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LASER FOCUSING OF HIGH-ENERGY CHARGED-PARTICLE BEAMS*

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ABSTRACT

It is shown that laser focusing of high-energy charged-particle beams using the inverse Cherenkov effect is well suited for applications with large linear colliders. Very high gradient (> 0.5 MG/cm) lenses result that can be added sequentially without AG cancellation. These lenses are well understood, have small geometric aberrations, and offer the possibility of correlating phase and energy aberrations to produce an achromatic final focus.

INTRODUCTION

The very large electromagnetic fields available in laser beams have led many people to consider their use in accelerating particles; in fact, experimental investigations are still under way.[1] In addition to accelerating particles, which can be done in various ways, it is also necessary to focus particles, and in some cases, such as the final focus for a large linear collider, very large focusing gradients may be required, which lasers might provide. The lasers required for focusing particle beams need not have the high efficiency required for acceleration and thus are likely to meet the repetition rate, control, and optical-quality specifications.

In this paper, I will extend the analysis of particle-beam focusing by laser beams begun in Ref. 2 to include the evaluation of phase, chromatic and geometric aberrations, and the effects of various errors and gas scatter. What emerges from this analysis is the conclusion that inverse Cherenkov focusing does provide accurately controlled, high-gradient, low-aberration focusing well matched to the transversely small, very short bunches required for a large linear collider; in addition, there is the possibility of an achromatic final focus more tolerant than conventional systems to beam energy spread from the linac. Inverse Cherenkov focusing will not, however, be useful for beams with extremely low emittances (ϵ normalized < $\pi*10^{-7}$ mrad) because of the effects of gas scatter.

If a particle interacts with the far field of a laser in vacuum with no other fields present, the strength of the interaction is proportional to $1/\gamma^2$ (γ is the relativistic factor of the particle) and is thus extremely small for the energy range of interest. In order to obtain a significant interaction, one can use near fields (gratings, droplets, foxholes, etc.),[3] add other fields (IFEL),[4] or add a medium that can be either active (plasma beatwave)[1] or passive (inverse Cherenkov).[5] I will concentrate on the use of the inverse Cherenkov effect for particle focusing because (1) it is known to work (at some level); (2) it is straightforward to analyze, (3) it preserves the laser virtues of fast, accurate control because no solid or active medium is near the particle, and (4) it is the only one I have studied.

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Adding a gas to the region of interaction between a laser beam and a particle beam has four effects: (1) it unbalances the E and B fields so that the interaction is now of order $(\varepsilon - 1)$ (ε is the dielectric constant of the gas) instead of order $1/\gamma^2$, (2) it reduces the speed of light so that the interaction length is limited because of the phase shift, (3) it limits the peak electric field to values below breakdown, and (4) it introduces emittance growth in the beam caused by gas scatter.

The value of the breakdown field is the least understood aspect of inverse Cherenkov focusing. The breakdown limit depends on many things, most importantly, the length of the laser pulse. A conservative value for the breakdown field, appropriate to a 20-ps pulse, would be 20 GV/m;[5] a more optimistic value, possibly applicable to a 1-ps pulse, would be 100 GV/m.

In the section below I present a model for the laser beam, which is a Gaussian solution of Maxwell's equations, and indicate how the Born expansion of the equation of motion for a particle in these fields proceeds. Then, I examine the linear lens portion of these solutions, evaluating the resulting focal length, the parameters required, and the laser power needed. In the following section, I examine the variation of focal length with phase and particle energy and point out the possibility of correlating these variations to form an achromatic lens. Next, I evaluate the various geometric aberrations and show that in most cases they are negligible. In the succeeding section, I examine a variety of effects including phase jitter, pressure fluctuations, and, most importantly, gas scatter and note that these effects are not a problem except for extremely low emittance beams. In the final section, I summarize my arguments and results.

THE BASIC INTERACTION

Let us first derive a model for a laser "waist," i.e., a laser beam converging to a focal spot and subsequently diverging. I begin with Maxwell's equations

$$\vec{\nabla} \times \vec{B} = \frac{\varepsilon}{c} \frac{\partial \vec{E}}{\partial t} \quad , \tag{1}$$

and

$$\vec{\nabla} \times \vec{E} = \frac{-1}{c} \frac{\partial \vec{B}}{\partial t} \quad , \tag{2}$$

where ε is the dielectric constant of the gas. Assuming a radial mode of a cylindrically symmetric beam, the fields have the components

$$B = B_\theta \hat{\theta} \quad , \tag{3}$$

and

$$\vec{E} = E_r \hat{r} + E_z \hat{z} \quad . \tag{4}$$

Substituting the forms of Eqs. (3) and (4) into Eqs. (1) and (2), it is easy to see that a set of solutions can be written as

$$E_r = \int_0^{\frac{\omega}{c}\sqrt{\varepsilon}} dk \; \Xi(k) \; \cos(kz - \omega t) \; J_1(Kr) \quad , \tag{5}$$

$$B_\theta = \frac{\varepsilon\omega}{c} \int_0^{\frac{\omega}{c}\sqrt{\varepsilon}} dk \; \frac{\Xi(k)}{k} \; \cos(kz - \omega t) \; J_1(Kr) \quad , \tag{6}$$

and

$$E_z = - \int_0^{\frac{\omega}{c}\sqrt{\varepsilon}} dk \; \frac{\Xi(k)K}{k} \; \sin(kz - \omega t) \; J_0(Kr) \quad , \tag{7}$$

where

$$K \equiv \sqrt{\frac{\varepsilon\omega^2}{c^2} - k^2} \quad , \tag{8}$$

and J_0 and J_1 are the Bessel functions of zeroth and first order, respectively. We determine the function $\Xi(k)$ from the initial-boundary value condition

$$E_r(r, z=0, t=0) = \sqrt{e} \;\; E_p\left(\frac{r}{w}\right) e^{\frac{-r^2}{2w^2}} \quad , \tag{9}$$

where w is the radius of the optical beam and E_p is the peak electric field. Substituting Eq. (5) into Eq. (9), one can show that

$$\Xi(k) \simeq kK\sqrt{e}\, E_p w^3 e^{(-w^2K^2)/2} \quad . \tag{10}$$

The fields given by Eqs. (5), (6), (7), and (10) describe a laser waist with a Gaussian decrease for large r, transverse fields that are linear in r for small r and that decrease away from z = 0 with a decay distance L, given by the Rayleigh range

$$L = \frac{2\pi w^2}{\lambda} \quad , \tag{11}$$

where

$$\lambda \equiv \frac{2\pi}{k_0} \tag{12}$$

is the optical wavelength.

I will now evaluate the interaction of a high-energy particle traveling near the optical axis with these fields using the Born approximation. The equations of motion of a relativistic particle are

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right) , \tag{13}$$

and

$$\frac{d\gamma}{dt} = \frac{q\vec{E} \cdot \vec{v}}{mc^2} \quad , \tag{14}$$

where

$$\vec{p} \equiv \gamma m\vec{v} \tag{15}$$

is the momentum and q is the particle's charge. Shifting the independent variable to z, the longitudinal position, the equations of motion can be written as

$$\frac{d\gamma}{dz} = \frac{q\,\vec{E}\cdot\vec{v}}{mc^2 v_z} \quad , \tag{16}$$

$$\frac{dt}{dz} = \frac{1}{v_z} \quad , \tag{17}$$

and

$$mv_z \frac{d}{dz}\left(\gamma v_z \frac{d\vec{x}}{dz}\right) = q\,\vec{E} + \frac{\vec{v}\times\vec{B}}{c} \quad . \tag{18}$$

We will expand the solutions in both $1/\gamma$ (and ignore $1/\gamma^2$ terms) and in the field strengths. The zeroth order solutions are

$$\gamma = \gamma_0 \quad , \tag{19}$$

$$v_z = c \quad , \tag{20}$$

$$t = \bar{t} + \frac{z}{c} \quad , \tag{21}$$

and

$$\vec{x}_\perp = \vec{x}_{\perp 0} \quad , \tag{22}$$

where $\bar{t}$ is the time the particle arrives at $z = 0$, $\vec{x}_{\perp 0}$ is the transverse position at $z = -\infty$, and γ_0 is the energy at $z = -\infty$. The first-order corrections to these trajectories (denoted with a subscript 1) satisfy the equations

$$t_1 = 0 \quad , \tag{23}$$

$$v_{z1} = 0 \quad , \tag{24}$$

$$\frac{d\gamma_1}{dz} = \frac{qE_z}{mc^2} \quad , \tag{25}$$

and

$$\frac{d^2\vec{x}_{\perp 1}}{dz^2} = \left(\frac{q}{\gamma_0 mc^2}\,\hat{r}\right)\left(E_r - B_\theta\right) , \tag{26}$$

where zeroth-order trajectories are used in the arguments of the fields. Substituting the zeroth order trajectories [Eqs. (19)-(22)] into the fields [Eqs. (5), (6), (7), and (10)], putting the results into the right-hand sides of Eqs. (25) and (26), and integrating, we find that

$$\Delta(\gamma) = \frac{2\pi q}{mc^2}\,\Xi\left(\frac{\omega}{c}\right) K_\omega \frac{c}{\omega} J_0(K_\omega r_0) \sin \omega\bar{t} , \tag{27}$$

and

$$\Delta\left(\frac{d\vec{x}_\perp}{dz}\right) = \frac{2\pi q\vec{x}_{\perp 0}}{\gamma_0 mc^2 r_0}\,\Xi\left(\frac{\omega}{c}\right) J_1(K_\omega r_0) \cos \omega\bar{t} \quad , \tag{28}$$

where $r_0 = |x_{\perp 0}|$ and $K_\omega = (\omega/c)\sqrt{\epsilon - 1}$, and where $\Delta(\bullet)$ denotes the change in the quantity $(\bullet)$ from $z = -\infty$ to $z = +\infty$. The Born expansion can be readily carried to higher order in a straightforward but tedious manner.

A LINEAR LENS

Because we are interested in focusing, let us ignore the energy change, Eq. (27) (which is small for the cases of interest), and expand the angular kick, Eq. (28), for small r_0; the result, using Eq. (10), is

$$\Delta \frac{d\vec{x}_\perp}{dz} = -\frac{\sqrt{e}\,\pi q\vec{x}_{\perp 0}}{\gamma_0 mc^2} \frac{\omega^3}{c} E_p w^3(\epsilon - 1)^2 e^{-\frac{w^2\omega^2(\epsilon - 1)}{2c^2}} \cos \omega\bar{t} \quad . \tag{29}$$

The net effect of the interaction is that the laser waist looks like a lens to the particle. Note that the strength of the lens first increases with $(\varepsilon - 1)$ (the quadratic factor), then decreases with $(\varepsilon - 1)$ (the exponential factor), and thus has a maximum. This maximum is achieved when

$$\varepsilon = 1 + \frac{\lambda^2}{\pi^2 w^2} , \tag{30}$$

where the angular kick has the form

$$\Delta \frac{d\vec{x}_1}{dz} = - 1.785 \left(\frac{\vec{x}_{10}}{w} \right) \frac{qE_p \lambda}{\gamma_0 \, mc^2} \quad \cos \omega t \quad . \tag{31}$$

The minimum focal length $(t = 0)$ of this lens is thus

$$F_\ell = 0.56 \; w \left(\frac{\gamma mc^2}{qE_p \lambda} \right) \quad . \tag{32}$$

As an example, let us assume

$\lambda = 10 \; \mu m$ (CO_2 Laser) ,

$w = 100 \; \mu m$,

$qE_p = 100 \; \frac{GeV}{m}$,

and

$\gamma_0 mc^2 = 1.5 \; TeV$.

We then find that

$$F_\ell = 84 \; m \quad , \tag{33}$$

a remarkably small focal length for such an energetic beam. This is equivalent to about 0.5 MG/cm focusing gradient.

Note that because of the symmetry of the focusing in x and y, we could, without AG cancellation, have a sequence of n of these interactions (laser waists) with the same relative phase along the particle path and reduce the focal length by a factor n. Of course, more interaction regions require proportionately greater laser power; the peak power required in a single interaction region is

$$P = 1.36\ cE_p^2 w^2 \quad . \tag{34}$$

For the previous example this is

$$P = 4.54 * 10^{12} W \quad . \tag{35}$$

A laser pulse of this power level lasting a picosecond would have a few joules of energy; this is a quite substantial but not unrealistic laser energy.

The laser waist in the above example is 0.2 mm in diameter and about 6 mm long. The dielectric constant required can be evaluated using Eq. (30) and is given by

$$\varepsilon - 1 \simeq 1.013 * 10^{-12} \quad , \tag{36}$$

a value that would result in about four atmospheres of hydrogen.

PHASE AND ENERGY ABERRATIONS - AN ACHROMATIC LENS

Examining Eq. (31), we see that the focal length varied proportionally to

$$F_\ell \propto \frac{Y_0}{\cos \omega t} \quad . \tag{37}$$

Let us first consider the variation cos ωt. The allowed variation in focal length ΔF_ℓ is limited by

$$\frac{\Delta F_\ell}{F_\ell} < \frac{1}{M} \quad , \tag{38}$$

where M is the magnification desired. Because a magnification of 30 is probably required to have an interesting lens, we see that the

relative variation in focal length is limited to about 3%. The variation in the cosine term will certainly exceed 3% unless the particle bunch is very short ($\approx 1\mu m$ for a CO_2 laser). Note, however, that such short bunches may be required[6] to avoid excessive beamstrahlung, so that this scheme is well suited to a large linear collider.

By adding harmonics to the laser, one can increase the phase extent that falls within the allowed focusing variation, but probably not by enough to justify the effort involved.

A magnification of 30 would also limit the allowed energy spread to less than 3%, a figure that might be hard to achieve if there are significant wakefield effects. Let us note, however, that only the ratio of the energy and the cosine term enters into the focal length, so that the numerator and denominator can vary by significantly more than 3% if this ratio has a small variation. That this might be the case can be seen from the fact that the energy spread will probably be determined by the accelerating fields, or, more likely, by wakefields, both of which effects result in a particle energy strongly correlated with longitudinal position. Because we are considering bunches of the order of a micron long, the particle energy varies systematically over this distance, as does the cosine term; thus, by a relative phase adjustment, it should be possible to achieve an adequately small variation in the focal length.

GEOMETRIC ABERRATIONS

If we keep the next order term in r_0 in the expansion of the Bessel function in Eq. (28), we find that the thin-lens geometric aberration can be expressed as

$$\Delta\left(\frac{d\vec{x}_1}{dz}\right)_{\text{Aberration}} \approx \frac{-r_0^2}{2\,w^2}\,\Delta\left(\frac{d\vec{x}_1}{dz}\right)_{\text{Linear}} \quad . \tag{39}$$

Requiring a relative variation in the focal length of less than 3% implies

$$\frac{r_0}{w} < 0.24 \quad . \tag{40}$$

For the example we have been considering, the particle beam radius is thus bounded by

$$r_0 < 24\ \mu m \quad , \tag{41}$$

a bound that is not stringent for the very high energy beams being considered (even the present Stanford Linear Collider (SLC) almost satisfies this at 50 GeV).

Thick-lens geometric aberrations can be evaluated by keeping the next (second) order in the Born expansion. As one would expect for such small lenses, thick-lens geometric abberations are four orders of magnitude smaller than the thin-lens terms for TeV beams and can thus be ignored.

One might be concerned that particles at different radial positions, deflected through different angles might have sufficiently different path lengths to shift their relative phases by an optical wavelength in a multiple lens system. A simple geometrical estimate, however, shows that this effect is completely negligible.

ERRORS AND OTHER EFFECTS

A system of n lenses will each, in general, have a random relative phase error, α_i, where

$$\langle \alpha_i \rangle = 0 \quad , \tag{42}$$

and

$$\langle \alpha_i^2 \rangle = \alpha \quad , \tag{43}$$

where the brackets denote average and where $\alpha \simeq 2\,\pi/30$ is probably achievable for the large focal spots (20 wavelengths) we have assumed. The net focal length from such a system of lenses is approximately given by

$$\frac{1}{F_\ell} \simeq \frac{1}{F_{\ell,\,min}} \sum_{i=1}^{n} \cos(\omega\bar{t} + \alpha_i) \quad . \tag{44}$$

When α is small, it is then easy to see that

$$\frac{1}{F_\ell} \simeq \frac{n}{F_{\ell,\,min}} \cos \omega\bar{t}\, 1\left(- \frac{\alpha^2}{2}\right) \tag{45}$$

and that the standard deviation is given by

$$\frac{\Delta F_{rms}}{F_\ell} \simeq \frac{\alpha|\sin \omega\bar{t}|}{\sqrt{n} \cos \omega t} \quad . \tag{46}$$

Thus, the average focusing force is reduced slightly (about 2%), and the standard deviation is small because of the $1/\sqrt{n}$ and because sin ωt is small for maximum focusing. As an example, if we let $\omega t = 30°$, $\alpha = 2\pi/30$, and assume 20 lenses, we find about a 3% standard deviation.

Inverse Cherenkov focusing is quite insensitive to pressure and temperature fluctuations because of the maximization of Eq. (29) with respect to ε and the resulting lack of first-order dependence on ε and thus the pressure and temperature on which ε depends.

Inverse Cherenkov focusing is limited by emittance growth caused by gas scattering of the particle beam. As a rough measure of this effect, we consider the ratio R of net scattering angle in the gas to the angle from emittance, which is given by

$$R \simeq \frac{16.4\sqrt{nd}\, r_{c\ell} r_b}{\varepsilon_n} , \tag{47}$$

where
$n \equiv$ gas density,
$d \equiv$ path length in gas,
r_e = classical radius,
r_b = beam radius, and
ε_n = normalized emittance.

If we assume

$n = 10^{26}\ m^{-3}$ (4 atmospheres hydrogen),
$d = 1$ m,
$r_b = 0.3\ \mu m$, and
$\varepsilon_n = 10^{-7}$ m rad,

we find that

$$R \simeq 1.4 , \tag{48}$$

a value that would yield a just barely acceptable 50% emittance growth. Thus ,inverse Cherenkov focusing is unlikely to be useful for beams with emittances smaller than about 10^{-7}m rad.

SUMMARY

I have analyzed a scheme to use inverse Cherenkov focusing for particle beams likely to be encountered in large linear colliders and I have shown that high-focusing gradients (~0.5 MG/cm) are achievable. The cylindrical symmetry of the focusing allows one to

use lenses sequentially without the usual AG cancellation and thus to achieve small focal lengths and fast, accurate control with lasers that seem practical both in performance specifications and in power requirements. This focusing is achieved in a system that is sufficiently well understood (and known to work) to determine that geometric aberrations are small and that tolerances on phases, pressure, and temperature are acceptable. In addition, there is an intriguing possibility of correlating phase aberrations and energy variation to produce an achromatic lens more tolerant to energy spread from the linac than are conventional systems. It should be noted,
however, that the inevitable gas scatter that accompanies inverse Cherenkov focusing precludes the use of particle beams with extremely low normalized emittances ($\leq 10^{-7}$ m rad).

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