LEGIBILITY NOTICE

A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

LA-UR -87-2909

LA-UR--87-2909

DE88 000516

TITLE PION DOUBLE CHARGE EXCHANGE AND NUCLEAR STRUCTURE

AUTHOR(S) Joseph N. Ginocchio, T-5

SUBMITTED TO Proceedings of the Workshop on "Pion-Nucleus Physics: Future Direction and New Facilities at LAMPF", August 17-21, 1987, Los Alamos National Laboratory, Los Alamos, NM

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

PION DOUBLE CHARGE EXCHANGE AND NUCLEAR STRUCTURE

Joseph N. Ginocchio

Theoretical Division, Los Alamos National Laboratory
Los Alamos, NM 87545

ABSTRACT

Pion double charge exchange to both the double-analog state and the ground state is studied for medium weight nuclei. The relative cross section of these two transitions and the importance of nuclear structure as a function of pion kinetic energy is examined.

INTRODUCTION

Pion double-charge exchange (DCX) on nuclear targets must take place on at least two nucleons. For this reason this reaction has been studied with the expectation that the DCX reaction will probe two-nucleon correlations in nuclei. Recent experiments at LAMPF indicate that this is indeed the case. If the DCX reaction (π^+,π^-) is long range, it will primarily excite in the residual nucleus the double-analog state (DIAS) of the ground state of the target nucleus, and the cross section will increase proportionally with the number of excess neutron pairs in the target. This means that the DIAS cross section will increase in the ratio 1:6:28 for the ^{42}Ca: ^{44}Ca; ^{48}Ca isotopes. If an $A^{-10/3}$ dependence for distortion is taken into account this ratio is 1:5:18. At pion energy T_π = 292 MeV this measured ratio[1] is 1:1.5:4.3 to within 20% and at T_π = 35 MeV this ratio[2] is 1:0.6:1.2. This large discrepancy indicates that the DCX reaction is probing correlations between the nucleons. In fact for low pion energies the large magnitude of the DCX reaction has prompted explanations in terms of six quark bags[3]. Recently it has been pointed out that most of the cross section for $^{42,48}Ca$[4] and in fact for all the Calcium isotopes[5] can be understood in terms of shell model correlations.

In this talk we shall focus on the "$f^n_{7/2}$" nuclei because these nuclei encompass isotopes of varying valence nucleon number n and isospin T and can be quite well described in terms of one spherical orbit. In the following Section we first discuss some properties of the DCX operator within the shell model formalism. After that we discuss the seniority model for the single-j shell. We then go on and apply this model to DCX and derive an analytic expression for the transition in terms of n and T and the long-range and short-range part of the pion DCX in two nuclei. For the Calcium isotopes $(T=\frac{n}{2})$ the ground state has seniority zero within the $f^n_{7/2}$ model and the seniority results are applicable. However for other isotopes which have both valence neutrons and protons filling the $f_{7/2}$-shell $(T\neq\frac{n}{2})$ seniority is not a good quantum number[6]. In the following Section we present the results for a realistic $f_{7/2}$-model. Then we calculate DCX transitions in these two models and compare with available experimental results. Finally we make some remarks of what the future of DCX could be.

THE DCX OPERATOR

The double-charge exchange operator for $0^+ \rightarrow 0^+$ transitions is usually calculated in the multipole form

$$F = \Omega \sum_L F_L (b_j^\dagger \tilde{a}_j)^{(L)} \cdot (b_j^\dagger \tilde{a}_j)^{(L)} / (2L+1) \tag{1}$$

where $\Omega = j + 1/2$ and $a_{jm}^\dagger$, $b_{jm}^\dagger$, ($\tilde{a}_{jm}$, $\tilde{b}_{jm}$) creates (destroys) a valence neutron and proton, respectively, in the spherical valence shell model orbital with single-nucleon angular momentum j and projection m. The coefficients F_L depend on the pion-nucleon dynamics, the pion energy, and the single-nucleon wavefunctions. If the pion-nucleon interaction is spin independent, only the even multipoles will contribute; if there is spin dependence the odd multipoles will appear as well, L=0,1,...,2j. However these 2j+1 complex amplitudes are not all independent. This can be seen by recoupling so that the pair of protons and pair of neutrons are each coupled to angular momentum J:

$$F = \sum_{J=0}^{2j-1} G_J (b_j^\dagger b_j^\dagger)^{(J)} \cdot (\tilde{a}_j \tilde{a}_j)^{(J)} \tag{2a}$$

where

$$G_J = -\Omega \sum_L \begin{Bmatrix} j & j & L \\ j & j & J \end{Bmatrix} F_L \quad . \tag{2b}$$

and $\{\}$ is a 6-j symbol.

Because of the Pauli principle only J even appears, J=0,2,...,2j-1, and hence there are really only $j + \frac{1}{2}$ independent complex amplitudes G_J. Furthermore, since the monopole operator is proportional to the isospin lowering operator $\hat{T}_-$;

$$\hat{T}_- = \Omega (b_j^\dagger \cdot \tilde{a}_j)^{(0)} \quad , \tag{3}$$

the monopole part of the DCX operator is only involved in the double-analog charge exchange transition. Hence for non-double analog charge exchange transitions there are only $j - \frac{1}{2}$ complex amplitudes.

For a single-j shell there is a particle-hole symmetry in the double charge exchange reaction if other configurations are ignored and the mass dependence of the pion distorted waves and the nuclear mean field are neglected. Making the particle-hole transformation

$$a^{\dagger}_{jm},\ b^{\dagger}_{jm} \rightarrow \tilde{\alpha}_{jm},\ \tilde{\beta}_{jm} \tag{4a}$$

$$\tilde{a}_{jm},\ \tilde{b}_{jm} \rightarrow -\alpha^{\dagger}_{jm},\ -\beta^{\dagger}_{jm} \tag{4b}$$

and using the anticommutation relations, the double-charge exchange operator becomes

$$F \rightarrow \bar{F} = \Omega \sum_{L} F_{L}\ (\alpha^{\dagger}_{j}\tilde{\beta}_{j})^{(L)} \cdot (\alpha^{\dagger}_{j}\tilde{\beta}_{j})^{(L)} \tag{5}$$

Hence we see that the double-charge exchange on the target nucleus with n_{π}, n_{ν} valence neutrons and protons is the same as the target nucleus with $\bar{n}_{\pi}$; $\bar{n}_{\nu}$ valence neutrons and proton holes where

$$\bar{n}_{\pi} = 2\Omega - n_{\pi} \tag{6a}$$

$$\bar{n}_{\nu} = 2\Omega - n_{\nu} \tag{6b}$$

which is equivalent to changing the <u>total</u> number of valence nucleons ($n = n_{\pi} + n_{\nu}$) to holes and keeping the same isospin:

$$\bar{n} = 4\Omega - n \tag{7a}$$

$$\bar{T} = T \tag{7b}$$

Of course this symmetry will not be exact because there will be mass dependence in the coefficients F_L from pion distortion and the shell model radial wavefunctions. Also there can be admixtures of other configurations in the nuclear wavefunction. However it would be of interest to do experiments on nuclear targets which are particle-hole conjugates, particularly for pion energies for which distortions are believed to be small and nuclear targets for which configuration admixtures are small. Even though this symmetry may be difficult to observe, it is useful for calculations since the nuclear structure matrix elements remain the same, only the F_L changes.

THE SENIORITY MODEL

The seniority model[7] has been extensively covered in the literature; we shall only briefly review the basic ideas in this Section. The basic assumptions of the seniority model are: (1) the dominant effective interaction between valence nucleons occurs for nucleons coupled to angular momentum zero and isospin one, and (2) the single-nucleon energies are degenerate or quasi-degenerate. Assumption (1) is not good for nuclei which have both valence neutrons and protons active as discussed in the next Section. In this Section we shall deal with a single spherical shell with angular momentum j so the assumption (2) is valid. However the results in this Section also apply to the seniority model with degenerate single-nucleon energies.

The ground state of the seniority model can be expressed as a product of $J^{\pi} = 0^{+}$, $T = 1$, pairs of nucleons outside a doubly-magic core. These pairs are:

$$S_1^{\dagger} = \frac{1}{2} \sum_m (-1)^{j-m} a_{jm}^{\dagger} a_{j-m}^{\dagger} \tag{8a}$$

$$S_0^{\dagger} = \frac{1}{\sqrt{2}} \sum_m (-1)^{j-m} a_{jm}^{\dagger} b_{j-m}^{\dagger} \tag{8b}$$

$$S_{-1}^{\dagger} = \frac{1}{2} \sum_m (-1)^{j-m} b_{jm}^{\dagger} b_{j-m}^{\dagger} \tag{8c}$$

where $a_{jm}^{\dagger}$ and $b_{jm}^{\dagger}$ create a valence neutron and proton, respectively, in the spherical valence shell with single-nucleon angular momentum j and projection m.

These three pair creation operators, $S_q^{\dagger}$, form an isospin triplet, where $q = -1,0,1$ is the isospin projection. For neutrons and protons outside the doubly-magic core denoted by $|\ 0 >$ the seniority zero ground state is

$$|\ n,T,T_z,v=J=0 > = \eta(n,T)\ (S^{\dagger}\cdot S^{\dagger})^{\frac{(n-2T)}{4}}\ (S^{\dagger})^{T,T_z}|\ 0 > \tag{9}$$

where n is the total number of valence nucleons outside the core and must be even for seniority zero and $\eta(n,T)$ is the normalization of the state. The core has isospin zero and hence the isospin T is carried by the valence nucleons. The isospin projection is $T_z = (N-Z)/2$ where N is the total number of neutrons and Z is the total number of protons in the nucleus. The term $(S^{\dagger})^{T,T_z}$ means that T pairs are coupled to isospin T and projection T_z. For $T_z = T$, which is true for most targets, $(S^{\dagger})^{T,T} = (S_1^{\dagger})^T$; i.e., a product of all neutron pairs. The four nucleon isoscalar product is

$$S^{\dagger}\cdot S^{\dagger} = (S_0^{\dagger})^2 - 2\ S_1^{\dagger}\ S_{-1}^{\dagger}\ . \tag{10}$$

The allowed isospin is $T = n/2,\ n/2 - 2,\ \ldots\ 1$ or 0, which includes all even-even nuclei.

Although the ground state of the seniority model as given by (9) is the product of n zero-coupled pairs, this <u>does</u> <u>not</u> mean that non-zero angular momentum pairs can not be extracted for this state. This seemingly paradoxical statement results from the fact that all nucleons are antisymmetrized with respect to each other. For example for the four-nucleon system with maximum isospin and projection $T=T_z=2$, the state (9) reduces to

$$|\ n=4,T=T_z=2,\ v=J=0 > = [2\ \Omega(\Omega - 1)\ ]^{-\frac{1}{2}}\ S_1^{\dagger}\ S_1^{\dagger}\ |\ 0 > \tag{11}$$

where $\Omega = j + \frac{1}{2}$, $(\Omega=\Sigma_j(j+\frac{1}{2})$ for the degenerate many j-shell case) the number of nucleons in the half-filled shell. However we can recouple the neutron creation operators to get

$$|\ n=4, T=T_z=2,\ v=J=0\rangle = \left[2\Omega(\Omega-1)\right]^{-\frac{1}{2}} \sum_{\substack{JM \\ J\ \text{even}\ M}} (-1)^M A^{\dagger}_{JM} A^{\dagger}_{J-M} |\ 0\rangle \tag{12a}$$

where $A^{\dagger}_{JM}$ is a normalized pair of neutrons coupled to angular momentum J and projection M,

$$A^{\dagger}_{JM} = \left[\frac{a^{\dagger}\ a^{\dagger}}{\sqrt{2}}\right]^{J}_{M} \tag{12b}$$

where the subscript j has been omitted. Therefore all angular momenta J and projection M exist with equal probability.

The matrix element of any two-nucleon operator can be calculated in terms of the two-nucleon matrix elements of the operator. For matrix elements between seniority zero states only, the matrix elements of a two-nucleon operator $V^{t}_{t_z}$, where t is the isospin tensorial rank, become:

$$\langle j^n T' T'_z v'=J'=0 | V^{t}_{t_z} | j^n T\ T_z v=J=0\rangle =$$

$$(-1)^{T+T'+T_z} \begin{pmatrix} T & t & T' \\ T_z & t_z & -T'_z \end{pmatrix} \sum_{2} C(nT'T;T_2J_2)(2J_2+1)\langle j^2 T_2 J_2 || V^t || j^2 T_2 J_2\rangle \tag{13}$$

where ( ) is the Wigner 3-j symbol and the double-barred matrix element refers to the isospin space. The coefficients $C(nT'T;T_2J_2)$ are complicated and given in detail in Ref. 8. However these coefficients have, in the seniority model, the feature that, for two-nucleon states with J_2 even > 0, which of course have $T_2=1$, the coefficients are all equal, and for J_2 odd, which have $T_2=0$, the coefficients are all equal. This is a special feature of the seniority model and is not true in general as we shall see later. Hence the matrix elements of any two-nucleon operator between seniority zero states in a single j-shell depend at most on three two-nucleon matrix elements:

$$\langle V^t\rangle_0 = \langle j^2\ T=1, J=0\ ||\ V^t\ ||\ j^2\ T=1, J=0\ \rangle \tag{14a}$$

$$\langle V^t\rangle_e = \sum_{J\ \text{even}>0} (2J+1)\ \frac{\langle j^2 T=1, J\ ||\ V^t\ ||\ j^2 T=1, J\rangle}{(\Omega-1)(2\Omega+1)} \tag{14b}$$

$$\langle V^t\rangle_o = \delta_{t,0} \sum_{J\ \text{odd}} (2J+1)\ \frac{\langle j^2 T=0, J\ ||\ V^t\ ||\ j^2 T=0, J\rangle}{\Omega\ (2\Omega+1)} \tag{14c}$$

where $\langle V^t\rangle_{e,o}$ are average matrix elements. Furthermore we see that for isovector or isotensor operators, the matrix elements depend on only <u>two</u> two-nucleon matrix elements since $\langle V^t\rangle_o$ vanishes in these cases.

All the results in this section apply to the seniority model with degenerate single-nucleon energies if we take as Ω,

$$\Omega = \sum_j (j+\frac{1}{2}) \quad . \tag{15}$$

DOUBLE-CHARGE EXCHANGE AND THE SENIORITY MODEL

The double-charge exchange operator in (1) changes two neutrons into two protons and hence is a two-nucleon isotensor operator, $t=2$ and $t_z = -2$. The two-nucleon matrix elements of F are

$$\langle j^2\ T=1\ J\ ||\ F\ ||\ j^2 T=1\ J\rangle = 2\sqrt{5}\ G_J \tag{16}$$

where the factor $\sqrt{5}$ is the 3-j symbol involved in going to the double-barred matrix element. From (14) and the fact that F is an isotensor operator, only two matrix elements will be involved for a transition between seniority zero states:

$$\langle F\rangle_0 = \sqrt{5} \sum_{L\ \mathrm{even}} F_L \tag{17a}$$

$$\langle F\rangle_e = \sqrt{5}\left[F_0 - \frac{\Omega+1}{(\Omega-1)(2\Omega+1)} \sum_{L>0} F_L\right] \tag{17b}$$

Hence only the monopole and the sum of the higher multipoles occur for the DCX between seniority zero states. The monopole piece has particular physical significance because it corresponds to successive single-charge exchanges through the intermediate analog state as seen in (3). For the same reason the monopole part of the DCX operator can not change the isospin of the target even though it is an isotensor operator. This statement is only valid for a single j-shell; for many j's there are many monopole operators and only one linear combination is proportional to the isospin generators. With this in mind we define the amplitudes:

$$A = F_0 \ , \tag{18a}$$

$$B = \sum_{L>0} F_L \ . \tag{18b}$$

The amplitude A is the long-range (monopole) part of the DCX reaction while B is the short-range part. Using the results of Ref. 8, the DCX matrix element for the transition from a target with $T_z = T = \frac{(N-Z)}{2}$, $v=0$, to the double-analog state, $T' = T$, $T_z' = T-2$, $v'=0$, is,

$$\langle j^n T, T_z=T-2,\ v=J=0|F|j^n T, T_z=T, v=J=0\rangle = \sqrt{T(2T-1)}\ \{A+XB\} \tag{19a}$$

where

$$X = \frac{1}{(\Omega-1)(2T+3)(2T-1)}\left[(n+3)(\Omega+1-n) + \frac{(n-2T)(n+2T+2)(3\Omega+2)}{2(2\Omega+1)}\right] \quad (19b)$$

We see from this expression that for a given number of valence nucleons the effect of the monopole amplitude A increases as the isospin increases but the importance of B with respect to A decreases; i.e., the pairs coupled to isospin zero in (9) do not contribute to the monopole part of the DCX. However for a fixed isospin the contribution of B with respect to A increases as the number of valence nucleons increase.

For targets with identical nucleons only ($T = \frac{n}{2}$) this reduces to

$$\langle j^n\, T = \tfrac{n}{2},\, T_z = \tfrac{n}{2} - 2,\, v{=}J{=}0 \,|\, F \,|\, j^n\, T = T_z = \tfrac{n}{2},\, v{=}J{=}0\rangle \quad (20)$$

$$= \sqrt{\frac{n(n-1)}{2}}\left[A + \frac{(\Omega+1-n)}{(\Omega-1)(n-1)}\, B\right]$$

which agrees with Ref. 5. This formula is valid for the Calcium isotopes. If B is zero, then the DCX cross section will increase in proportion to the number of neutron pairs = T(2T-1). However the fact that B, the short-range part, does not vanish produces the observed cross section[5] as discussed in the Introduction.

The DCX transition to the ground state, $T'=T-2$, depends on B only because, as we mentioned previously, the monopole term can not change isospin. This matrix element is given by,

$$\langle j^n T'=T-2, T_z'=T', v'=J'=0 \,|\, F \,|\, j^n T, T_z=T, v=J=0\rangle = \sqrt{T(2T-1)}\; Y\, B \quad (21a)$$

where

$$Y = \frac{\Omega}{4(\Omega-1)(2\Omega+1)(2T-1)}\left[\frac{(T-1)(n+2T+2)(n-2T+4)(4\Omega+4-n-2T)(4\Omega+2-n+2T)}{(2T+1)}\right]^{\frac{1}{2}}. \quad (21b)$$

All of these expressions have a particle-hole symmetry which is consistent with the relation derived in (7). This means that the DCX reactions will be the same for particle-hole related nuclei, <u>except</u> for the dependence of the pion dynamics on atomic mass; i.e. the mass dependence of the amplitudes A and B.

In Table I the seniority zero matrix elements are tabulated for $j=\frac{7}{2}$, i.e., $\Omega=4$. We see that the value of X varies substantially for the different isotopes. We also see that for the same target the value of Y is always larger than X indicating that non-analog transitions are more sensitive to the non-monopole amplitude than the analog transition.

However for nuclei with $T<n/2$, i.e. nuclei with both protons and neutrons filling the valence shells, seniority will not be a good quantum number. We shall study more realistic transitions for these nuclei in the next section.

Table I. Values of X and Y for $j=\frac{7}{2}$, $\Omega=4$

$n, \bar{n}$	T	Nuclei	$\sqrt{T(2T-1)}$	X	Y	$\sqrt{T(2T-1)}\,Y$
2	1	^{42}Ca, ^{54}Fe	1	1		
4	2	^{44}Ca, ^{52}Cr	2.4495	0.1111	0.5132	1.2571
6	3	^{46}Ca, ^{50}Ti	3.8730	-0.0667	0.3556	1.3771
6	1	^{46}Ti, ^{50}Cr	1	1.4741		
8	4	^{48}Ca	5.2915	-0.1429	0.2199	1.1638
8	2	^{48}Ti	2.4495	0.1675	0.6184	1.5147

DCX IN A MORE REALISTIC $(f_{7/2})^n$ MODEL

The seniority quantum number is not conserved for neutrons and protons filling the same major shell[6], that is for nuclei with isospin $T < \frac{n}{2}$. We can use more realistic $(f_{7/2})^n$ eigenfunctions to calculate the DCX transition[6,9-11]. The double analog transitions from the Calcium isotopes will remain the same since $T = \frac{n}{2}$ in both the initial and final state. However, the transition to the ground state of the Titanium isotopes will change since these state have lower isospin, $T = \frac{n}{2} - 2$. The result for these transitions is:

$$\langle n, T=T_z=\frac{n}{2} - 2, J=0 | F | n, T=\frac{n}{2}=T_z, v=J=0 \rangle = \sum_{L=0} \beta_L(n) F_L \qquad (22a)$$

where

$$\beta_L(n) = \frac{1}{2}\sqrt{\frac{n}{\Omega(\Omega-1)}} \left\{ \sqrt{(\Omega-1)(2\Omega+2-n)}\ \alpha_0(n) + 2\Omega\sqrt{n-2} \sum_{J>0} \begin{Bmatrix} j & j & L \\ j & j & J \end{Bmatrix} \sqrt{2J+1}\ \alpha_J(n) \right\} \qquad (22b)$$

where α_J is the amplitude of the pair of protons coupled to angular momentum J in the residual Titanium nucleus.

Because these transitions are non-analog the monopole coefficient $\beta_0 = 0$, which also provides a check on the calculation.

Hence in general the non-analog DCX will depend on three amplitudes rather than on only one (that is only B) as in the seniority model. We use the amplitudes tabulated in Ref. 12 and reproduced in Table II for the Titanium isotopes. In Table III we give the resulting β_L for the Calcium isotopes as targets.

From Table II we note that in the Titanium isotopes the J=0 and J=2 proton pairs account for more than 95% of the ground-state wavefunctions. This result is consistent with recent models of nuclear collective motion[13-15] which assume that J=0 and 2 pairs dominate the low-lying states of nuclei. If $\alpha_4 = \alpha_6 = 0$ identically, then $\beta_4 = \beta_6$. Because of this the ratio β_4/β_6 in Table III is almost the same (~0.93 - 0.97) for all Calcium isotopes so that in practice the non-analog DCX depends on approximately two complex amplitudes for the Calcium isotopes.

Table II. Values of α_J for Titanium Isotopes

J	$\alpha_J(^{44}Ti)$	$\alpha_J(^{46}Ti)$	$\alpha_J(^{48}Ti)$
0	0.7608	0.8224	0.9136
2	0.6090	0.5420	0.4058
4	0.2093	0.0861	0.0196
6	0.0812	-0.0127	-0.0146

Table III. Values of β_L for Calcium Targets

L	$\beta_L(^{44}Ca)$	$\beta_L(^{46}Ca)$	$\beta_L(^{48}Ca)$
2	0.7976	0.6574	0.4922
4	1.1284	1.2617	1.1923
6	1.2056	1.3530	1.2343

For targets for which $T < \frac{n}{2}$, seniority will be broken in both the intial and final state. Hence both the <u>analog</u> and non-analog transitions will change. For $(f_{7/2})^n$ the only remaining targets for which the β_L need to be calculated are ^{46}Ti and ^{48}Ti; particle-hole symmetry gives the rest. The calculations are in progress.

DCX CROSS SECTIONS

Since the DCX operator (1) is a two-nucleon operator, once the DCX reaction is known for two nucleons coupled to angular momentum J = 0,2,..., 2j-1 in a shell model orbit j, then it can be determined for many nucleons in that orbit. For the seniority model Equations (19), (20) are used; for a more realistic model (22) plus Table III are used. In order to get an estimate of the difference of the two models we have used the DCX scattering model of Ref. 16. In this model the DCX proceeds through two successive pion single-charge exchanges with closure over the intermediate nuclear states and with pion distortions taken into account. The F_L have been calculated in this model and the nuclear structure results of the proceeding section have been used to calculate the differential cross sections.

The measured cross section[1] at pion energy of 292 MeV and scattering angle $\theta=5°$ are reproduced in column two of Table IV. In the third column is a fit[1] to the seniority model treating A and B as parameters in the DIAS formula given in Equation (19). In the last column are the calculated results in both the seniority model (19, 20) and the more realistic model (22). For the DIAS transition the two models are the same for these isotopes which all have $T=\frac{n}{2}$. However, for the ground state transition the seniority model is given on the top, the more realistic model on the bottom.

Table IV. DCX cross sections at $\theta=5°$ and $T_\pi=292$ MeV.

Transition	$\frac{d\sigma}{d\Omega}_{exp}(\theta=5°)$ (μb/sr)	$\frac{d\sigma}{d\Omega}_{fit}(\theta=5°)$ (μb/sr)	$\frac{d\sigma}{d\Omega}_{th}(\theta=5°)$ (μb/sr)	
$^{42}Ca \to {}^{42}Ti$(DIAS)	0.404±0.061	0.404	0.330	
$^{44}Ca \to {}^{44}Ti$(DIAS)	0.600±0.096	0.562	0.771	
$^{48}Ca \to {}^{48}Ti$(DIAS)	1.746±0.290	1.714	2.026	
$^{50}Ti \to {}^{50}Cr$(DIAS)	0.968±0.201	1.025	1.033	
$^{52}Cr \to {}^{52}Fe$(DIAS)	0.574±0.111	0.562	0.440	
$^{44}Ca \to {}^{44}Ti$(GS)	0.014±0.014	0.306	$\begin{pmatrix} .072 \\ .033 \end{pmatrix}$	SENIORITY REALISTIC
$^{48}Ca \to {}^{48}Ti$(GS)	≤0.051	0.262	$\begin{pmatrix} .046 \\ .013 \end{pmatrix}$	"
$^{50}Ti \to {}^{50}Cr$(GS)	≤0.066	0.367	$\begin{pmatrix} .057 \\ .017 \end{pmatrix}$	"
$^{52}Cr \to {}^{52}Fe$(GS)	≤0.028	0.306	$\begin{pmatrix} .041 \\ .019 \end{pmatrix}$	"

The seniority model fit to the DIAS (third column) predicts too large a cross section to the non-DIAS ground state transition which is consistent with the fact that seniority is not a good quantum number.

The calculated results show good agreement with the double analog transitions (DIAS). The transitions to the ground state have large experimental errors, in fact the measured cross sections are primarily upper limits. For two targets, ^{44}Ca and ^{52}Cr, which are particle-hole conjugates of each other, the seniority results are larger than the upper limits, while for the remaining ground state transitions the seniority results are within the upper bounds. However the realistic wavefunction results are all within the upper limits. The change in the cross sections with the realistic wavefunctions is quite large, in all cases the cross sections are reduced by a factor of 2-4 from the seniority model.

In Table V the measured cross sections (column two) for a lower energy, $T_\pi = 35$ MeV, but a larger angle, $\theta = 40°$, are compared to the calculated cross in the realistic $(f_{7/2})^n$ model.

Table V. DCX Cross Sections

Transition	T_π=35 MeV, θ=40° $\frac{d\sigma}{d\Omega}_{exp}$ (μb/sr)	$\frac{d\sigma}{d\Omega}_{th}$ (μb/sr)	θ=5° T_π=35 MeV $\frac{d\sigma}{d\Omega}_{th}$ (μb/sr)	T_π=50 MeV $\frac{d\sigma}{d\Omega}_{th}$ (μb/sr)
$^{42}Ca \rightarrow {}^{42}Ti$(DIAS)	2.0±0.5	0.815	1.718	1.485
$^{44}Ca \rightarrow {}^{44}Ti$(DIAS)	1.1±0.3	0.659	1.024	0.740
$^{44}Ca \rightarrow {}^{44}Ti$(GS)		0.375	0.962	1.023
$^{48}Ca \rightarrow {}^{48}Ti$(DIAS)	2.4±0.7	1.025	0.869	0.312
$^{48}Ca \rightarrow {}^{48}Ti$(GS)		0.249	0.712	0.950

At the lower energy the DIAS cross sections are larger than at T_π = 292 MeV, and the trend with atomic mass is different. The calculated values are about a factor of two smaller than the measured values, but are a factor of three better than the results taking the monopole alone. Otherwise the calculated values do give the mass dependence correctly. The fact that the calculated values at this energy do not agree as well indicates that other configurations may play a role, or other sources of correlations may also be involved.

The calculations also indicate that the ground state transitions will be comparable to the DIAS rather than an order of magnitude smaller as at T_π = 292 MeV. This is particularly true at the smaller angles (column four) and at T_π = 50 MeV (column five) for which the ground state transition is predicted to be larger than the DIAS! This means that the DCX can probe the change of the ground states of nuclei as a pair of neutrons are converted into a pair of protons. Recent models of nuclear collective motion[13-15] suggest that low-lying collective states of nuclei are composed primarily of coherent pairs of neutrons and protons coupled to angular momentum $J^\pi = 0^+, 2^+$. This DCX reaction could test these models.

FUTURE

In the near future measurements on the $(f_{7/2})^n$ nuclear targets should be completed at both high and low energies. At high energies the DCX seems to be dominated by the monopole part of the reaction (i.e., the amplitude A). For this reason the nuclear structure plays little role, and hence the DCX reaction mechanism can be tested. At lower energies (~50 MeV) and small angles the higher multipoles are important and hence the nuclear structure is important. Also the pion distortions seem to be smaller. These conditions imply that the nuclear structure can be probed successfully. For medium and light weight nuclei the shell model technology is available, so it is feasible at this time to combine both experiment and theory together to see if it is necessary to include subnuclear degrees of freedom[3] in order to understand DCX on nuclei at $T_\pi \sim 50$ MeV. Of particular interest is to measure the transition to the ground state in (π^+,π^-) DCX as well as the analog transition. By the same token we should do the (π^-,π^+) to the ground state since this gives the same information as the (π^+,π^-) ground state to ground state, and in some cases may be easier to do.

If we are able to understand the DCX on the light and medium weight nuclei, then the real challenge for the future will be to study heavy nuclei. The transition from spherical to deformed nuclei is thought to be caused by the neutron-proton interaction. The DCX (π^+,π^-) $[(\pi^-,\pi^+)]$ reaction is such a special probe for studying the nature of this transition in nuclear ground states since it replaces a neutron (proton) pair with a proton (neutron) pair keeping the atomic mass the same, and it has the ground-state spectrum of an even-even nucleus in both the initial and final state. Our initial calculations suggest that at low energies ($T_\pi \sim$ 50 MeV) and small angles, the ground state will be strongly excited in these reactions. These features are particularly relevant today because of recent models of nuclear collective motion[13-15] which assume that the low-lying states of nuclei are predominately composed of monopole and quadrupole pairs of neutrons and protons. The pion DCX can be a valuable tool for testing these models.

ACKNOWLEDGEMENTS

I would like to thank W. R. Gibbs and W. B. Kaufmann for many discussions and the results of their DCX calculations.

REFERENCES

1. J. D. Zumbro *et al.*, to be published in PRC.
2. Z. Weinfeld *et al.*, Los Alamos preprint LA-UR-87-1073 to be published in PRC.
3. G. E. Miller, Phys. Rev. Lett. 53, 2008 (1984).
4. E. Bleszynski, M. Bleszynski, and R. Glauber, Proceedings of the International Conference on Particles and Nuclei, Kyoto, April, 1987.

5. N. Auerbach, W. R. Gibbs, and E. Piasetzky, Los Alamos preprint LA-UR-86-4283, to be published in PRL.
6. J. N. Ginocchio, Nucl. Phys. 63, 449 (1965).
7. A. de-Shalit and I. Talmi, Nuclear Shell Theory (Academic Press, New York, 1963).
8. N. Auerbach, W. R. Gibbs, J. N. Ginocchio, and W. B. Kaufmann, in preparation.
9. J. N. Ginocchio and J. B. French, PL7, 137 (1963).
10. J. D. McCullen, B. Bayman, and L. Zamick, Phys. Rev. 134B, 515 (1964).
11. J. N. Ginocchio, PR144, 952 (1966).
12. J. D. McCullen, B. F. Bayman, and L. Zamick, Princeton University Technical Report NYO-9891 (1964); for ^{48}Ti this report treats the neutrons as holes while we use a phase convention consistent with treating the neutrons as particles.
13. A. Arima and F. Iachello, Adv. in Nuclear Physics, ed. by J. W. Negele and E. Vogt (Plenum, New York, 1984), Vol. 13, p. 139.
14. J. N. Ginocchio, Ann. of Phys. 126, 234 (1980).
15. C. L. Wu *et al.*, Phys. Lett. B168, 313 (1986).
16. W. R. Gibbs, W. B. Kaufmann, and P. B. Siegel, Proceedings of LAMPF Workshop on Pion Double Charge Exchange, ed. by H. Baer and M. Leitch, LA-10550-C, p. 90 (1985).