must be some naturally small numbers inherent in c.

Dimension 4 SUSY operators will be eliminated by demanding a discrete symmetry R parity. d = 5 operators on the other hand can lead to reasonable nucleon decay rates.[3] All d = 5 operators (consistent with R-parity) conserve B-L.

The spectrum of states in the minimal low energy N = 1 SUSY theory is given in Table 1.

Table 1.

ordinary particles		super partners	
spin 1/2		spin 0	
$q_i = \begin{pmatrix} u \\ d \end{pmatrix}_i$	$\bar{u}_i$ -4/3 $\bar{d}_i$ 2/3	$\tilde{q}_i = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_i$	$\tilde{\bar{u}}_i$ -4/3 $\tilde{\bar{d}}_i$ 2/3
$l_i = \begin{pmatrix} \nu \\ e \end{pmatrix}_i$	$\bar{e}_i$ +2	$\tilde{l}_i = \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_i$	$\tilde{\bar{e}}_i$ +2
spin 1	$g,\ W^{\pm}, Z^{o}\ \gamma$	spin 1/2	$\tilde{g}, \tilde{W}^{\pm},\ \tilde{\gamma}^{o}$

spin 0

$$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}^{+1} \qquad \bar{h} = \begin{pmatrix} \bar{h}^- \\ \bar{h}^0 \end{pmatrix}^{-1}$$

$Q = I_3 + Y/2$ (hypercharge is given in superscripts)
$i = 1,2,3$ number of generations
$\alpha = 1,\ldots, 4$ number of neutralinos
$\tilde{w}^{\pm}$, $\tilde{\gamma}^{\alpha}$ are superpositions of gaugino and higgsino states.

It will be convenient to use the compact superspace notation to describe the baryon violating operators. We thus define a general chiral superfield

$$\Phi(x_\mu,\theta_\alpha) = \phi(x) + \sqrt{2}(\theta\psi(x)) + (\theta\theta)\, F(x)$$

where $\phi(x)$, $\psi(x)$ are a complex scalar and left-handed fermi field, respectively, and $F(x)$ is a complex auxiliary field. θ_α is a two-component left-handed Weyl spinor. Note Φ transforms as a Lorentz scalar and satisfies commutation relations. For gauge fields we introduce the chiral SUSY field strength (in the Wess-Zumino gauge)

$$W_\alpha(x_\mu,\theta_\beta) = -i\lambda_\alpha(x) + [D(x) - i\,\sigma^{\mu\nu}F_{\mu\nu}(x)]\theta_\alpha + (\theta\theta)\,\sigma^\mu\,\partial_\mu\,\bar{\lambda}(x)$$

where $\lambda_\alpha(x)$, $F_{\mu\nu}(x) \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$ are the gaugino and gauge field strength, respectively and $D(x)$ is an auxiliary field. The superfield content of the minimal low energy theory is given in Table II.

Table II.

quarks, leptons
squarks, sleptons

$$Q_i = \begin{pmatrix} U \\ D \end{pmatrix}_i \qquad \bar{U}_i \quad \bar{D}_i$$

$$L_i = \begin{pmatrix} V \\ E \end{pmatrix}_i \qquad \bar{E}_i$$

gauge bosons, gauginos
W^3_α, W^2_α, W^1_α for SU_3, SU_2, U_1 resp.

Higgs bosons, Higgsinos

$$H = \begin{pmatrix} H^+ \\ H^\circ \end{pmatrix} \quad \bar{H} = \begin{pmatrix} \bar{H}^- \\ \bar{H}^\circ \end{pmatrix}$$

The baryon number violating operators consistent with $SU_3 \times SU_2 \times U_1 \times$ SUSY are given in Table III.

Table III.

Baryon and/or Lepton number violating operators

$d = 6$

$$R_{even} \begin{cases} \frac{b_1}{M^2} \int d^4\theta \; Q^*_{\alpha i} \bar{U}_j \; Q^*_{\beta k} \; \bar{E}_\ell \; \epsilon^{\alpha\beta} & \Delta B = \Delta L \\ \frac{b_2}{M^2} \int d^4\theta \; Q^*_{\alpha i} \bar{U}_j \; L^*_{\beta k} \; \bar{D}_\ell \; \epsilon^{\alpha\beta} & \Delta B = \Delta L \end{cases}$$

$d = 5$

$$R_{even} \begin{cases} \frac{b_3}{M} \int d^2\theta \; Q_i Q_j Q_k L_\ell & \Delta B = \Delta L \\ \frac{b_4}{M} \int d^2\theta \; \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_\ell & \Delta B = \Delta L \end{cases}$$

$$\int d^2\Theta \; Q_i Q_j Q_k \bar{H} \qquad \Delta B = 1 \quad \Delta L = 0$$

$$\int d^4\theta \; Q_i Q_j \bar{D}^*_k \qquad \Delta B = 1 \quad \Delta L = 0$$

$$\int d^4\Theta \; Q_i \bar{U}_j L^*_k \qquad \Delta B = 0 \quad \Delta L = -1$$

$d = 4$

$$\int d^2\Theta \; \bar{U}_i \bar{D}_j \bar{D}_k \qquad \Delta B = -1 \quad \Delta L = 0$$

$$\int d^2\Theta \; Q_i \bar{D}_j L_k \qquad \Delta L = 1 \quad \Delta B = 0$$

$$\int d^2\Theta \; \bar{E}_i L_j L_k \qquad \Delta L = 1 \quad \Delta B = 0$$

In order to avoid catastrophic nucleon decay rates we shall require our theory to have the discrete symmetry

$$\Phi(x,\theta) \to \eta_\Phi \, \Phi(x,\theta)$$

$$W_\alpha(x,\theta) \to W_\alpha(x,\theta)$$

where

$$\eta_\Phi = -1 \text{ for } \Phi = Q, \bar{U}, \bar{D}, L, \bar{E}; \; \eta_\Phi = +1 \text{ otherwise} \quad .$$

This is the so-called family reflection symmetry.[4] It is equivalent to the discrete symmetry

$$(-1)^{3(B-L)} \quad .$$

Note that the discrete symmetry, R-parity,[5] given by

$$\Phi(x,\theta) \to \eta_\Phi \, \Phi(x,-\theta)$$

$$W_\alpha(x,\theta) \to - W_\alpha(x,-\theta)$$

with η_Φ as above is equivalent to the discrete symmetry

$$(-1)^{3(B-L)} \cdot (-1)^F$$

where F is fermion number. Under R-parity all ordinary particles are R even and all superpartners are R odd.

Since $(-1)^F$ is conserved in any Lorentz invariant theory we see that R-parity and the Family Reflection symmetry are identical. Imposing this symmetry eliminates all the unwanted d = 4 operators. It also eliminates some d = 5 operators. All remaining operators satisfy, as previously stated, $\Delta B = \Delta L$.

The remaining dimension 5 operators

$$F_{ijkl} = \int d^2\theta \; Q_{\alpha ai} \, Q_{\beta bj} \, Q_{\gamma ck} \, L_{\delta l} \times \epsilon^{\alpha\beta} \, \epsilon^{\gamma\delta} \, \epsilon^{abc}$$

$$\bar{F}_{ijkl} = \int d^2\theta \; \bar{U}^a_i \, \bar{U}^b_j \, \bar{D}^c_k \, \bar{E}_l \, \epsilon_{abc}$$

where $(\alpha\beta\gamma\delta)$ (a,b,c) and (i,j,k,l) are SU_2, SU_3 and generation indices resp., satisfy

$$F_{iiji} \equiv 0 \quad , \quad \bar{F}_{ijii} \equiv 0 \text{ for } i = j.$$

<u>Therefore the dominant nucleon decay modes are expected to have strange particles in the final state.</u>[3]

Note, when SUSY is broken at a scale of order m_W then dimension 6 SUSY operators with SUSY breaking insertions can lead to lower dimensional operators in terms of component fields. For example with the insertion of the SUSY breaking term $m^2\theta^2\bar{\theta}^2$, responsible for scalar masses, into the dimension 6 operators we obtain a dimension four (four scalar) operator of the form

$$\frac{bm^2}{M^2_{GUT}} \; \phi^2 \phi^{*2} \quad .$$

Such terms contribute negligible nucleon decay rates since they require at least two loops worth of dressing in order to obtain an effective four fermi operator.[6]

Minimal SU_5 SUSY GUT

The spectrum of states in the minimal SU_5 SUSY GUT is given in Table IV.

Table IV.

$$10_i \supset \begin{pmatrix} \bar{U} & Q \\ & \bar{E} \end{pmatrix}_i \qquad \bar{5}_i \supset \begin{pmatrix} \bar{D} \\ L \end{pmatrix}_i$$

$$W^5_\alpha$$

$$H_5 = \begin{pmatrix} H_3 \\ H \end{pmatrix} \qquad \bar{H}_5 = \begin{pmatrix} \bar{H}_3 \\ \bar{H} \end{pmatrix}$$

24

SU_5 breaking

The 24 is necessary to break SU_5 to $SU_3 \times SU_2 \times U_1$ at M_{GUT}. It is also used to give mass to the color triplet Higgs H_3, $\bar{H}_3$ at a scale $\sim 0\ (M_{GUT})$. The Higgs doublets H and $\bar{H}$ obtain mass $\mu \sim O(m_W)$. This is accomplished by fine tuning parameters. In other scenarios this splitting may be accomplished more naturally.[7] The low energy theory $E \ll M_{GUT}$ includes only those states given in Tables I and II.

The standard renormalization group analysis from M_{GUT} to m_W determines the parameters M_{GUT} and $\alpha_5(M_{GUT})$ in terms of $\alpha_3(m_W)$ and $\alpha_{EM}(m_W)$ and predicts the value of $\sin^2\hat{\theta}_W(m_W)$. We find (at two loops, assuming a common SUSY threshold at M_W)[8]

$$M_{GUT} \cong 6 \times 10^{16}\left(\frac{\Lambda_{\overline{ms}}}{1\text{Gev}}\right) \text{Gev}$$

$$\alpha_5(M_{GUT}) \sim 1/25$$

and

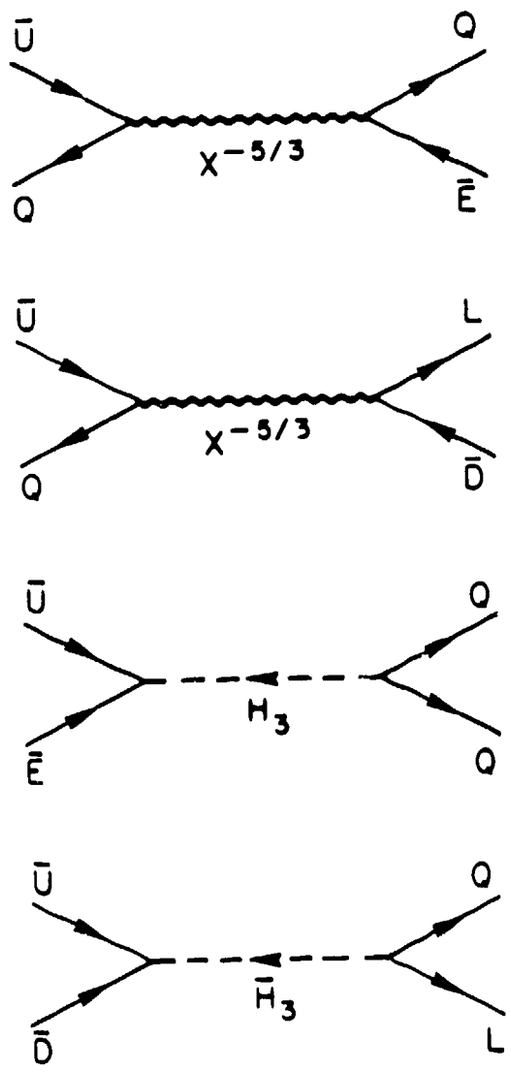
$$\sin^2\hat{\theta}_W(m_W) = .233 \begin{smallmatrix} + .006 \\ - .001 \end{smallmatrix}$$

with the upper (lower) error given by $\Lambda^{--}_{ms} = .1(.4)$ Gev and the central value given by $\Lambda^{--}_{ms} = .3$ Gev. Note that M_{GUT} in SUSY SU_5 is about 40 times larger than M_{GUT} in ordinary SU_5, and $\sin^2\hat{\theta}_W$ is about 10% larger than the ordinary SU_5 prediction $\sin^2\hat{\theta}_W(m_W) = .209 \pm {}^{.007}_{.001}$. It is worth re-remarking that the experimental value of $\sin^2\hat{\theta}_W(m_W)$ appears to be increasing. Recent data from CDHSW, CHARM, CCFRR and FMM on $\nu_\mu N$ deep inelastic scattering give $\sin^2\hat{\theta}_W(m_W) = .232 \pm .004 \pm .006$ and from UA2 using the W and Z masses $\sin^2\hat{\theta}_W(m_W) = .232 \pm .004 \pm .008$. These results have significantly smaller errors than previous experiments.[9]

Nucleon decay

Dimension 6 operators.

There are only two such operators which are given in Table III. They can result from either gauge boson exchange or scalar Higgs triplet exchange. (see Fig. 1.)



Gauge exchange.

Since M_{GUT} has increased in SUSY SU_5 we find that τ_N due to gauge exchange alone is unobservably long.

$$\tau_N = 2 \left(\frac{\Lambda\overline{ms}}{.1\ \text{Gev}}\right)^5 10^{\ 34\pm1}\text{years}$$

Hence the standard dominant decay mode $p \to \pi^o e^+$ is suppressed.

Higgs exchange.

Higgs scalar exchange can be important for Higgs masses of order 10^{10-11} Gev. These processes are naturally suppressed due to small Yukawa couplings. Although it is possible to have such light Higgs scalars, naturally, in a SUSY SU_5 model (see Geometric Hierarchy),[10] they are not light in the minimal version of SUSY SU_5. They have mass of order M_{GUT}. The dominant decay modes for Higgs scalar exchange are

$$P \to K^o\mu^+ \ , \ K^+\bar{\nu}_\mu$$

$$n \to K^o\bar{\nu}_\mu \quad .$$

Dimension 5 operators

There are two relevant dimension 5 operators in an R-parity invariant low energy theory. They are given in Table III. They are obtained by exchanging the color triplet Higgs fermions H_3 and $\bar{H}_3$ (see Fig. 2), using the following Yukawa interactions.

$$L_{Yukawa} \supset \int d^2\theta \ \Big(H_3\{D\left(V^T \frac{m_u}{v}\right) U + U\left(\frac{m_u}{v} V\right) D$$

$$+\bar{E}\left(V^T \frac{m_u}{v}\right) \bar{U} + \bar{U}\left(\frac{m_u}{v} V\right) \bar{E}\}$$

$$+ \bar{H}_3\{-V\left(\frac{m_d}{\bar{v}}\right) D + E\left(\frac{m_d}{\bar{v}} V^\dagger\right) U + \bar{D}\left(\frac{m_d}{\bar{v}} V^\dagger\right) \bar{U}\}\Big)$$

where V_{ij} is the Cabibbo-Kobayashi-Maskawa matrix, (m_u, m_d) are diagonal mass matrices and $(v,\bar{v})$ given by $v = \langle H_0 \rangle$, $\bar{v} = \langle \bar{H}_0 \rangle$ are weak vacuum expectation values.

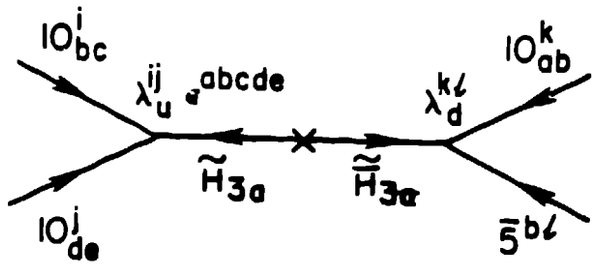


Fig. 2.

We thus obtain the effective dimension 5 operators

$$\mathcal{L}_{\Delta B} \sim \frac{\Delta m_{3\bar{3}}}{M^2} \left\{ \left[D\left(V^T \frac{m_u}{v}\right) U + U\left(\frac{m_u}{v} V\right) D \right] \times \left[V\left(\frac{m_d}{\bar{v}}\right) D + E\left(\frac{m_d}{\bar{v}} V^\dagger\right) U \right] \right.$$
$$\left. + \left[\bar{E}\left(V^T \frac{m_u}{v}\right) \bar{U} + \bar{U}\left(\frac{m_u}{v} V\right) \bar{E} \right] \times \left[\bar{D}\left(\frac{m_d}{\bar{v}} V^\dagger\right) \bar{U} \right] \right\}_{F \text{ term}}$$

where $\Delta m_{3\bar{3}}$ is the $H_3 - \bar{H}_3$ mixing mass term and M is the average $H_3 - \bar{H}_3$ mass. In the minimal SUSY SU_5 model $M \equiv \Delta m_{3\bar{3}}$. Since H_3 and $\bar{H}_3$ are the SU_5 partners of the Higgs doublets H and $\bar{H}$ the couplings are known and are small, i.e. the couplings are proportional to the quark masses. This is yet another reason for the dominance of strange particle final states.

The dimension 5 operators are quadratic in scalar and in fermi fields. They are dressed at the weak scale by gaugino exchange in order to obtain effective four fermi operators contributing to nucleon decay.

The low energy theory; $E \ll M_{GUT}$

SUSY Breaking

The gaugino, Higgsino, squark and slepton masses and mixing angles determine the parameters in the effective 4-fermi baryon number violating operators. A brief review of the low energy SUSY mass spectrum will suffice for our purposes.[11]

Gauginos-Higgsinos.

Assuming a common SUSY breaking mass term for gauginos at a scale $\mu \sim 2 \times 10^{18}$ Gev we find the SU_3, SU_2, U_1 gaugino majorana mass terms at m_W in the ratio $M_3 : M_2 : M_1 = \alpha_3 : \alpha_2 : \frac{5}{3}\alpha_1$.

The gluino mass is M_3. The charged and neutral winos and Higgsinos mix at m_W. The mass matrices are given in Table V.

Table V.

$$\begin{array}{c|cc} & \tilde{w}^+ & \tilde{h}^+ \\ \hline \tilde{w}^- & M_2 & g_2 v \\ \tilde{\bar{h}}^- & g_2 \bar{v} & \epsilon \end{array}$$

$$\begin{array}{c|cc|cc} & \tilde{w}^3 & \tilde{b} & \tilde{h}^o & \tilde{\bar{h}}^o \\ \hline \tilde{w}^3 & M_2 & 0 & g_2 v & -g_2 \bar{v} \\ \tilde{b} & 0 & M_1 & g_1 v & g_1 \bar{v} \\ \hline \tilde{h}^o & g_2 v & g_1 v & 0 & \epsilon \\ \tilde{\bar{h}}^o & -g_2 \bar{v} & g_1 \bar{v} & \epsilon & 0 \end{array}$$

$$m_W^2 \equiv g_2^2 \frac{(v + \bar{v}^2)}{2}, \quad m_Z^2 = m_W^2/\cos^2\theta_W$$

In the limit $M_2 \sim \epsilon < m_W$ the charged states have predominantly a Dirac mass of order m_W. The neutralinos have approximate mass eigenstates given by a photino $\tilde{\gamma}$, with mass $m_{\tilde{\gamma}} \sim \frac{8}{3}\frac{\alpha}{\alpha_3} m_{\tilde{g}}$ or $m_{\tilde{\gamma}} \sim \frac{1}{5} m_{\tilde{g}}$; a Higgsino with mass $\sim\epsilon$ and a Dirac Zino with mass $\sim m_Z$. The experimental limit on gluino masses from UA1 is now $m_{\tilde{g}} \geq 60$ Gev. Thus $M_2 \geq 20$ Gev which is still compatible with the above limit.

Squarks and Sleptons

Squark and slepton masses come from several different sources. For definiteness let us focus on the top squark mass matrix. It is given in Table VI.

Table VI.

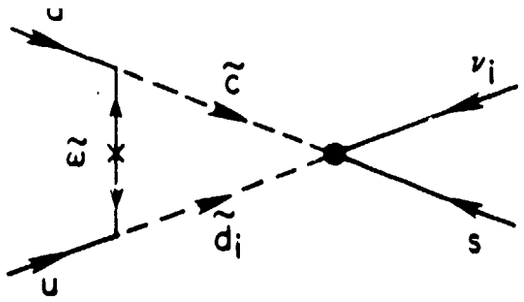
$$\begin{array}{c|cc} & \tilde{t} & \tilde{\bar{t}}^{*} \\ \hline \tilde{t}^{*} & m_{\tilde{t}}^{2} + \frac{\left(g_2^2 - \frac{1}{3}g_1^2\right)}{4}\left(v^2 - \bar{v}^2\right) + m_t^2 & Am_t \\ \tilde{\bar{t}} & Am_t & m_{\tilde{\bar{t}}}^{2} + \frac{g_1^2}{3}\left(v^2 - \bar{v}^2\right) + m_t^2 \end{array}$$

There are four different contributions. $m_{\tilde{t}}$ and $m_{\tilde{\bar{t}}}$ are SUSY breaking terms which are assumed to be equal at M. At m_W this contribution is ~ 3 times larger for squarks than for sleptons due to radiative corrections coming from gluinos. For the top squark $m_{\tilde{t}}$ and $m_{\tilde{\bar{t}}}$ may also receive large corrections due to a large Yukawa coupling λ_t. The A parameter is a soft SUSY breaking term for cubic scalar interactions. The terms proportional to $(v^2-\bar{v}^2)$ come from SUSY gauge interactions --so-called D terms. This contribution differs for up and down squarks. Finally m_t is the supersymmetric contribution.

Nucleon decay

The dominant four fermi operators result from the processes depicted in Figs. 3, 4 and 5.

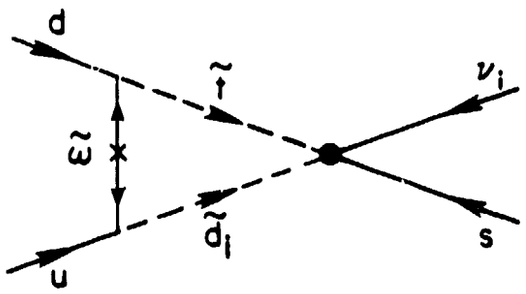
In order to make contact with experiment we must take into account two additional effects. We must find the renormalization group corrections to these effective four fermi operators from the physics between M_{GUT} and m_N (the nucleon mass). This is made in two stages, a) from M_{GUT} to m_W for which we may renormalize the effective SUSY dimension 5 operators and b) from m_W to m_N which is identical to the corrections occuring in non-SUSY theories.[12] Note that we are assuming in this analysis that the SUSY threshold occurs at m_W. Secondly, we must take matrix elements of the effective four fermi operators between the nucleon and appropriate final states. This requires knowledge of strong QCD effects which introduces large theoretical uncertainties. We shall rely on the chiral Lagrangian approach to obtain an estimate of the amplitudes[13] (see Fig. 6). This introduces the strong interaction parameter β describing the three quark fusion process of (Fig. 6b). Theoretical estimates of β are in the range $.003 < \beta < .03$ Gev^3.



$\alpha_2 m_c m_{d_i} \; V_{cs} V_{cd} V^*_{ud_i}$

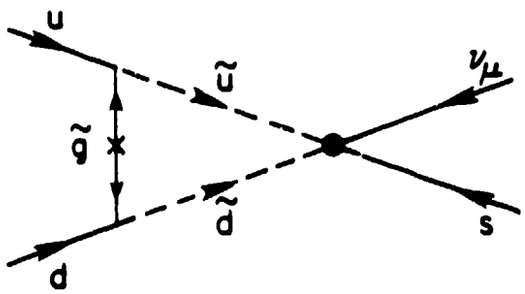
m_b may dominate

(b)



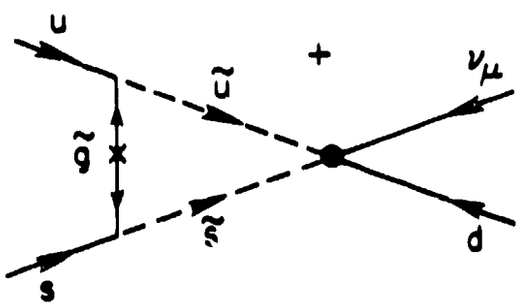
$\alpha_2 m_5 m_{d_i} \; V_{ts} V_{td} V^*_{ud_i}$

(c)



$\alpha_3 m_u m_s \; V_{ud} \; \frac{\Delta^2_{\tilde{u}\tilde{d}}}{m^2}$

$\Delta^2_{\tilde{u}\tilde{d}} = m_W^2\left(\frac{v^2-\bar{v}^2}{v^2+\bar{v}^2}\right)$



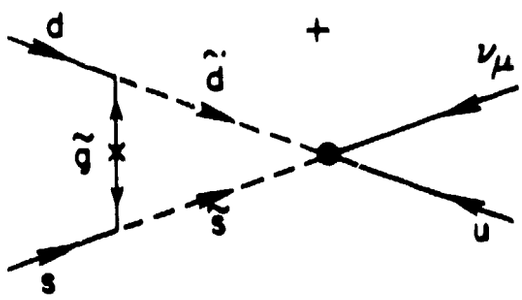
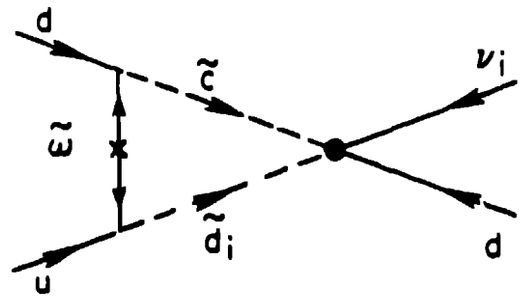


Fig. 3

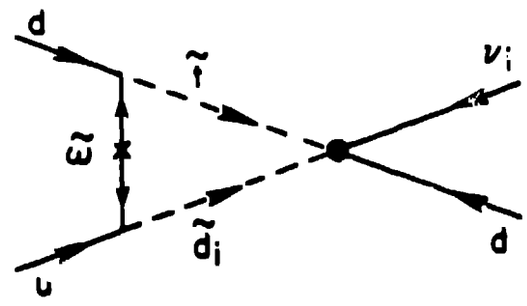
$p \to \pi^{+} \nu_i \qquad n \to (\pi^{-}, \eta) \nu_i$

(a)



$\alpha_2 m_c m_{d_i} \; V^2_{cd} V^*_{ud_i}$

(b)



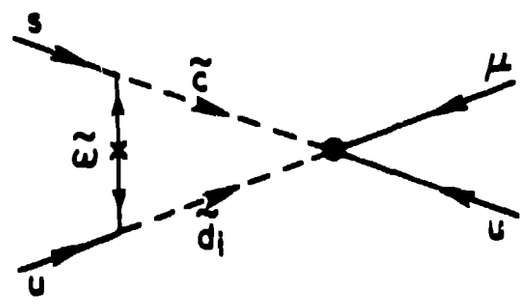
$\alpha_2 m_t m_{d_i} \; V^2_{td} V^*_{ud_i}$

Fig. 4

$p \to K^{o} \mu^{+}$

(a)

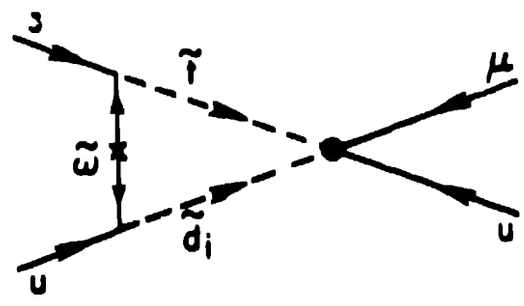
$n \to K^{o} \pi^{-} \mu^{+}$



$\sum_{i=1}^{3} \alpha_2 \left[m_c m_s \; V_{cd_i} V^*_{ud_i} V_{cs} V^*_{us} \right.$

(b)

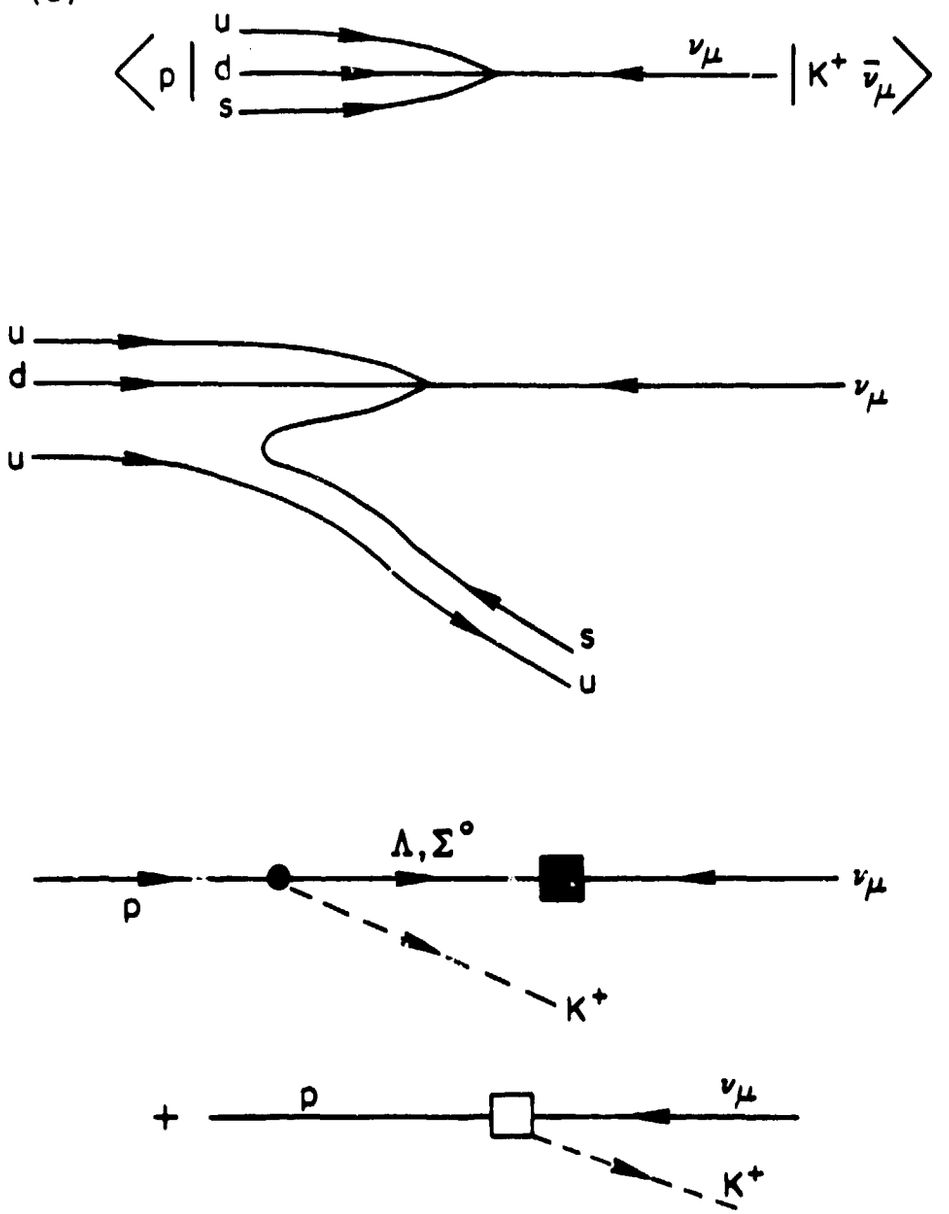
$\left. + \delta_{d_i,d} \; m_u m_s \; V_{ud} V^*_{cs} \right]$



$0 = \sum_{i=1}^{3} \alpha_2 \; m_t m_s \; V_{td_i} V^*_{ud_i} V_{ts} V^*_{us}$

Fig. 5

(a)



(b)

$$\beta \equiv \langle \Sigma^\circ | \, uds \, | 0 \rangle$$

Fig. 6

Table VII.

Branching ratios for some dominant nucleon decay modes

Chadha and Daniel[14]

$p \to K^+(\bar{\nu}_\mu, \bar{\nu}_\tau)$	90%
$p \to \pi^+(\bar{\nu}_\mu, \bar{\nu}_\tau)$	10%
$p \to K^0\mu^+$	$\sim .6\times10^{-3}$
$p \to \pi^0\mu^+$	$\sim .3\times10^{-3}$
$n \to K^0(\bar{\nu}_\mu, \bar{\nu}_\tau)$	95%
$n \to \pi^0(\bar{\nu}_\mu, \bar{\nu}_\tau)$	3%
$n \to \eta(\bar{\nu}_\mu, \bar{\nu}_\tau)$	2%
$n \to K^0\bar{\nu}_e$	$\sim .3\times10^{-3}$

In Table VII we present branching ratios for nucleon decay due to the processes given in Figs. 3,4,5, assuming $\tilde{w}$ dominance. Several comments are in order.

A) The sum of graphs in Fig. 3c has been shown to vanish for degenerate squarks.[15] However in supergravity theories the up-down squark mass difference is given by $\Delta^2_{\tilde{u}\tilde{d}} = m_w^2\left(\frac{v^2-\bar{v}^2}{v^2+\bar{v}^2}\right)$ which can be of order the squark masses.

As a result the ratio of amplitudes

$\frac{A_{\tilde{w}}(p\to K^+\bar{\nu})}{A_{\tilde{g}}(p\to K^+\bar{\nu})}$ can be of order one.[16]

B) It has been shown by Arnowitt, Chamseddine and Nath[17] that for a range of parameters with, in particular $m_t \geqq 40$ Gev that the amplitudes of Figs. 3a and 3b can be arranged to cancel. If one then also suppresses the gluino exchange diagrams of Fig. 3c by increasing the average squark mass one finds the dominant nucleon decay modes are

$$n \to \pi_0 \bar{\nu}$$

$$p \to \pi^+ \bar{\nu} \quad .$$

C) I have ignored neutralino exchanges since these are typically negligible except in the case that the aforementioned cancellations occur. In this case neutralino exchange can be suppressed with

$$\frac{v^2 - \bar{v}^2}{v^2 + \bar{v}^2} \sim 1.$$

D) Finally, for large top quark masses, the graphs of Fig 7 become important. Large top masses lead to large $\tilde{t} - \tilde{\bar{t}}$ mixing for $A \sim m$ as needed in Fig. 7b or to large $\tilde{t} - \tilde{u}$ mixing due to large renormalization group corrections to $m^2_{\tilde{t}}$ and $m^2_{\tilde{\bar{t}}}$ which are proportional to the top quark Yukawa coupling λ_t. The ratio of rates from Fig. 7(A and C) to Fig. 4a is of order[18]

$$\frac{\Gamma_{gluino}(p \to K^0 \mu^+)}{\Gamma_{\tilde{w}}(p \to K^+ \bar{\nu})} \cong \frac{1}{10}\left(\frac{\alpha_3 m_t}{\alpha_2 m_c}\frac{V_{ts}}{V^2_{cd}}\frac{V_{td}}{}\left(\frac{\Delta^2_{\tilde{u}\tilde{t}}}{m^2}\right)\frac{m_{\tilde{g}}}{m_{\tilde{w}}}\right)^2 .$$

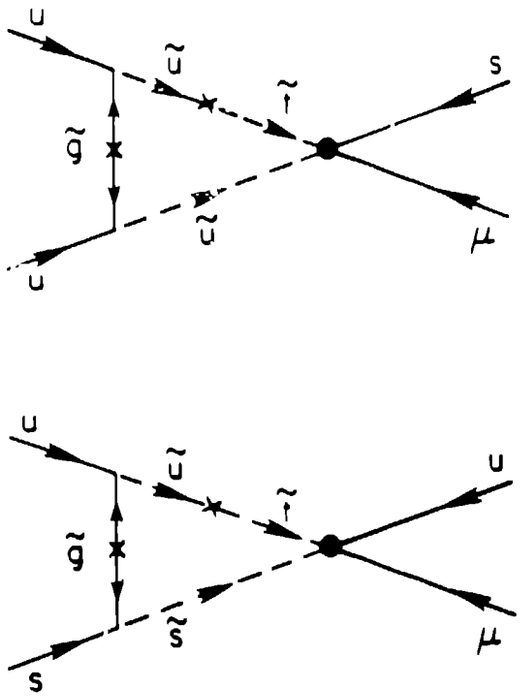
Assuming $\frac{V_{ts}\ V_{td}}{V^2_{cd}} \sim \sin^4\theta_c$, and using $m_{\tilde{g}}/m_{\tilde{w}} \sim \alpha_3/\alpha_2$ we obtain $\Delta^2_{\tilde{u}\tilde{t}}/m^2 \sim 1$ and

$$\frac{\Gamma_{gluino}(p \to K^0 \mu^+)}{\Gamma_{wino}(p \to K^+ \bar{\nu})} \cong 1$$

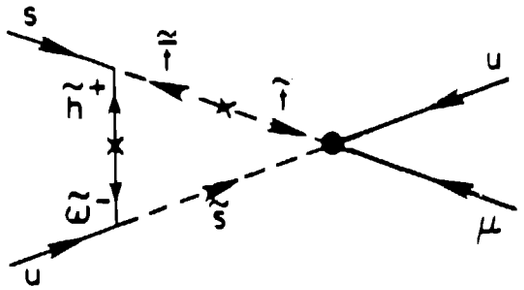
for $m_t = 40$ Gev.

Note Fig. 7c illustrates that the right handed operator of the form $\bar{U}_i \bar{U}_j \bar{D}_k \bar{\mu}_\ell$ can contribute to nucleon decay in this limit.

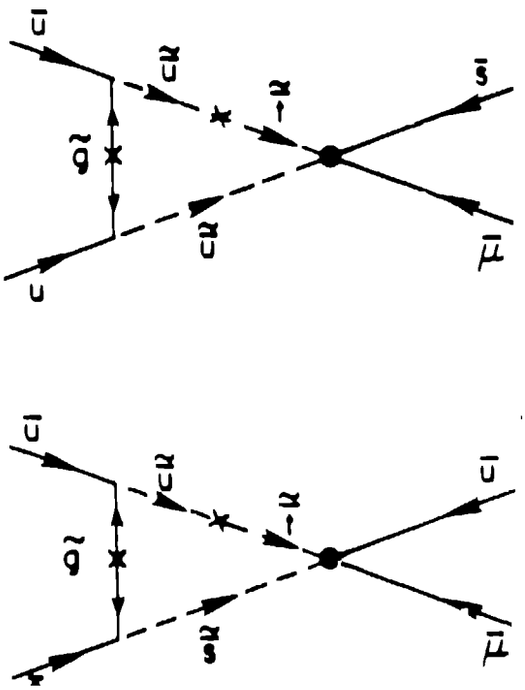
(a)



(b)



(c)



The dominant decay mode of the proton is $P \to K^+\bar{\nu}_\mu$. The decay rate for this mode (assuming $\tilde{w}$ dominance) is given by [17]

$$\Gamma(p \to K^+\bar{\nu}_\mu) \geq \frac{\beta^2}{M^2}\frac{m_N}{32\pi f_\pi^2}\left(1-\frac{m_K^2}{m_N^2}\right)^2 A_L^2\left(A_S^L\right)^2 \left| 1 + \left(\frac{m_N}{m_\Lambda}\right)(D+F)\right|^2 \left|A_{K\bar{\nu}_\mu}\right|^2$$

where

$$A_{K\bar{\nu}_\mu} = \frac{\alpha_2^2\, m_c\, m_s \sin^2\theta_c}{2m_w^2 \sin 2\theta_h}\,[F(\tilde{c};\tilde{s};\tilde{w}) + F(\tilde{c};\tilde{\mu};\tilde{w})]$$

and

$$\tan\theta_h \equiv \frac{\langle h^0\rangle}{|\langle \bar{h}^0\rangle|} \quad .$$

D, F and f_π are parameters in the chiral Lagrangian. We take

$D = .76$

$F = .48$

$f_\pi = 139$ Mev.

Finally, the function F (a result of the loop integration) is given by

$$F(m_1, m_2, \tilde{w}) \equiv \cos^2\theta\, f(m_1, m_2, m_+) + \sin^2\theta\, f(m_1, m_2, m_-)$$

where

$$\tan\theta = \frac{(M_2+\epsilon) - \sqrt{(M_2+\epsilon)^2 + 4m_w^2}}{2m_w}$$

$$m_\pm = \frac{1}{2}[M_2-\epsilon \pm \sqrt{(M_2+\epsilon)^2 + 4m_w^2}]$$

and

$$f(m_1, m_2, m_3) = \frac{m_3}{m_2^2-m_3^2}\left[\frac{m_2^2}{m_1^2-m_2^2}\,\ln\frac{m_1^2}{m_2^2} - \frac{m_3^2}{m_1^2-m_3^2}\,\ln\frac{m_1^2}{m_3^2}\right] \quad .$$

Note this is not the most general result for the masses $m_\pm$ and mixing angles but is valid in the limit $\tan\theta_h = 1$. $m_\pm$ are the charged wino eigenstates (see Table V) and θ is the appropriate mixing angle. The result for $\Gamma(p \to K^+\bar{\nu}_\mu)$ is clearly dependent on the low energy supersymmetric mass spectrum and the mass M of the heavy color triplet Higgs fermions. If we insert some characteristic values for the experimental limit for $\tau_{p\to K^+\bar{\nu}_\mu}$ we can obtain a lower bound for M.

We find

$$M \cong 10^{17\pm1}\left(\frac{M_2}{20\ \text{Gev}}\right)\left(\frac{100\ \text{Gev}}{m_{\tilde{q}}}\right)^2\ \text{Gev}$$

for $m_{\tilde{q}} \sim m_{\tilde{\ell}} >> M_2 \sim \epsilon$.

Since we expect $M \sim M_{GUT}$ and $M_{GUT} \sim 10^{16}$ Gev in the minimal low energy supersymmetric model, we conclude that nucleon decay should be observable in SUSY GUTS.

In fact, once the low energy supersymmetric spectrum is observed, then it may be possible to rule out the minimal SU_5 SUSY GUT on the basis of nucle decay. There is then only one free parameter in the theory (the color triplet Higgs mass M) and this parameter cannot be much greater than M_{GUT}, as remarked previously.[19]

$M_{GUT} \sim M_{pl}$.

It might be preferable to have a theory for which $M_{GUT} \sim M_{pl}$. This appears to be the case in superstring theories. Moreover it has the aesthetic advantage of containing the ultimate desert with no new physics between m_W and M_{pl}. It may also be forced upon us by experiment. It is worth pointing out that in order to have $M_{GUT} \sim M_{pl}$ one must necessarily have additional light states.

A particularly interesting possibility occurs in models where SUSY breaking and the GUT symmetry breaking have a common origin. In this case one finds the additional states in Table VIII.[20]

Table VIII.

	$SU_3 \times SU_2 \times U_1$
$\underset{\sim}{8}$	(8, 1, 0)
$\underset{\sim}{3}$	(1, 3, 0)
$\bar{E}$	(1, 1, 2)
E	(1, 1, -2)

Using the standard renormalization group analysis one finds[20]

$$M_{GUT} \sim 10^{19\pm2}\ \text{Gev}$$

$$\sin^2\hat{\theta}_w \sim .230 \pm .005$$

where the errors are theoretical and include the standard threshold uncertainties.

Conclusion

Nucleon decay in SUSY GUT's has a characteristic signature. One expects

$$p \to K^{+}\bar{\nu}_{\mu}$$

$$n \to K^{0}\bar{\nu}_{\mu}$$

to be the dominant decay modes. Unlike the case of ordinary SU_5 GUT's however, the predictions in SUSY theories depend on the details of the as yet unobserved "low energy" supersymmetric spectrum and also on the mass M of color triplet Higgs fermions with $M \sim M_{GUT}$. There are ranges in parameter space for which the decay modes

$$P \to \pi^{+}\bar{\nu}_{\mu}$$

$$\to K^{0}\mu^{+}$$

$$n \to \pi^{0}\bar{\nu}_{\mu}$$

$$\to \eta\bar{\nu}_{\mu}$$

may be significant or may even dominate. Assuming the "low energy" SUSY spectrum is fixed experimentally one may then confidently calculate (in a minimal SU_5 SUSY GUT) the nucleon decay rate as a function of M. This free parameter has an upper bound obtained by requiring the theory to be perturbative at the scale M_{GUT}. Hence an absolute theoretical upper bound for τ_N may in principle be obtained and compared with experiment.

We remind the reader that in order to obtain models for which M_{GUT} is of order M_{pl}, as may be desired for example in superstring theories, one necessarily expects additional "low energy" states (see Table VIII) beyond the minimal low energy SUSY spectrum (see Tables I & II).

Finally we note that minimal SUSY GUT's typically predict a value for $\sin^2\hat{\theta}_w(m_w)$ of order .233. This is significantly higher than minimal non-SUSY GUT's. It is a nice test for the simple models discussed here.

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Footnote

F1. This analysis demonstrates that, given a GUT scale at $\sim 10^{15}$ Gev, dimension 5 operators must be suppressed. If we would have considered the $d = 5$ operators first, we would have concluded that $M \geqq 10^{32}$ Gev which is much larger than M_{pl} and thus unnatural.