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ANTIMATTER: ITS HISTORY AND ITS PROPERTIES

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ABSTRACT

We review the conceptual developments of quantum theory and special relativity which culminated in the discovery of and understanding of antimatter. In particular, we emphasize how quantum theory and special relativity together imply that antimatter must exist. Our modern understanding of antimatter is summarized in the CPT theorem of relativistic quantum field theory. The implications of this theorem have never been contradicted by any experiment ever done.

I. INTRODUCTION

Given quantum mechanics and special relativity, antimatter's existence is a consequence.¹ However, traces of it can be seen in non-relativistic quantum mechanics and special relativity, independently.

In this survey we begin with a discussion of the discovery of quantum mechanics, and how the interpretation of the wave function was a clue towards the later discovery of antimatter. Schrödinger did not understand his complex wave function. In fact, at first he thought that only the modulus of the wave function was physically significant.

Only later was it realized that the wave function is a complex probability amplitude. This is a key. The probabilistic nature of quantum mechanics only means we have lost classical determinism. But the complex nature of the wave function allows the existence of antimatter. At the time this hint was missed.

Next, after reviewing special relativity and why its strong-reflection (time and space reflection) symmetry does not quite imply antimatter, we discuss the search for a relativistic quantum theory. The Klein-Gordon equation was the first to be discovered, but it failed for the hydrogen-atom spectrum. But later there was triumph with the Dirac equation. This equation has as its basis the desire to take the square-root of the special-relativistic, energy-momentum-mass relation.

The success of the Dirac equation was dramatic. But it had two extra components besides those for the electron. After some confusion over whether these other solutions could represent the proton, it was realized that they had to correspond to a particle of the same mass as the electron but with opposite charge. Imagine the amazement when in

1932 Anderson found this particle in a cloud chamber.

Then began a fascinating period of detective work, as the positive and negative muons, the three pions, and, most significantly to the community, the antiproton were discovered. Finally, in 1957, Lüders² systematized earlier work and published his paper on the CPT theorem. (C=charge conjugation, P=parity, T=time reversal.) This theorem states that for every particle there will be an antiparticle with the same inertial mass, the opposite charge, and the same total decay rate. These properties have been obeyed by every particle ever discovered. This theorem is the foundation of quantum field theory as a description of particle physics up to and including the "standard model" of the strong and electroweak interactions. Even with the discovery of P and CP violation, there is no suggestion of a violation of CPT invariance.

So where does gravity fit in?

Independently of quantum theory, Einstein developed general relativity as a classical (non-quantum) theory. The gravitational field is a tensor field. In its standard, classical form, general relativity does not specifically contain the concept of antimatter. Antimatter is simply another form of energy and has the corresponding weight. Therefore, antimatter must behave in the same way as matter in a classical, general-relativistic, gravitational field.

But even in the era predating modern attempts to unify gravity with the other (quantized) forces of nature, the question was raised of whether antimatter had to have the same weight as matter. In the 1950's there was speculation that antimatter could possibly be repelled by matter, so-called "antigravity".³ Quantum field theory tells us, "No!" However, that was not the end. Modern, quantum field

theories, which attempt to unify all the forces of nature, tell us that the gravitational acceleration of antimatter can be different than that of matter. That fascinating story is the topic of a separate discussion in this Proceedings. You are referred there for the details.⁴

II. The Discovery of Quantum Mechanics.

The great, intuitive breakthrough in atomic physics was Bohr's description of the hydrogen atom in his "old quantum theory," formulated in 1912.⁵ This theory quantized classical orbits. Limited though it was, it correctly predicted the energy levels of hydrogen as

$$E_n = -R/n^2, \quad n = 1, 2, 3 \dots \quad (1)$$

$$R = me^4/(2\hbar^2). \quad (2)$$

In 1926, Schrödinger's wave-mechanics version of quantum theory explained this result from a fundamental viewpoint.⁶ In this new quantum theory, one makes a substitution for the energy, momentum, and position. They become operators in a wave equation:

$$E_{\text{class}} \rightarrow i\hbar(d/dt), \quad (3)$$

$$p_{\text{class}} \rightarrow -i\hbar \nabla, \quad (4)$$

$$x_{\text{class}} \rightarrow x. \quad (5)$$

The classical energy equation, kinetic energy plus potential energy equals the total energy,

$$E = p^2/(2m) + V(r) \quad (6)$$

is now written in the form

$$E_n \Psi = i\hbar(d/dt)\Psi = [-(\hbar^2/2m)\nabla^2 - V(r)]\Psi \quad (7)$$

The solution to this differential equation yields the same energy levels, E_n , as those obtained by Bohr.

However, one of the aspects of this new theory of operators is that xp is no longer equal to px . In particular,

$$[x, p] = xp - px = i\hbar \quad (8)$$

This equation is the commutator which implies the Heisenberg Uncertainty Relation:

$$(\Delta x)^2(\Delta p)^2 \geq \hbar^2/4 \quad (9)$$

The implication of this relation is that one can never know both the position and the momentum of a particle with infinite precision. This is the place where classical determinism disappears in modern quantum theory.

In the effort to understand this, Schrödinger discovered, what in modern language are called, the coherent states of the harmonic oscillator.^{7,8} These are the wave-function solutions of the quantum

equations of motion. They follow the motion of a classical particle as well as possible. However, wave functions are complex. Schrödinger did not understand what this meant. Below is a reproduction of the translation into English of Schrödinger's remarks.⁸

$$(8) \quad \psi = e^{i\nu t} - \frac{A^2}{4} e^{i\nu t} + 4x e^{2i\nu t} - \frac{x^2}{2}.$$

Now we take, as is provided for, the real part of the right-hand side and after a short calculation obtain

$$(9) \quad \psi = e^{\frac{A^2}{4} - (x - A \cos 2\nu t)^2} \cos \left[\pi\nu t + (A \sin 2\nu t) \cdot \left(x - \frac{A}{2} \cos 2\nu t \right) \right].$$

The *second* factor in (9) is in general a function whose absolute value is small compared with unity, and which varies very rapidly with x and also t . It ploughs many deep and narrow furrows in the profile of the first factor, and makes a *wave group* out of it, which is represented—schematically only—in Fig. 2.

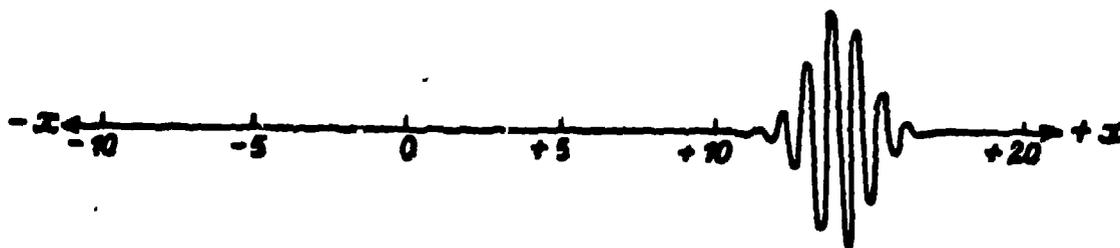


FIG. 2.—Oscillating wave group as the representation of a particle in wave mechanics.

As one can read, Schrödinger originally thought that only the "real part" of the wave function was physically significant. He wanted to ignore the imaginary part, the par. which turns out to be critical to the understanding of anti-matter. It allows for C conjugation.

It was with the work of Born that the physical significance of the wave function was understood. The wave function is a probability amplitude. Its modulus-squared is the probability density. Since wave functions are only amplitudes, their phases are significant in a relative sense, but not in an absolute sense. This is experimentally seen in quantum interference experiments.

III. Special Relativity.

In 1905 Einstein produced his special theory of relativity.⁹ It describes the kinematics of all of known physics in situations where gravity can be ignored. For a free particle, this theory says that the relationship between mass and (only) kinetic energy is no longer the classical

$$E = (1/2)mv^2 \quad (10)$$

but rather is the new relationship

$$E^2 = (mc^2)^2 + (pc)^2 \quad (11)$$

Special relativity has part of the physics that is needed for antimatter. In particular, there is a symmetry called strong reflection.¹⁰ This involves letting all four coordinates (space and time) be reflected through the origin. The effect of this inversion on the equations of classical electrodynamics is to change the sign of the electric charge. For each solution, then, strong-reflection allows the existence of another. This other solution is similar to what we will call an "antiparticle" solution. However, as we observe below, it is only when quantum theory is introduced that true-antiparticle

solutions appear, that are required.

Now, starting with Bohr's old quantum theory, Sommerfeld had "added" special relativity and had derived the "Sommerfeld formula" for the hydrogen-atom energy levels:¹¹

$$W_{n,\varphi} = mc^2 f(n \equiv \rho + \phi, \phi) , \quad (n, \rho) \leq 0, 1, 2, \dots \quad \phi \leq 1, 2 \dots \quad (12)$$

$$\equiv mc^2 \cdot R [1/n^2 + (\alpha^2/n^4)\{n/\phi - 3/4\} \dots] , \quad (13)$$

where

$$f(N, L) = [1 + \alpha^2 / \{N - L + [L^2 - \alpha^2]^2\}^{1/2}]^{-1/2} , \quad (14)$$

$$\alpha = e^2 / (\hbar c) . \quad (15)$$

W vs. E denotes that the rest-mass energy has been included in the eigenvalues. ρ and ϕ are radial and angular quantum numbers, of the "quantized orbit" Bohr type. Eq. (13) agreed with the energy levels of the Bohr atom to the level of the principle quantum number, n . The next term, which includes the angular quantum number, ϕ , agreed with the hydrogen atom fine-structure splittings. But from quantum mechanics it was known that the physical interpretation of ρ and ϕ was incorrect, even though phenomenologically they gave the correct energy eigenvalues.

Therefore, an immediate goal in quantum mechanics was to try to add special relativity to Schrödinger's operator ideas. The first attempt was the Klein-Gordon equation,¹² which is the quantum-mechanical form of Eq. (11), with the electromagnetic

potential inserted:

$$[i\hbar(d/dt) - v(r)]^2\psi = [c^2p^2 + m^2c^4]\psi \quad (16)$$

The solution for the energy levels is

$$W_{n,\ell} = mc^2 f(n, \ell+1/2) \quad , \quad \ell \leq n + 1 \quad (17)$$

$$\equiv mc^2 - R [1/n^2 + (\alpha^2/n^4)\{ n/(\ell+1/2) - 3/4 \} \dots] \quad (18)$$

ℓ being the angular momentum quantum number.

This result did not agree with the hydrogen atom. We now know that the Klein-Gordon equation describes particles with internal spin-0. Thus, it should be the equation for the pi-mesic atom: a negative pi-meson bound to a nucleus. Ironically, because of technical difficulties, the verification of the spectra of Eq. (18) for the pi-mesic atom did not occur until 1978.¹³ This was long after the situation was understood¹⁴ both theoretically and also from other experiments.

The next step was the equation of Pauli, which incorporated the concept of spin-1/2 electrons. (Spin was the famous discovery of Goudsmit and Uhlenbeck.¹⁵) Pauli gave the Hamiltonian (energy operator) of the hydrogen atom as

$$H = (-\hbar^2/2m)\nabla^2 + V(r) + (2m^2c^2r)^{-1}(dV/dr) \mathbf{L}\cdot\mathbf{S} \quad (19)$$

where L is the angular momentum operator. S is the spin operator, represented by a (2×2) matrix. Therefore, there are two solutions to the Schrödinger equation, corresponding to spin-up or -down.

The Pauli equation gave agreement with the hydrogen spectra approximation of Eq. (13), with ϕ being replaced by $(j + 1/2)$. "j" is the total angular-momentum quantum number from $J = L + S$. This replacement explained Sommerfeld's semi-ad hoc rule that $\phi \geq 1$. But the Pauli equation obviously was a half-way house to complete understanding. For instance, no one could understand where spin came from. If one took the known "size" of the electron and the value of the angular momentum that the spin value represented, then the edge of the electron would be moving (classically) faster than the speed of light!

IV. The Dirac Equation and Antimatter

The resolution of all this came with the Dirac equation. Note that Eq. (11) can be written as

$$mc^2 = [E^2 - (pc)^2]^{1/2} . \quad (20)$$

Dirac wanted to be able to avoid the analogous square root implicit in the Klein-Gordon form of quantum mechanics. Therefore, he searched for some mathematical way in which the quantum operator form of Eq. (20) could be described by

$$mc^2 = [E^2 - p^2c^2]^{1/2} \quad (21)$$

$$= [(E\gamma_0 - \mathbf{p} \cdot \boldsymbol{\gamma}c)^2]^{1/2} , \quad (22)$$

so that one could write the equation

$$mc^2\psi = \{[(E - V(r))\gamma_0 - \mathbf{p}\cdot\boldsymbol{\gamma}c]\psi \quad . \quad (23)$$

Amazingly, in 1928 Dirac found a solution with the correct mathematical properties. The four γ operators in Eq. (23) were (4 x 4) matrices. Therefore, there were four solutions to the Dirac equation, corresponding to

$$(+E \text{ spin up, } +E \text{ spin down, } -E \text{ spin down, } -E \text{ spin up}). \quad (24)$$

The last two solutions have negative energies. Dirac was so scared of these solutions that when he first attacked the hydrogen atom with his equation he only looked for an approximate solution.¹⁶ It corresponded to the results of Pauli. Later, Darwin and Gordon exactly solved the Dirac equation for the hydrogen atom, and they obtained the correct energy levels as¹⁷

$$W_{n,j} = mc^2 f(n, j+1/2) \quad (25)$$

$$\cong mc^2 - R [1/n^2 + (\alpha^2/n^4)\{ n/(j+1/2) - 3/4 \} \dots] \quad (26)$$

As obtained in the Pauli equation, j is the total angular momentum quantum number, corresponding to the operator $\mathbf{J} = \mathbf{L} + \mathbf{S}$.

V. Antimatter, the Negative-Energy States

Now began the fascinating fight to understand the negative-energy solutions of Dirac. For details on what follows, consult the excellent

articles on the history of the Dirac equation and on the early stages of experimental particle physics.¹⁸

A summary can be started with Dirac's above-mentioned fear of the negative-energy solutions. Obviously something was right since the hydrogen atom worked so well. Dirac had to think of some physical explanation of them.

The particles described by the solutions of the Dirac equation were "fermions." Such particles have the property that only one of them at a time can occupy any energy state. In 1930 Dirac¹⁹ proposed that all of the negative energy states are filled with particles, forming what is now known as the "Dirac sea." This state was called the ground state since it had the lowest possible energy. An excitation out of this sea leaves a "hole" in it. It has a positive energy and opposite electric charge to the positive-energy solution. But what were these new particles described by the holes? Dirac suggested that they were protons.

This got Dirac into trouble. Bohr had rejected the physical validity of Dirac's equation.²⁰ Bohr felt Dirac's proposal could not be the ultimate answer since there was no correspondence principle (a well-defined, large-energy, classical limit) for spin, and also because negative energies were "absurd." Later, Oppenheimer pointed out that the holes could not be protons because they had the wrong mass (the hole states had to have the same mass as the positive energy solutions). Also, if they were the protons, they would have decayed.²¹

Faced with these criticisms, Dirac modified his holes to have the same mass as the electrons, and boldly wrote,²² *"A hole, if there were one, would be a new kind of particle, unknown to experimental physics,*

having the same mass and opposite charge to an electron. We may call such a particle an anti-electron. ... Presumably the protons will have their own negative-energy states ... an unoccupied one appearing as an anti-proton."

The stage was set for Carl Anderson,²³ who in 1932 reported the discovery of the anti-electron or positron, as it is now called. He was using a cloud chamber in Millikan's lab. However, this chamber had a piece of lead in it and a magnetic field perpendicular to the vertical. Therefore, high-energy cosmic rays hit the lead, made electron-positron pairs, and the two particles curved in opposite directions in the magnetic field. This showed that the two tracks came from particles with the same momentum but opposite charges. Antimatter had been discovered!

VI. The Understanding of Antimatter

In the following years, we came to understand antimatter.

First, in 1935, Yukawa proposed²⁴ that the strong force must be mediated by a particle of about 100 MeV rest-mass energy because it obviously was short ranged. This meant the potential for the strong force was not of the Newton-Coulomb $1/r$ form, but rather was

$$V(r) = g [e^{-r/\lambda}] / r \quad . \quad (27)$$

In 1937 Anderson and others²⁵ found a particle with a mass of about 200 times that of the electron. But it lived much too long for it to be associated with the strong force. Interestingly, however, it too came in species with both charges.

This part of the story was laid to rest in 1947, when a University of Bristol group found the following processes:²⁶

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \rightarrow e^- + \bar{\nu}_e + \bar{\nu}_\mu + \nu_\mu, \quad (28)$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \nu_\mu + \bar{\nu}_\mu. \quad (29)$$

The π -mesons were the Yukawa particles that mediate the strong force. The μ particles were the ones found by Anderson and collaborators in 1937. These muons are charged leptons which decay weakly into the electron species of the same charge. Thus, we see that the pions, muons, and electrons come with both particle and antiparticle species. The neutrinos (ν) are the particles (and antiparticles) first postulated by Pauli to conserve energy in the beta-decay of neutrons. Eventually they and their antiparticles were all experimentally shown to exist.

In the 1950's, these and other ideas were systematized in the CPT theorem for quantum field theory.^{2,27} In it, three quantum-mechanical transformations, P, T, and C, are combined. The last of these, C, has as its basis the complex nature of the solutions of quantum mechanics. C changes the "charges" of a particle. In the simplest case this is done by complex-conjugating the wave function and equation. CPT in quantum theory is similar to strong-reflection in classical theory.²⁸ But in quantum theory the complex nature of the fields and equations means that CPT is equivalent to strong-reflection times complex conjugation of the fundamental fields and equations. This new feature, the inherent

complex nature of the system, is what requires the negative-energy solutions, and hence predicts the existence of antimatter.

In a graphic form, the theorem says that if one were to take a motion picture of a physical process, and if one then were to run the film backwards (T), look at it in a mirror and rotate oneself by 180° (P), and change the "charges" or "internal quantum numbers" of the particles, then one would not be able to tell the difference in the laws of physics seen. Put another way, every particle has an antiparticle with

- i) the same (inertial) mass
- ii) the same total lifetime
- iii) the opposite electric charge
- iv) the opposite magnetic moment
- v) the opposite internal quantum numbers.

This theorem has been verified in every experiment ever done. It is a foundation of modern quantum field theory, and indeed, one does not know how to formulate a mathematically consistent relativistic field theory that does not satisfy this theorem.¹ Even ideas of the separation of matter from antimatter in the early universe are based upon CP violation (and a presumed countermanding T violation), not CPT violation. We have observed and understand the existence of P violation, CP violation, C violation, and we hope to observe T violation.³⁰ But we do not foresee CPT violation, at least in the short term.

VI. The Discovery of the Antiproton

Returning to 1955, the Bevatron was completed at Berkeley with just enough energy (6.2 GeV) to create antiprotons. This was done by

accelerating protons to the maximum energy, colliding them with nuclei, and observing the process

$$p + \bar{p} \rightarrow 3p + \bar{p} . \quad (30)$$

The actual detection method is described in Ref. 31.

Now, one of us (MMN), being a quantum mechanic and raised after all this was done, always thought, "Why did the discovery of the antiproton earn a Nobel Prize? They should have gotten it if they hadn't found the antiproton!" Then, in preparing this and other discussions related to our antiproton gravity work, we came across and read the 1956 Scientific American article on the discovery of the antiproton.³¹ There it said, *"At this time (1955) several long-standing bets on the existence of the antiproton started to be paid. The largest we know of was for (1955) \$500."*

To us, of our generation, this is simply amazing. We find it absolutely clear that antiparticles exist. We do not see how one can conceive of there not being an antiparticle for every type of particle. It is always difficult to understand the past with one's present viewpoint, and for us this was no exception.

The best recent analogy to this we can think of is that there were those who doubted that the W's and the Z would be discovered at the SPS, the SPS being the accelerator at CERN built to discover them. But that had nothing to do with an antiparticle. That only had to do with there being a correct unification of electromagnetism and the weak interactions. Not finding the W's and Z would be like proton decay not

being seen, which supposedly was the key to unifying the electroweak and the strong interactions³² in the "standard model."^{32,33}

VII. Conclusion

Of course, one must always test for CPT violation. Somewhere it may break down. In fact, there are ideas floating around about how this might happen by a small amount for phenomena on a cosmological scale.³⁴ (We ourselves have been guilty of such types of speculation.³⁵)

However, except for the case of gravity, CPT is experimentally proven to be correct with precisions ranging up to parts in 10^9 , depending upon the interaction and the phenomenon involved.³⁶ Since the proposed antiproton gravity experiment would be the first involving antimatter, at present we can experimentally say nothing about CPT and gravity. CPT violation would imply a different gravitational interaction than expected. However, as is noted elsewhere in these Proceedings,⁴ that is not necessary. Indeed, new gravitational forces from quantum theory are a more likely possibility to induce unexpected results.

But an important thing to remember is that if any of these speculated violations of CPT turn out to be correct, they would be small, and would have NO effect - period - end of report - NO effect on present-day applied-physics experiments. All such experiments are dealing with the every-day earth. As such they are governed by the electromagnetic interactions which hold both us and also magnets together. Electromagnetism is the interaction for which CPT has been tested to the highest accuracy. Further, quantum electrodynamics is

the quantum theory whose fundamental predictions have been tested to the highest accuracy.³⁷ Finally, recall that it was the fundamental electrically charged particle, the electron, whose antiparticle, the positron, was first discovered and comprehended.

We understand antimatter just as well as we understand matter. Our only problem is that we don't know how to handle antimatter in a matter world. The opposite would be the case for antipeople in an antimatter world, if there are any.

That brings up a final point. Who decides what is matter and what is antimatter? Is it all relative, as our simplist view of the equations of physics might indicate? Or, is nature really telling us something by our not seeing any evidence of antimatter galaxies in the universe? There are some ideas that this "baryon asymmetry" (we do not see antimatter galaxies) is not just a local fluctuation. These ideas hold that baryon asymmetry is a real effect due to CP or CPT violation being much more significant in the early universe.³⁸ If this is correct, then antimatter is not a relative concept. Dirac would have been proven correct.

REFERENCES

1. R. P. Feynman, in Elementary Particles and the Laws of Physics, The 1986 Dirac Memorial Lectures (Cambridge Univ. Press, Cambridge, 1987).
2. G. Lüders, Ann. Phys. (NY) **2**, 1 (1957).
3. P. Morrison, Am J. Phys. **26**, 358 (1958).

4. M. M. Nieto and B. E. Bonner, in these Proceedings, on looking for new gravitational forces using antiprotons.
5. N. Bohr, *Phil. Mag.* 26, 1, 476, 857 (1913).
6. E. Schrödinger, *Ann. Phys.* 79, 361 (1926).
7. E. Schrödinger, *Naturwiss.* 14, 664 (1923).
8. Translation of Ref. 7 above, in Collected Papers on Wave Mechanics (Blackie & Son, London, 1928), p.41.
9. A. Einstein, *Ann. Phys.* 17, 891, 905, (1905); 18, 639 (1905).
10. E. C. G. Stückelberg, *Helv. Phys. Acta* 14, 322, 588 (1941); R. P. Feynman, *Phys. Rev.* 74, 939 (1948).
11. A. Sommerfeld, Atomic Structure and Spectral Lines (Methuen, London, 1934), Chap. V, p. 251. This translation of the 5th German edition gives a physical feeling for the old quantum theory.
12. H. Kragh, *Am. J. Phys.* 52, 1024 (1984), discusses the 1926 "equation with many fathers."
13. K.-C. Wang, F. Boehm, et al., *Phys. Lett. B* 79, 170 (1978); L. Delker, et al., *Phys. Rev. Lett.* 42, 89 (1979).
14. M. M. Nieto, *Am. J. Phys.* 47, 1067 (1979).
15. S. A. Goudsmit, *Phys. Today* 29, No. 6, 40 (1976); G. E. Uhlenbeck, ibid. p. 43. Published on the 50th anniversary of the discovery of spin.
16. P. A. M. Dirac, *Proc. Roy. Soc.* 117, 610 (1928).
17. C. G. Darwin, *Proc. Roy. Soc.* 118, 654 (1928); W. Gordon, *Zeit. Phys.* 48, 11 (1928).
18. D. F. Moyer, *Am. J. Phys.* 49, 944, 1055, 1120 (1981) is a three article series on the discovery of the Dirac electron. Also see, H.

Kragh, Arch. Hist. Exp. Sci 24, 31 (1981). L. M. Brown and L. Hoddeson, Phys. Today 35, No. 4, 36 (1982), discuss the early history of elementary particle physics.

19. P. A. M. Dirac, Proc. Roy. Soc. A 126, 260 (1930).
20. See p. 1056 of the second article in Ref. 18.
21. J. R. Oppenheimer, Phys. Rev. 35, 562 (1930).
22. P. A. M. Dirac, April 1931 letter to Van Fleck, quoted in p. 1060 of the second article in Ref. 18.
23. C. D. Anderson, Science 76, 238 (1932).
24. H. Yukawa, Proc. Phys.-Math. Soc. Jpn. 17, 48 (1935).
25. S. H. Neddermeyer and C. D. Anderson, Phys. Rev. 51, 884 (1937); J. C. Street and E. C. Stevenson, Phys. Rev. 51, 1005 (1937); Y. Nishina, M. Takeuchi, and T. Ichimiya, Phys. Rev. 52, 1198 (1937).
26. C. M. G. Lattes, et al., Nature 159, 694 (1947).
27. R. F. Streater and A. S. Wightman, PCT, Spin & Statistics, and All That (W. A. Benjamin, New York, 1964), and references therein.
28. J. S. Bell, Proc. Roy. Soc. A 231, 479 (1955); W. Pauli, in Niels Bohr and the Development of Physics, edited by W. Pauli (Pergamon, New York, 1955), p. 30.
29. J. J. Sakurai, Invariance Principles and Elementary Particles (Princeton Univ. Press, Princeton, N.J., 1964), Chap. 6, p. 136.
30. J. Miller, in these Proceedings, on tests of CP violation (and fundamental symmetry experiments) at LEAR.
31. E. Segrè and C. E. Wiegand, Sci. Am. 194, No. 6, 37 (1956).
32. S. Weinberg, Dædalus, Proc. Am. Acad. Arts Sci. 106, No. 4, 17 (1977); S. Weinberg, in Ref. 1.

33. T. Goldman, in these Proceedings, on the "standard model."
34. J. F. Donoghue, B. R. Holstein, and R. W. Robinett, *Gen. Rel. and Grav.* 17, 207 (1985).
35. T. Goldman and M. M. Nieto, in Physics with Antiprotons at LEAR in the ACOL Era, edited by U. Gasta'di, R. Klapisch, J. M. Richard, and J. Tran Thanh Van (éditions Frontières, Paris, 1985), p. 639.
36. Particle Data Group, "Review of Particle Properties," *Phys. Lett. B* 170, 1 (1986). Sec. 16, on tests of conservation laws.
37. S. S. Schweber, *Rev. Mod. Phys.* 58, 449 (1986).
38. A. G. Cohen and D. B. Kaplan, Harvard University preprint HUTP-87/A061.