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LA-UR -87-2096

JUL 1 0 1987

CONF 8703136 1

LA-UR--87-2096

DE87 011762

TITLE HEAVY MULTIQUARK STATES

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SUBMITTED TO Proceedings of the workshop, "The Elementary Structure of Matter", les Houches, France, March 24 - April 2, 1987.

Heavy Multiquark States

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I. Introduction

In a number of papers [1-5] it has been suggested that the dimeson $(Q^2\bar{q}^2)$ is stable against breakup into the two $(Q\bar{q})$ mesons provided the mass m of Q is large. Since some of these papers make purely phenomenological assumptions about the nature of the interaction in the four quark system, the physical basis for the result is not obvious. We have recently argued [6] that for sufficiently large m the dimeson *must* be bound, and in Section II we show how this result follows from minimal assumptions that are consistent with Quantum Chromodynamics.

To actually decide if a particular dimeson is bound, it is necessary to make an assumption about the form of the confining interaction. Our method for deriving the static part of the potential energy is discussed in Section III; it is the fact that the two-body and four-body systems are treated on the same footing that enables us to calculate and compare the energies of single mesons and dimesons. In Section IV the method for solving the four-body problem, and the results, are presented.

II. Qualitative discussion of dimesons

We shall consider the four-body system $(Q^2\bar{q}^2)$ composed of two quarks and two antiquarks for two special cases of the quark masses

A. $m \to \infty$ with $\bar{m}$ fixed

A very simple physical picture emerges in this limit because all the relative momenta in the problem remain finite *except* for that between the two heavy quarks. Their relative motion is dominated by their Coulomb attraction in the color $\bar{3}$ state, and their relative wave function becomes hydrogenic with a reduced mass $\mu = m/2$ and an effective coupling constant $\alpha_{EFF} = 2\alpha_s/3$. The Bohr radius of this pair is

$$a = \frac{3}{m\alpha_s} \tag{2.1}$$

and the energy associated with their relative motion is

$$E(Q^2) = -\frac{1}{9}\, m\alpha_s^2 \ . \tag{2.2}$$

We know that none of the other energies (apart from rest mass) in the four-body system is proportional to m for the very same reason that the energy of a single meson ($Q\bar{q}$) is not proportional to m, namely, the kinetic energy is governed by $\bar{m}$ in the large m limit. The only assumption about the confining potential that is needed to complate this argument is the trivial one that it remains finite when two particles come close together. Finally,

$$2M(Q\bar{q}) - M(Q^2\bar{q}^2) \simeq \frac{1}{9}\, m\alpha_s^2 + O(m^\circ) \tag{2.3}$$

which proves that for sufficiently large m there must be a bound exotic dimeson.

B. $m = \bar{m}$

All the relative momenta are comparable in this case, and since the $Q\bar{q}$ Coulomb attraction in the color singlet state is stronger (2x) than the QQ attraction in the $\bar{3}$ state, the $Q\bar{q}$ pairing is the preferred one. While this does not prove that there is no bound dimeson for this system, it is consistent with the fact that we have not found any [6].

III. The Potential Energy

To derive the static part of the potential energy for a system of quarks and/or antiquarks we use a Born-Oppenheimer approximation to the MIT bag model. The quarks are treated as static, localized sources of the glue field, and the latter is required to satisfy the bag model boundary condition $n^\mu F_{\mu\nu} = 0$ on the surface. After solving for the glue field and the correct bag surface, the energy is regarded as the potential energy in which the quarks move. In Fig. 1 this approximation is contrasted with another extreme approximation to the bag model, the 'cavity' approximation, which has been used to describe systems containing only light quarks. In that approach it is the boundary condition on the quark field that determines their allowed modes; and the total energy is regarded as the actual mass of the hadron.

CAVITY APPROXIMATION		BORN-OPPENHEIMER APPROXIMATION
u,d	s	c,b,

CAVITY APPROXIMATION	BORN-OPPENHEIMER APPROXIMATION
FIXED spherical cavity.	FIXED quark positions.
Boundary condition on QUARK field.	Boundary condition on GLUE field.
Quarks in lowest mode.	Solve for glue field and bag boundary.
E - Hadron mass.	E = static potential energy.

Fig. 1 Comparison of two extreme approximations to the MIT Bag model

If one tries to treat heavy quarks in the cavity approximation it is necessary to mix high mode numbers into the wave function because the small increase in kinetic energy can be more than compensated for by the gain in color Coulomb attraction. On the other hand, if light quarks are treated in the Born-Oppenheimer approximation, there may be important non-static terms in the potential energy. We assume that this is not the case and will calculate systems like $(b^2\bar{u}^2)$ just using the static potential.

We start at the classical level with a set of quarks at position $\vec{r}_i$ with color charges F_i^a enclosed in a bag described by surface parameters S_α. Since color magnetic moments are neglected at this stage, the only boundary condition is $\hat{n}\cdot\vec{E}^a = 0$, and this Neumann problem can be solved for an arbitrary surface. Adding together the color electrostatic energy and the bag volume energy gives

$$W(\vec{r}_i, F_i^a, S_\alpha) = \int dV \left[\frac{1}{2} \sum_a \vec{E}^{a\,2} + B \right] . \tag{3.1}$$

Since the surface parameters are not dynamical variables they must be eliminated, leading to

$$\frac{\partial W}{\partial S_\alpha} = 0 , \tag{3.2}$$

which is called the equation of constraint or the pressure balance condition. When the solution of this equation is inserted into (3.1) the result is the potential energy

$$V = W\Big(r_{ij},\ F_i\cdot F_j,\ S_\alpha(r_{ij}, F_i\cdot F_j) \Big) . \tag{3.3}$$

Since the bag forms around the quarks, the potential energy is translation invariant.

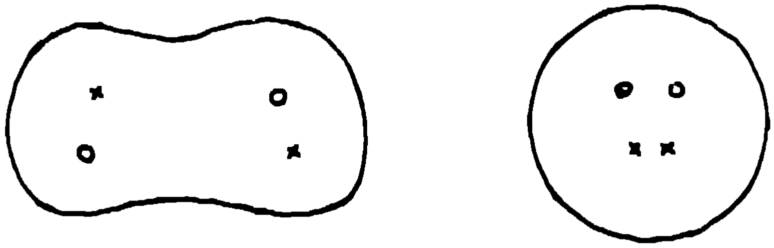
After the kinetic energy of the quarks is added to the potential energy (3.3) quantization is carried out, and this leads to V becoming a matrix in color space. For the system of two quarks and two antiquarks it is a 2×2 matrix since there are two independent color singlet states.

To actually implement the program described in (3.1) - (3.3) requires solutions of the bag model with deformed surfaces. If we were dealing with molecular-type states in which one $(Q\bar{q})$ pair were rather well separated from the other, then bag deformation would be an important consideration [see Fig. 2]. As the states discussed in this paper are not of this type, we expect a spherical approximation to the bag to be good for the parts of the wave function having large probability, and since an analytic Green's function is known for a sphere it is possible to write down an analytic expression [7] for W in (3.1). If the interparticle separations are small, $r_{ij} \lesssim 1$ Fm, then it is sufficient to keep just the dipole term from the homogeneous part of the Green's function, and this leads to [7]

$$V = \sum_{i>j}^{N} \alpha_s \frac{F_i \cdot F_j}{r_{ij}} + \frac{k}{\sqrt{2}} \left[(\vec{D}^a)^2 \right]^{1/2} \tag{3.4}$$

where

$$\vec{D}^a = \sum_{i}^{N} F_i^a \, \vec{r}_i \tag{3.5}$$



(a) (b)

Fig. 2. Bag shapes for the two-quark two-antiquark system. (a) A deformed bag that would be important for a molecular type state. (b) If all interparticle spacings are comparable a spherical bag is expected to provide a good approximation

is the color dipole moment operator. The string tension is given by

$$k = \left(\frac{32\pi}{3} B\alpha_s\right)^{1/2} \tag{3.6}$$

and the bag radius, which is also an operator in color space, has the expression

$$R = \left[\frac{3\alpha_s}{4\pi B} (\vec{D}^a)^2\right]^{1/6} . \tag{3.7}$$

There are two noteworthy features of this potential energy. First of all, (3.4) has the same structure for any number N of quarks and/or antiquarks; we can use it, therefore, to calculated both mesons and dimesons to see if the latter are stable against breakup into the former. The second point to note is that the second term in (3.4), which is the confining potential, is a many-body operator. We have argued on theoretical grounds that the confining interaction must be a many-body potential [8]. In addition, phenomenology does not tolerate a sum of two-body potentials because it gives rise to unphysical van der Waals forces between hadrons.

When (3.4) is specialized to the $(Q\bar{q})$ system it becomes

$$V^{(2)} = -\frac{4}{3}\frac{\alpha_s}{r} + \left(\frac{2}{3}\right)^{1/2} kr , \tag{3.8}$$

which has the same Coulomb plus linear structure as the potential derived from lattice gauge theory and also the phenomenological potentials that have been used to fit ψ $(c\bar{c})$ and Υ $(b\bar{b})$ spectra. Note that there is a confining term even at small distances, but the slope is only ~0.8 of its value at large distances, r > 1 Fm, where the bag develops into a tube of flux [7,9].

For the $(Q^2\bar{q}^2)$ system the 2×2 matrix $V^{(4)}$ determined from (3.4) is written out in [10] in the singlet-singlet, octet-octet representation defined by the couplings

$$\begin{aligned} \psi_1 &= |[(12)^1(34)^1]^1\rangle , \\ \psi_8 &= |[(12)^8(34)^8]^1\rangle . \end{aligned} \tag{3.9}$$

where particle 1 and 3 are quarks, and 2 and 4 are antiquarks.

Since some of the four-body wave function extends into the region where the spherical approximation to the bag breaks down, we use some physical arguments to write down the potential there. The most important region is the one in which one $(Q\bar{q})$ pair starts to separate from the other. If the separation R between the (12) pair and the (34) pair becomes large, in the representation (3.9) we require that

$$V^{(4)} \rightarrow \begin{pmatrix} V^{(2)}(r_{12}) + V^{(2)}(r_{34}) & \mathcal{O} \\ \mathcal{O} & C_8 R \end{pmatrix} . \tag{3.10}$$

The significance of the various matrix elements is as follows. The diagonal element in the singlet-singlet state is the sum of the two potential energies *within* the individual $(Q\bar{q})$ pairs with no interaction between them. The diagonal element in the octet-octet state is the confining part of the bag model potential energy between two octets. Concerning the off-diagonal element $\mathcal{O}$, it must fall off sufficiently rapidly with distance so that there are no van der Waals forces, and this means at least exponentially. We actually use a Gaussian form, $\exp(-R^2/d^2)$, to make a smooth transition from (3.4) to large distances, so $\mathcal{O}$ is itself a Gaussian and we expect the parameter d to be approximately 1 Fm. The details are given in [11].

IV. Meson and Dimeson Energies

Given the potential energy described in Section III, the next step is to solve the Schrödinger equation for the meson and dimeson problems and compare their energies. We did this initially using the nonrelativistic expression for the kinetic energy of the quarks, but since the light quarks (m = 0.35 GeV) are quite relativistic, all the results reported here were obtained with $\Sigma\,(\vec{p}_i^{\,2} + m_i^2)^{1/2}$ as the kinetic energy operator. These calculations that use the static potential energy and the relativistic expression for the kinetic energy are referred to as "semirelativistic".

To actually solve for the energy of the ground state of each system we have performed variational calculations and also Green's function calculations. The latter also start with a trial wave function, ψ_v, but then let it evolve in (imaginary) time, and the ground state energy is projected out via

$$E_o = \lim_{\tau\to\infty} \frac{\langle\psi_v|He^{-\tau H}|\psi_v\rangle}{\langle\psi_v|e^{-\tau H}|\psi_v\rangle} . \tag{4.1}$$

The Monte Carlo technique for doing this is described in [11].

The variational wave function for a single meson is chosen to be

$$\psi_v = \exp(-\alpha r) , \tag{4.2}$$

and for the dimeson

$$\begin{aligned}\psi_v = &\{\exp[-\alpha_c(r_{12} + r_{34}) - \alpha_{13}r_{13}] + [1\leftrightarrow 3]\}\,\psi_3 \\ &+ c_m\{\exp[-\alpha_c(r_{12} + r_{34}) - \alpha_{13}r_{13}] - [1\leftrightarrow 3]\}\,\psi_6 ,\end{aligned} \tag{4.3}$$

where α, α_c, α_{13}, and c_m are the variational parameters. The color states ψ_3 and ψ_6 are defined by the couplings

$$\begin{aligned}\psi_3 &= |[(13)^{\bar{3}}\,(24)^3]^1\rangle \\ \psi_6 &= |[(13)^6(24)^{\bar{6}}]^1\rangle\end{aligned} \tag{4.4}$$

and are antisymmetric and symmetric, respectively, under the interchange of either the two quarks or the two antiquarks. According to the discussion in Section II we expect the term in (4.3) containing ψ_3 to be dominant for large quark mass, and that is why it was assigned a symmetric spatial wave function.

The correlations that are built into (4.2) and (4.3) contain sufficient flexibility to describe, on the one hand, the expected large m limit in which [see (2.1)]

$\alpha_{13} = \bar{m}\alpha_s/3$; and also the limit of two separated mesons, which corresponds to $\alpha_{13} = 0$ and $\alpha_c = \alpha$, with $c_m = -\sqrt{2}$ making each meson a color singlet. An additional correlation between the two light quarks would complicate the variational calculation, but is not essential for the Green's function calculation.

For the bag model parameters we choose $\alpha_s = 0.370$ and $B^{1/4} = 0.245$ GeV, which leads to the string tension having the value $k = 1.07$ GeV/Fm. In conjunction with the quark masses $m_c = 1.364$ GeV and $m_b = 4.781$ GeV, this results in a good fit to the $c\bar{c}$ and $b\bar{b}$ spectra.[6] In Table I we show the masses of four mesons obtained by solving the semi-relativistic Schrödinger equation with the potential (3.8).

Table I. The masses of some mesons. α is the variational parameter in (4.2), and E is the eigenvalue of the semirelativistic Schrödinger equation with the potential (3.8). An estimate has been made of the hyperfine splitting in the T-η_b system. All energies are in GeV. The parameters are $\alpha_s = 0.370$, $B^{1/4} = 0.245$ GeV, $m_c = 1.364$, $m_b = 4.781$, and $m_u = m_d = 0.350$

Quark Content	$\alpha(\mathrm{Fm}^{-1})$	E	M	M_{XPT}
$c\bar{c}$	4.1	0.32	3.05	$\frac{3}{4}(\psi) + \frac{1}{4}(\eta_c) = 3.07$
$b\bar{b}$	8.2	-0.13	9.43	$\frac{3}{4}(T) + \frac{1}{4}(\eta_b) = 9.44$
$c\bar{u}$	3.3	0.54	2.25	$\frac{3}{4}(D^*) + \frac{1}{4}(D) = 1.97$
$b\bar{u}$	4.0	0.43	5.56	$\frac{3}{4}(B^*) + \frac{1}{4}(B) = 5.31$

As the hyperfine interaction is not included at this stage the comparison with experiment is made in terms of the appropriately weighted masses of the vector and pseudoscalar particles. Since the mass of the η_b has not yet been measured, an estimate of the hyperfine splitting has been made for that system.

It is seen in Table I that the calculated ground state masses of the $(c\bar{u})$ and $(b\bar{u})$ systems are too large. If the mass of the light quark is reduced from the value 0.350 GeV that was used there, then the discrepancy with experiment is reduced, but even with $m_u = 0$ the mass of $(b\bar{u})$ is still too large by about 0.13 GeV. While this indicates some deficiency in the present approach to mixed light-heavy systems, nevertheless, we proceed to calculate the dimeson systems and expect that there is some cancellation of the error in the energy difference.

Table II shows the results of our calculations for the energies of three dimesons. These were obtained from (4.1) again using the semirelativistic Hamiltonian described in Section III. The dimeson binding energy is with respect to the two separated mesons

$$\text{Binding Energy} = M(Q\bar{q}) + M(Q\bar{q}') - M(QQ\bar{q}\bar{q}') \quad . \qquad (4.5)$$

The column labelled "No Hyperfine" omits this interaction in both the mesons and the dimeson, while the column "With Hyperfine" includes it in *both*. Note that before the hyperfine interaction is turned on there is very little difference in the binding energies of the various dimesons, there being a spread of only 10 MeV between the values in the three rows. This already shows that the b quark is not heavy enough for the term in (2.3) that is linear in m (the Coulomb term) to completely dominate. This is consistent with the known fact that the confining potential is quite important in the $(b\bar{b})$ system.

After the hyperfine interaction is turned on every one of the dimesons in Table II becomes less bound; this is because the hyperfine attraction in the mesons is larger than in the dimeson. In fact, those containing two c quarks become unbound and those with one b and one c are marginal.

Table II. Dimeson energies. E is the eigenvalue of the semirelativistic Schrödinger equation; the statistical uncertainty in the Green's function Monte Carlo calculation is ±10 MeV. The binding energy as defined in (4.5) is shown without and with the inclusion of the hyperfine interaction. The final column gives the spin quantum numbers of the lowest energy dimeson state.

		Binding Energy				
Quark Content	E	No Hyperfine	With Hyperfine	Spin State (S_{QQ}	$S_{\bar{q}\bar{q}'}$,	S)
$bb\bar{u}\bar{d}$	0.77	0.09	0.07	1	0	1
$bb\bar{u}\bar{u}$			0.03	1	1	0
$bc\bar{u}\bar{d}$ $bc\bar{u}\bar{u}$	0.87	0.09	0.00	1	1	0
$cc\bar{u}\bar{d}$ $cc\bar{u}\bar{u}$	0.99	0.08	(-0.07)	1	1	0

The final column of Table II contains the spin quantum numbers of the two heavy quarks, S_{QQ}, the two light antiquarks, $S_{\bar{q}\bar{q}'}$, and the total spin S of the lowest energy state. A dimeson containing two *distinct* light quarks ($\bar{u}$ and $\bar{d}$) does not have a Pauli principle restriction on their spin state and may, therefore, have a lower energy than the corresponding dimeson with identical light quarks. This is indeed the case for the dimesons containing two b quarks, for which ($bb\bar{u}\bar{d}$) is bound by approximately 40 MeV more than ($bb\bar{u}\bar{u}$). The details of the hyperfine calculation are given in [11].

V. Conclusion

We have shown that for sufficiently large quark mass m and fixed antiquark mass $\bar{m}$ the dimeson $(Q^2\bar{q}^2)$ *must* be stable against all strong decays, due to the color Coulomb attraction of the two quarks in the color $\bar{3}$ state. Eq. (2.3) is the mathematical statement of this result.

Using the confining potential derived from a Born-Oppenheimer approximation to the MIT bag model, we have obtained the ground state energy of a number of dimesons. Those containing two b quarks and two light antiquarks are indeed energetically bound against decay into two mesons, but the binding energy is not great. In the most favorable case, $(bb\bar{u}\bar{d})$ is bound by 70 MeV with respect to the two mesons $(b\bar{u})$ + $(b\bar{d})$. The corresponding system containing two c quarks is not bound, and the mixed system with one b and one c is borderline. The numerical results show that the mass of the b quark is not large enough for the bb Coulomb attraction to completely dominate the energy. For the t quark this would indeed be the case.

This work was performed in collaboration with J. Carlson and J. A. Tjon, and a more complete account will be found in [11]. This research was supported by the U. S. Department of Energy.

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