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# THE HYBRID MONTE CARLO ALGORITHM AND THE CHIRAL TRANSITION

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In this talk I describe tests of the Hybrid Monte Carlo Algorithm for QCD done in collaboration with Greg Kilcup and Stephen Sharpe. We find that the acceptance in the global Metropolis step for Staggered fermions can be tuned and kept large without having to make the step-size prohibitively small. We present results for the finite temperature transition on  $4^4$  and  $4 \times 6^3$  lattices using this algorithm.

## INTRODUCTION

The Hybrid Monte Carlo algorithm (HMC) [1] is an exact fermion algorithm of the kind proposed by [2]. The basic evolution through phase space is controlled by the Hybrid algorithm (HA) [3]. The HA is an efficient blend of molecular dynamics (fast movement through phase space) [4] and the Langevin algorithm (ergodicity) [5] [6]. It has the major drawback that the equations of motion have to be discretized. The ensuing finite difference equations involve a finite step-size  $\epsilon$  in fictitious evolution time. So to get the final answer one has to take the limit  $\epsilon \rightarrow 0$ . This extrapolation can be potentially disastrous since the velocity through phase space tends to zero in this limit. The solution is to use the Hybrid evolution to propose link changes of the entire lattice and then do a global accept/reject (Monte Carlo). The potential achilles heel in such global Monte Carlo's is that the acceptance rates can tend to zero, and careful tests are needed. The tests we have carried out so far on QCD show promise. In this talk I describe the method and present our data.

### 1) Hybrid Monte Carlo for QCD with Fermions

The partition function for QCD in Euclidean space can be written in a number of equivalent forms:

$$Z = \int D\psi D\psi^\dagger DU \exp(S_G + \bar{\psi}(\mathcal{D} + m)\psi) \quad (1a)$$

$$Z = \int DU \det(\mathcal{D} + m) \exp(S_G) \quad (1b)$$

$$Z = \int DUD\psi D\psi^\dagger \exp(S_G - \psi^\dagger \frac{1}{\mathcal{D} + m} \psi) \quad (1c)$$

$S_G$  is the gauge action (which in the present study is the simple Wilson action).  $\mathcal{D}$  is the fermion covariant derivative.  $U$  the link variables and  $m$  the quark mass.

For staggered fermions

$$M|U|_{ij} = m\delta_{ij} + \frac{1}{2} \sum_{\mu} \eta_{i,\mu} [U_{i,\mu} \delta_{i,\mu-j} - U_{i-\mu,\mu}^\dagger \delta_{i,\mu-j}] \quad (2)$$

This operator is not positive definite (in the limit  $m \rightarrow 0$ ,  $M^\dagger = -M$ ), so we work with the effective fermion action

$$S_{eff} = \psi^\dagger (M^\dagger M)^{-1} \psi = \psi^\dagger \frac{1}{-\mathcal{D}^2 + m^2} \psi \quad (3)$$

where  $M \equiv (\mathcal{D} + m)$  is the Dirac operator and  $\psi$  are the pseudo-fermions. However, since

$$\int D\psi D\psi^\dagger e^{-\psi^\dagger (M^\dagger M)^{-1} \psi} = \det M \det M^\dagger \quad (4)$$

using  $M^\dagger M$  corresponds to twice the number of flavors. For staggered fermions this doubling is removed by noting that

$$\int D\psi D\psi^\dagger e^{\psi^\dagger (M^\dagger M)^{-1} \psi} = \det M \quad (5)$$

where  $\psi_e$  is defined only on even sites. There still remains the standard doubling problem so even with eqn (5), the fermion contribution corresponds to four flavors. We prefer staggered fermions because our first goal is to investigate the chiral transition.

The bottleneck in fermion simulations is that direct evaluation of either  $\det(\mathcal{D} + m)$  or  $\frac{1}{-\mathcal{D}^2 + m^2}$  is prohibitively slow. In Ref. [7] we describe our results for the chiral transition using an exact method based on form 1b (EDA). The largest lattice that can be handled with this method on a Cray-XMP is  $4^4$ . This is because six columns of  $M^{-1}$  (the inverse of the Dirac operator) have to be calculated at each link update to get the change in  $\det(\mathcal{D} + m)$ . The possibility that Hybrid Monte Carlo algorithm is faster arises because in it we are required to calculate  $\frac{1}{-\mathcal{D}^2 + m^2}$  acting on a vector only once per sweep of the entire lattice. However, there is a catch. What one really gets is an unbiased estimator of the fermion contribution. Thus, one may need a much larger statistical sample to get accurate results.

Our goal is to compare the two exact algorithms and to subject the HMC algorithm to detailed tests. The present study is limited to the  $\Phi$  version of the Hybrid algorithm proposed by Gottlieb *et al.*[8]. Since the details of the  $\Phi$ -algorithm are published, we just write down the answer and discuss it.

### 2) The $\Phi$ -Hybrid Algorithm

To set up a molecular dynamics evolution to simulate a system described by a Hamiltonian  $H$ , one discretizes Hamilton's equations of motion. For QCD, we need to first identify the variables conjugate to  $U$  and  $\varphi$ .

The Hamiltonian for the  $\Phi$ -algorithm is

$$H_\Phi = \frac{1}{2} \text{Tr} \sum P_{i,\mu}^2 + \frac{\beta}{N} \text{Tr} \sum (1 - U_p) + \varphi_i^T (M^T M)^{-1} \varphi_i \quad (6)$$

where  $P_{i,\mu}$  are the momenta conjugate to  $U_{i,\mu}$  and  $U_p$  is the plaquette. The  $\varphi$  fields have no dynamics in this algorithm and thus no conjugate momenta. In the modified partition function

$$Z = \int D\varphi DUDP e^{-H_\Phi} \quad (7)$$

the conjugate momenta are gaussian variables and can be integrated over. Thus correlation functions of  $U$  and  $\varphi$  are the same as with eqn. (1)

To preserve  $U$  as an element of  $SU(3)$ , the evolution of  $U$  must take the form

$$U(t + \epsilon) = e^{i\epsilon P} U(t) \quad (8)$$

where  $P$  is a traceless antihermitian matrix. From this it follows that

$$U(t) = e^{iP} U(t) \quad (9)$$

where  $P$ , the momenta conjugate to  $U$  are represented by

$$P_{i,\mu} = \sum_{\alpha=1}^8 r_{i,\mu}^\alpha \lambda_\alpha \quad (10)$$

Here  $\lambda_\alpha$  are generators of  $SU(3)$  normalized to  $\text{Tr}(\lambda_\alpha \lambda_\beta) = 2\delta_{\alpha\beta}$  and  $r_{i,\mu}^\alpha$  are eight independent real gaussian random variables with variance  $\frac{1}{2}$ . This gives the desired distribution for  $P$  i.e.  $e^{-\frac{1}{2} \text{Tr} P^2}$

The effective fermion action is represented by gaussian noise [6]. Let

$$\varphi = M^{-1} r \quad (11)$$

where  $\{r\}$  are again gaussian variables with variance  $\frac{1}{2}$ . then the  $\varphi$  have the desired distribution

$$P(r) = e^{-r^2} \Rightarrow P(\varphi) = e^{-\varphi^T (M^T M)^{-1} \varphi} \quad (12)$$

As pointed out by Gottlieb *et al.*[8], for a viable molecular dynamics update it suffices to write the evolution equations for  $U$  and  $P$  keeping  $\varphi$  fixed. These equations should preserve  $H_\Phi$  and the differential volume element in configuration phase space. This is satisfied by the equation of motion for  $U$  given in eqn. (9). The equation for  $P$  is determined by the requirement that  $H_\Phi$  be a constant of motion. The answer is as follows. For links starting at even sites,

$$\begin{aligned} i\dot{P}_{i,\mu} = & \left[ -\frac{\beta}{3} U_{i,\mu} V_{i,\mu} \right. \\ & + 2U_{i,\mu} \left( \sum_{\nu} U_{i,\mu\nu} T_{i,\mu\nu} \right. \\ & \left. \left. - \sum_{\nu} U_{i,\mu\nu}^* T_{i,\mu\nu} \right) \right]_{TA} \quad (13) \end{aligned}$$

while for links starting at odd sites

$$\begin{aligned} i\dot{P}_{i,\mu} = & \left[ -\frac{\beta}{3} U_{i,\mu} V_{i,\mu} \right. \\ & + 2U_{i,\mu} \left( \sum_{\nu} T_{i,\mu\nu} U_{i,\mu\nu} \right. \\ & \left. \left. - \sum_{\nu} T_{i,\mu\nu} U_{i,\mu\nu}^* \right) \right]_{TA} \quad (14) \end{aligned}$$

where  $X \equiv (M^T M)^{-1} \varphi$ ,  $T_{i,\mu} \equiv X_i X_i^*$  and  $TA$  stands for the traceless antisymmetric part. Also the staggered fermion phases are implicit in the  $U$

Given eqns. (9,13,14), the discretized leap-frog update scheme we use is as follows (it differs from that given in [8] in the order in which  $P$  and  $U$  are updated):

- [1] At time  $t = 0$ , refresh the momentum  $P$  and generate the  $\varphi$  fields. The  $U$  are assumed to have some initial value.
- [2] Update  $U(t)$  to  $U(t + \frac{\epsilon}{2})$  using

$$U(t + \frac{\epsilon}{2}) = e^{i\frac{\epsilon}{2}P(t)}U(t)$$

We approximate the exponential by a fourth order polynomial and reunitarize the resulting matrix.

- [3] Calculate  $\dot{P}$  at  $t + \frac{\epsilon}{2}$  using eqns. (13) and (14). This is the most time consuming part of the calculation because it requires calculating  $(M^\dagger M)^{-1}\varphi$ . We do this using the conjugate gradient algorithm.
- [4] Calculate  $P(t + \epsilon) = P(t) + \dot{P}\epsilon$
- [5] Update  $U$  to  $t + \frac{3\epsilon}{2}$  using  $P(t + \epsilon)$ . This is the same as step 2. Thereafter steps 2 to 5 are repeated  $n \equiv nrd$  times. At the end one has  $P(t + n\epsilon)$  and  $U(t + (n - \frac{1}{2})\epsilon)$ . To complete the leap frog step, calculate

$$U(t + nt) = e^{i\frac{\epsilon}{2}P(t+n\epsilon)}U(t + (n - \frac{1}{2})\epsilon)$$

One now has  $U(t + n\epsilon)$ ,  $P(t + n\epsilon)$  and the pseudo-fields  $\varphi$  which were held constant throughout the evolution. At this time the  $P$  and  $\varphi$  are refreshed and the evolution repeated. To make measurements, all expectation values are calculated as time averages after the system has equilibrated.

Starting with an equilibrium configuration, the molecular dynamics evolution preserves the equilibrium. The refreshing of  $P$  makes the algorithm ergodic and also pushes an arbitrary starting configuration towards equilibrium.

To summarise, the errors in the  $\Phi$ -Hybrid algorithm arise in the discrete approximations to eqns. (8,13,14) and from the incomplete convergence in the conjugate gradient iterations. They can all be systematically reduced; the finite step size error by going to higher order difference equations and the exponentiation error by including more terms in the expansion. In our tests, we limited ourselves to the lowest order leap-frog integration and approximate the exponential by a fourth order expansion. This algorithm has the desired

property of being reversible i.e. starting with  $U', -P'$  and evolving back gives  $U, -P$ . This, as is shown later, is essential in the Monte Carlo algorithm.

### 3) Global Metropolis Step

In the Metropolis method [9] one proposes small changes in the configurations, and accepts these changes with a probability

$$\text{prob} = \min \left( 1, \frac{P(U' \rightarrow U)e^{-H(U')}}{P(U \rightarrow U')e^{-H(U)}} \right) \quad (15)$$

Here  $U'$  is the trial configuration,  $H(U')$  its action,  $P(U \rightarrow U')$  is the probability of proposing the change given that one is at  $U$ , and  $P(U' \rightarrow U)$  is the reverse probability. If these two probabilities are equal, as they are in most algorithms for pure gauge  $SU(3)$ , then changes which lower the action are always accepted, while those which increase it are accepted only conditionally.

For the  $\Phi$ -Hybrid algorithm described above the "Action/Hamiltonian" is  $H_\Phi(U, P)$  given in eqn. (6). Thus the Hybrid Monte Carlo algorithm consists of the following steps: Given a configuration  $U$ , refresh the momentum  $P$  and generate the pseudo-fields  $\varphi$ . Then evolve to  $U', P'$  using the Hybrid algorithm. Accept/reject the complete change with probability

$$e^{-\delta H_\Phi} = \frac{e^{-H_\Phi(U', P')}}{e^{-H_\Phi(U, P)}} \quad (16)$$

at the end of a leap-frog sequence.

There are two key points that allow the use of the Monte Carlo step. 1) for the leap frog algorithm described above

$$P(U, P \rightarrow U', P') = P(U', -P' \rightarrow U, -P) \quad (17)$$

because the evolution is reversible and it is area preserving. We have ignored errors arising from approximating the exponential by a polynomial since these can be made arbitrarily small by going to higher orders. 2) Since one needs to calculate  $\delta H_\Phi$  for the global accept/reject, the fermion dynamics should be representable by an effective fermion action, like  $\varphi^\dagger (M^\dagger M)^{-1} \varphi$  here. For staggered fermions this last point is satisfied only for a multiple of four flavors. Thus we do not have a  $n_f = 2$  or 3 exact algorithm based on a global Metropolis step yet.

#### 4) Tuning the $\Phi$ Hybrid Monte Carlo Algorithm

In the Monte Carlo step,  $\delta H_\Phi$  consists of three pieces. The first two -  $\Delta P^2$  and  $\Delta S_G$  - are calculated to machine precision. The change in the fermion action is given by an unbiased estimator with a single pseudo-field. However, since the same pseudo-field is used to calculate both the old and the new  $S_{eff}$ , we expect the fluctuations to be correlated and reduce the variance in  $\Delta S_{eff}$ . In fact we believe that one gets reasonable acceptance rates only for changes (value of  $\epsilon$ ) which preserve the correlations. We are currently making tests to check this. Second, in the calculation of  $S_{eff}$ , biased error creeps in because the conjugate gradient algorithm is run to limited precision. Again we would like to quantify whether the bias cancels in  $\Delta S_{eff}$ .

The final distribution of configurations is determined by the two parameters  $\beta$  and  $m_q$  used in calculating  $\delta H_\Phi$ . This Monte Carlo step is totally independent of the parameters  $\beta_h$  and  $(m_q)_h$  used in the Hybrid evolution. This freedom leads to a possibility for optimisation in the parameters  $\beta_h$  and  $(m_q)_h$  [1]. The final goal is fast decorrelation with the correct equilibrium distribution. Thus errors due to 1) finite step size, and 2) incomplete convergence of conjugate gradient can be regarded as simply shifting the optimal values of  $\beta_h$  and  $(m_q)_h$ . Showing that this is true is a major goal of this study. In the next sections we show our results for optimisation in the pure gauge theory and  $n_f = 4$  QCD.

##### 4.1) Pure Gauge

A naive criterion for optimisation is the acceptance rate. We find that in the  $\Phi$ -Hybrid algorithm, the  $\epsilon$  error tends to order the system i.e.  $\langle plaq \rangle_{Hybrid} > \langle plaq \rangle_{Metropolis}$ . We would like to stress that, contrary to popular belief, small step size error does not always lead to disordering. The direction of error depends on the algorithm and the model being studied and conceivably also on the couplings. In our runs so far we find that the peak in the acceptance occurs for  $\beta_h < \beta$  as shown in Figs. 1 and 2. We used lattices of size  $4^4$  and  $6^4$  at  $\beta = 5.6$  and data was accumulated over 2500 Metropolis steps starting from the same thermalised lattice. The noteworthy features in the data are

- 1) The acceptance peak moves towards  $\beta_h = \beta$  when the number of steps ( $nmd$ ) in the leap frog algorithm are increased for fixed  $\epsilon$ .

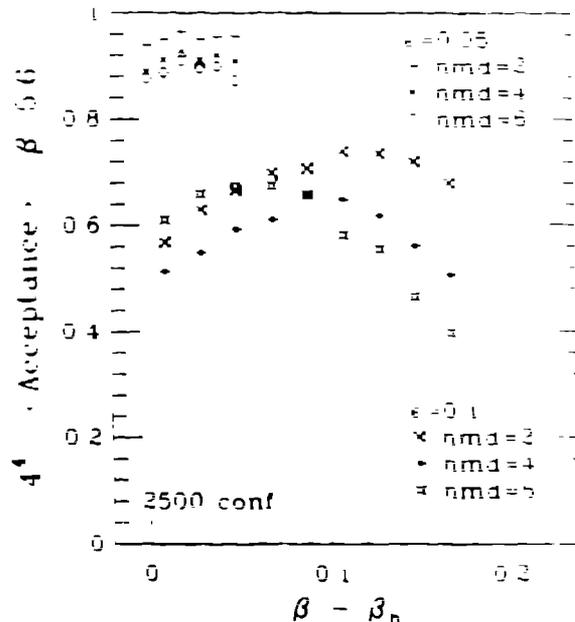


Fig 1 Acceptance for the pure gauge Hybrid Monte Carlo algorithm on a  $4^4$  lattice at  $\beta = 5.6$ . The runs were done for two values of  $\epsilon$  and for a variety of leap-frog steps  $nmd$ . The data is over 2500 Metropolis steps.

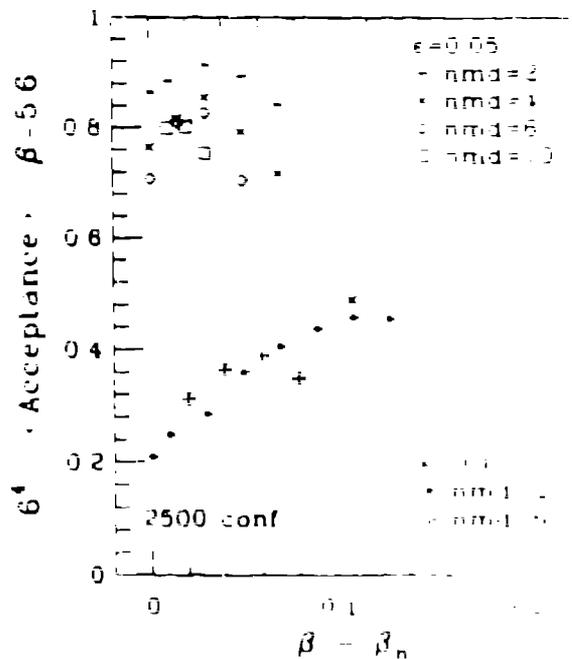


Fig 2 Same as Fig 1 but on a  $6^4$  lattice

- [2] The height of the acceptance peak decreases slowly with  $nmd$ .
- [3] The shape of the acceptance curve depends on  $nmd$ . It becomes more sharply peaked as  $nmd$  is increased.
- [4] The height of the acceptance curve falls rapidly with  $\epsilon$ .

This test suggests that a global Metropolis algorithm is practical for QCD. We are now testing optimization with respect to decorrelations by using a  $9^4$  lattice and measuring the correlation coefficients for block loops. The acceptance at  $\beta = \beta_A = 6.0$  with  $\epsilon = 0.04$  and  $nmd = 5$  is  $\approx 58\%$ .

#### 4.3) QCD with Fermions

We have geared our tests and optimization of the exact  $\Phi$  algorithm towards reproducing the  $n_f = 4$  results for the chiral transition [7]. This was done for two reasons; 1) we have a large body of data to compare against and 2) it would be a very stringent test to reproduce metastable phases. The results presented are part of an ongoing study of the finite temperature transition. So conclusions are preliminary.

The data for how to optimise  $\beta_A$  and  $(m_q)_A$  was collected by dividing the run at a given  $\beta$  and  $m$  into sections of at least 500 time steps (a time step is  $\frac{1}{4}$  sweeps). The results are similar to the pure gauge case. Acceptance peak occurs when both  $\beta_A$  and  $(m_q)_A$  are chosen to disorder the system i.e.  $\beta_A < \beta$  and  $(m_q)_A > m$ . Since it is a two parameter optimisation, we cannot show a figure but a rough estimate of the location of the peak is  $\beta_A = \beta - 0.02$  and  $(m_q)_A = m + 0.006$  for  $\beta$  in the range 5.2 to 5.5 and  $m$  in the range 0.2 to 0.5 on a  $4 \times 6^3$  lattice. The acceptance varied between  $\approx 55\%$  and  $87\%$  in our tests. A notable feature of the acceptance rate is that it showed an interesting phase dependent behavior at  $m = 0.1$ . We discuss this later.

#### 5) Results for the Finite Temperature Transition

The results for the finite temperature transition are of two kinds; 1) to make comparisons between the two exact algorithms we repeated the calculations done with EDA at  $m = 0.05, 0.1$  and  $0.2$  on a  $4^4$  lattice using HMC. 2) To study the finite volume dependence, we looked for the chiral transition at  $m = 0.1$  and  $0.2$  on  $4 \times 6^4$  lattices using hMC.

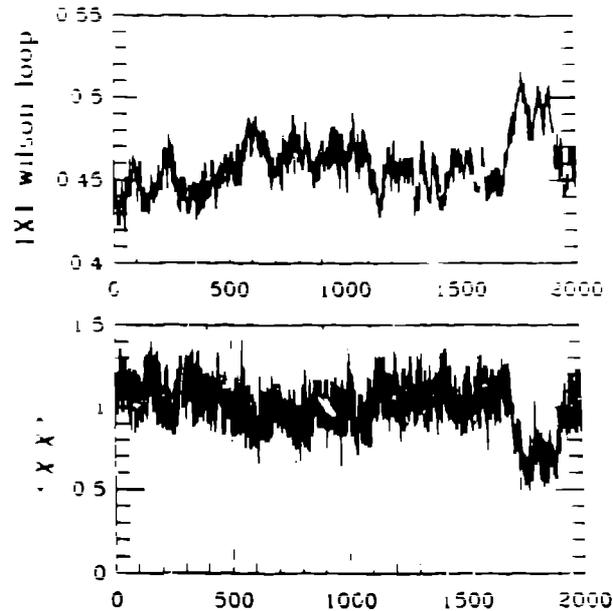


Fig.3 Time history of a)  $1 \times 1$  Wilson loop and b)  $\langle \bar{\chi} \chi \rangle$  on a  $4^4$  lattice at  $\beta = \beta_A = 5.04$  and  $m = m_A = 0.1$ . This data is using the  $\Phi$ -Hybrid algorithm.

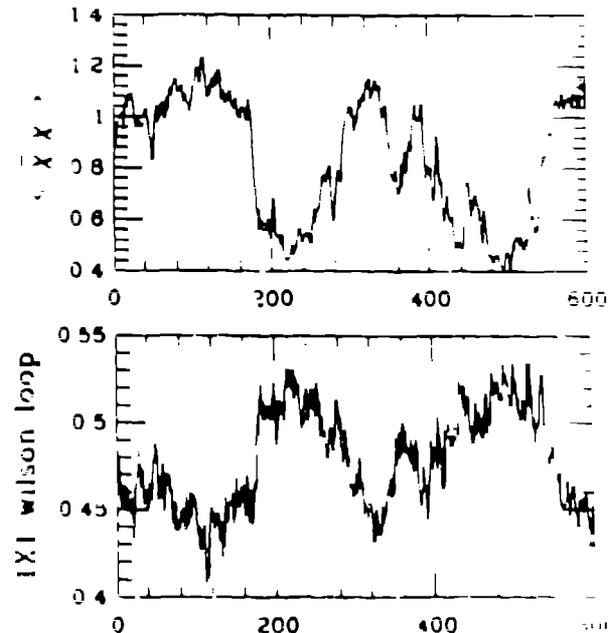


Fig.4 Time history of a)  $1 \times 1$  Wilson loop and b)  $\langle \bar{\chi} \chi \rangle$  on a  $4^4$  lattice at  $\beta = 5.04$  and  $m = 0.1$ . This data is using the exact determinant algorithm.

### 5.1) $m = 0.1$

In Figs. 3 and 4 we show the time history data at  $m = 0.1$  for the two algorithms on  $4^4$  lattices. We find that the  $\Phi$  HMC algorithm reproduces the discontinuity in the chiral order parameter  $\langle \bar{\chi}\chi \rangle$  and Wilson loops. There is a difference in the number of sweeps the system spends in a given phase. This, however, is not a physical parameter.

The behavior of the chiral transition on  $4 \times 6^3$  lattices is shown in Fig. 5. The run parameters were  $m = m_\lambda = 0.1$  and  $\beta = 5.13 = \beta_\lambda + 0.02$ . The data again supports the presence of a first order transition. Flip-flops are present and a histogram shows a clear 2-peak structure. The discontinuity in Wilson loops,  $\langle L \rangle$ ,  $\langle \bar{\chi}\chi \rangle$  and in the convergence after a fixed number of conjugate gradient steps is correlated. Note that increasing the spatial volume to  $6^3$  gives  $\beta_c \approx 5.13$ .

The difference from the  $4^4$  data is that the discontinuity in  $\langle \bar{\chi}\chi \rangle$  is somewhat smaller. The main change is to increase the value in the symmetric phase from  $\approx .5$  to  $\approx .6$ . We have not yet determined whether this is a finite volume effect or whether the couplings need tuning because they are not set at point of maximum discontinuity. However, if true, then we expect much smaller discontinuity at higher masses based on extrapolations of data as shown in fig. 6. This feature is very relevant to our  $m = 0.2$  data discussed below.

The acceptance rate shows interesting behavior in the two phases in our  $4 \times 6^3$  run. It is 55% (35%) in the hot (cold) phase. This can be due to two reasons: 1) the  $\epsilon$  dependent shifts in the couplings (in the hybrid preprocessor) are different in the two phases and 2) we are not exactly at the transition so the free energy in the two phases is not equal. We are making further runs to clarify this feature.

### 5.2) $m = 0.2$

The data on the  $4^4$  lattices for the two algorithms is similar as shown in Figs. 7 and 8. The data on the  $4 \times 6^3$  (fig. 9) lattice is substantially different. Even though there are indications of flip-flops, the discontinuity is too small to allow an unequivocal statement.

These results suggest that if the flip flops are real (it still is a first order transition) then our previous estimate of  $m_{c, \beta}$  (the value of  $m$  up to which the chiral

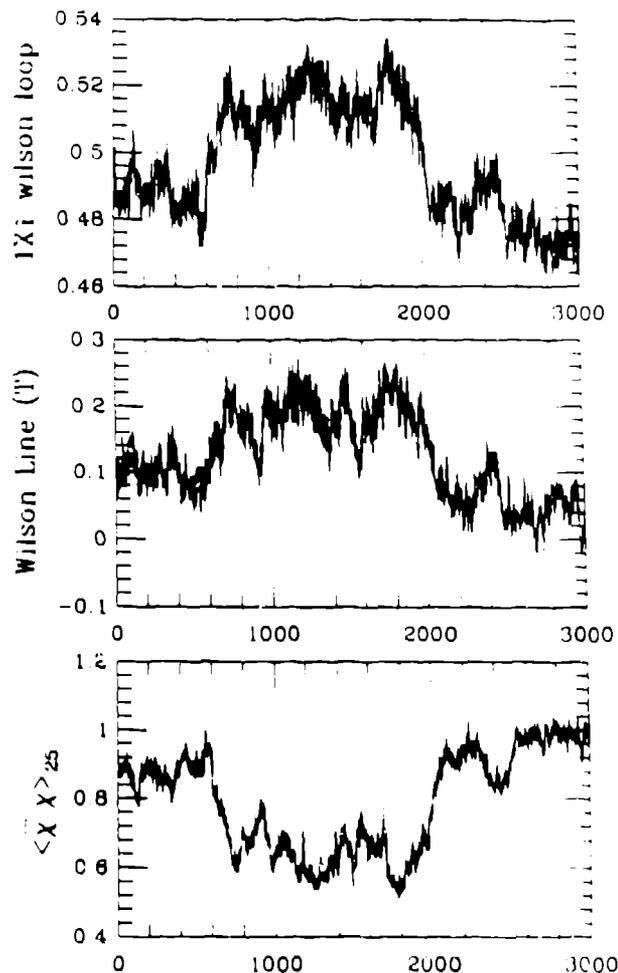


Fig.5 Time history of a)  $1 \times 1$  Wilson loop b) Wilson line and c)  $\langle \bar{\chi}\chi \rangle$  on a  $4 \times 6^3$  lattice at  $\beta = 5.13$ ,  $\beta_\lambda = 5.11$  and  $m = m_\lambda = 0.1$ .

nature of the transition dominates) from  $4^4$  lattices is changed significantly on increasing the spatial volume at heavier quark masses. This interpretation of a large finite volume effect is supported by the data at  $m = 0.1$ .

From these tests we also learned a reason why  $\langle \bar{\chi}\chi \rangle$  may not have been a good probe of the order of the transition when using approximate algorithms. In particular, in simulations using molecular dynamics or Langevin algorithms, a noisy estimate of  $\langle \bar{\chi}\chi \rangle$  is made using pseudofermions. This should be compared to the exact algorithm, where during each sweep  $\langle \bar{\chi}\chi \rangle$  is calculated 0 times on each site. In tests on our  $4^4$  lattices we

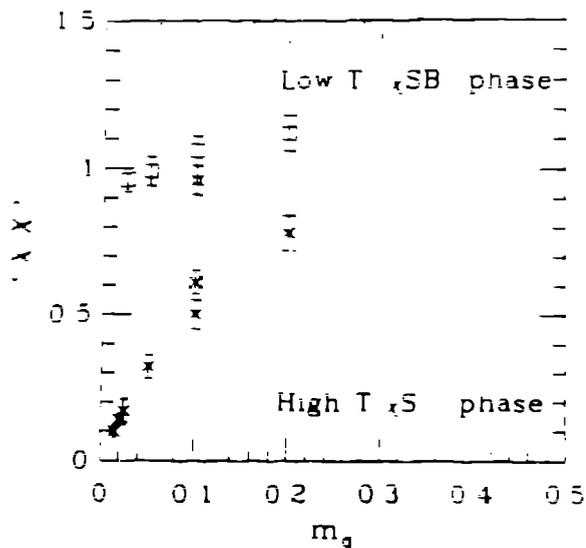


Fig.6 Plot of  $\langle \bar{X}X \rangle$  versus  $m_q$  at the transition from  $4^4$  lattices using EDA. We also show data from HMC on a  $4 \times 6^3$  lattice at  $m_q = 0.1$  (fancy symbols)

find that the noisy estimator gives a distribution centered about the exact  $\langle \bar{X}X \rangle$ , but with a variance which increases with decreasing  $m_q$ . For  $m_q = 0.1$  the standard deviation is  $\approx 0.25$ , which is comparable to the discontinuity. Thus for this and larger masses, large statistics would be required to resolve a first order transition using the noisy estimator. The cure is that one needs to make a more accurate measurement of observables like  $\langle \bar{X}X \rangle$  by using many  $\varphi$  fields. An example of this is shown in Fig. 10 where we show a)  $\langle \bar{X}X \rangle$  evaluated with a single  $\varphi$  field; b) the same data as in (a) but averaged over bins of 5 measurements and c)  $\langle \bar{X}X \rangle$  evaluated using 25  $\varphi$  fields. The noise in a) and b) makes it hard to distinguish flip-flops from fluctuations.

To summarize, we feel that the systematics of the HMC algorithm are still not fully understood. Therefore, we are at present making more detailed tests to understand the algorithm and to determine whether the metastability at  $m_q = 0.2$  survives in the infinite volume limit.

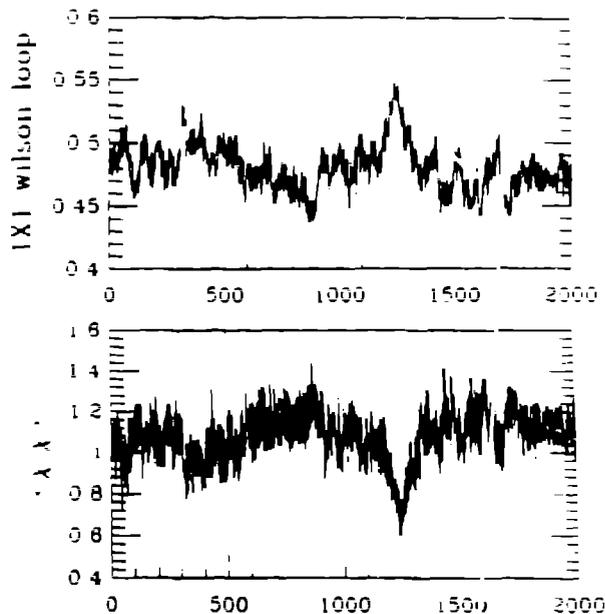


Fig.7 Time history of a)  $1 \times 1$  Wilson loop and b)  $\langle \bar{X}X \rangle$  on a  $4^4$  lattice at  $\beta = \beta_A = 5.18$  and  $m = m_A = 0.2$ . This data is using the  $\Phi$ -Hybrid algorithm.

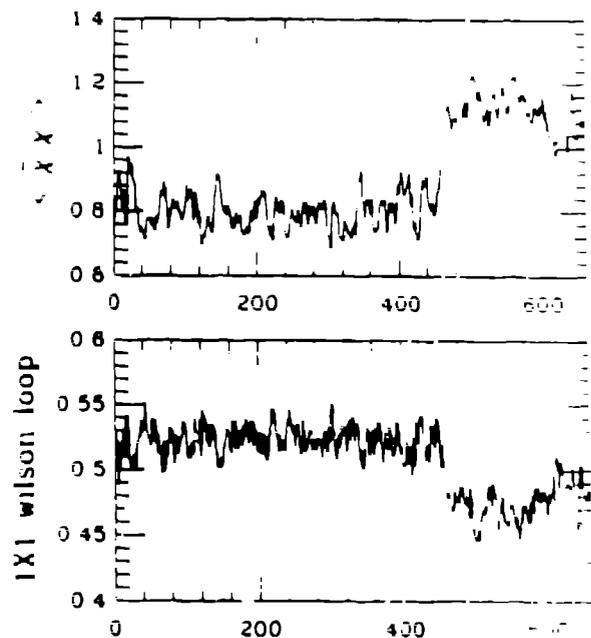


Fig.8 Time history of a)  $1 \times 1$  Wilson loop and b)  $\langle \bar{X}X \rangle$  on a  $4^4$  lattice at  $\beta = 5.18$  and  $m = 0.2$ . This data is using the exact determinant algorithm.

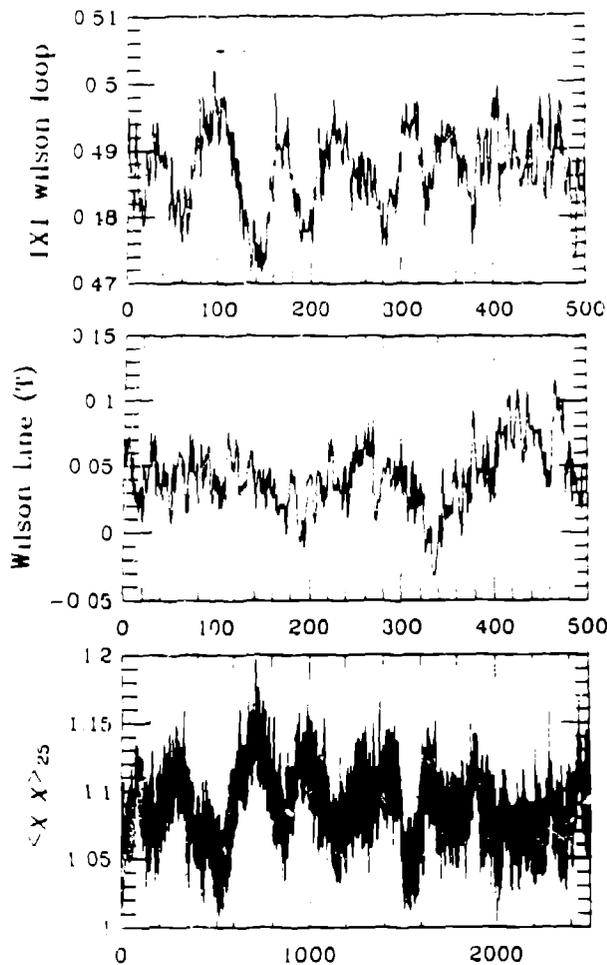


Fig.9 Time history of a)  $1 \times 1$  Wilson loop b) Wilson line and c)  $\langle \bar{\chi} \chi \rangle$  on a  $4 \times 6^3$  lattice at  $\beta = 5.22$ ,  $\beta_h = 5.24$  and  $m = m_h = 0.2$ .

#### 6) Status of the Finite Temperature Transition

There is a consensus that for  $N_f = 4$  and 4 flavors of staggered fermions, there is a strong first order chiral transition at small masses, i.e.  $m = 0.025$ . On  $4^4$  lattices this transition has been shown to survive up to  $m = 0.2$  and there is preliminary evidence that it exists for all masses [7]. In this talk I have shown evidence for a first order transition at  $m = 0.1$  on a  $4 \times 6^3$  lattice. The signal at  $m = 0.2$  is not clear.

Data on  $4 \times 8^3$  lattices is debated. Fukugita *et al.*[10], show evidence for metastability at  $m = 0.1$  while Gottlieb *et al.*[11] and Karsch *et al.*[12] see a clear signal only at  $m = 0.025$ .

Lastly, on a  $6 \times 10^3$  lattice, Kovacs *et al.*[13] find a first order transition at  $m = 0.025$  but not at  $m = 0.05$ .

The status of the chiral transition using approximate algorithms for  $n_f = 2$  is one of conflicting evidence. For  $n_f = 2$ , Gottlieb *et al.*[11] see no clear

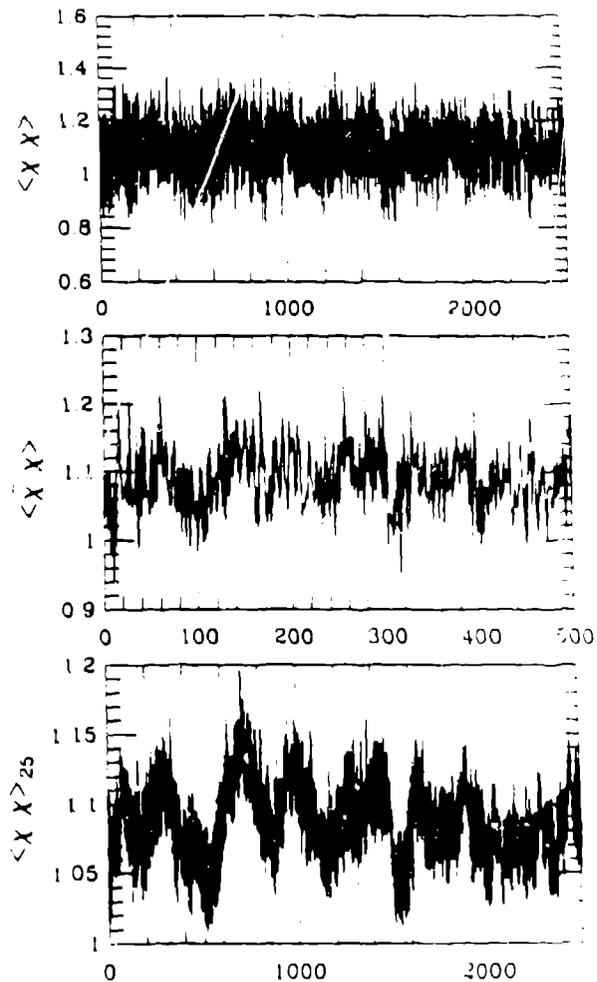


Fig.10 Time history of  $\langle \bar{\chi} \chi \rangle$  evaluated with a) a single scalar field  $\phi$  b) same data as in (a) but in bins of 5 and c) with 25 scalar field  $\phi$ . The comparison highlights the size of fluctuations in the unbiased estimator.

evidence of a transition for  $m_q a = 0.0125 - 0.2$ . In contrast, Kogut and Sinclair [14] find a first order transition for  $m_q a = 0.0125$ , but no transition for  $m_q a = 0.025$ . However, Gavai *et al.*[15] do find a transition for  $m_q a = 0.025$ , while Fukugita *et al.*[16] see one for  $m_q a = 0.1$ . We should also mention that Gavai *et al.*[17] find that the transition is first order along an interpolation between  $n_f = 2$  and  $n_f = 3$  where two quarks are held at  $m_q a = 0.025$ , while the mass of the third is varied between 0.025 and  $\infty$ .

Calculations for  $n_f = 4$  with an exact algorithm have only been done on a  $4^4$  lattice at  $m = 0.02$ . We again find a strong first order transition [7].

To end on an optimistic note, I believe that we should have much cleaner data by the next meeting. The nature of the finite temperature transition for two light and one strange flavor will be resolved in the near future.

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