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TITLE EXOTIC DECAYS OF LIGHT MESONS

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EXOTIC DECAYS OF LIGHT MESONS

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Abstract

We discuss the information one could obtain on physics beyond the minimal standard model from studies of the rare and forbidden decays of π^0, η and η'.

1. Introduction

My task in this talk is to try to assess what one could learn about physics beyond the minimal standard model [1] from studies of the decays of the light neutral non-flavored mesons. By "exotic decay" we shall understand a decay mode or a decay mechanism which is not allowed in the minimal standard model. Our attention will be restricted to the decays of π^0, η and η' [2], which appear to be the best tools for the issues we consider.

We shall focus on η-decays, for which the question of sensitivity to possible new physics has not been investigated yet to the same degree as for the π^0. To do so is timely, especially in view of the recent discovery at Saclay [3] of the possibility of high-flux tagged η-beams. Along with the discussion of η-decays we shall review briefly, for completeness and for comparison, the pertinent decays of the π^0. Where it appears useful, we shall consider also the decays of the η'.

The π^0, η and η' have both nonleptonic decay modes and decay modes involving leptons. We shall consider the nonleptonic decay modes only in connection with studies of CP-violation, since in CP-conserving observables the theoretical uncertainties would not allow a new contribution to be discerned. Among the decays involving leptons there are "neutral current decays" (decays in which the total charge of the leptons is zero) and "charged current decays." The π^0 has only neutral current decays. Neutral current decays consist of forbidden decays (e.g. $\eta \to \mu e$) and allowed decays. The forbidden decays probe the existence of interactions that violate the symmetries of the minimal standard model. The allowed decays receive a contribution from the electromagnetic interactions (in the case of decays involving charged leptons) and a first-order contribution from the neutral-current interaction of the minimal standard model. They can probe the existence of additional flavor-conserving neutral current interactions. New neutral current couplings are present in many extensions of the minimal standard model, generated for example by the exchange of new neutral gauge-bosons or Higgs bosons, or by the exchange of leptoquarks. The neutral current decays of the π^0 are sensitive only to isovector neutral current interactions. These involve only the u- and the d-quark. The neutral current decays of the η (and η') probe isoscalar neutral current interactions. In addition to couplings involving the u- and d-quark, isoscalar neutral current interactions may contain neutral currents involving the s-quark and also heavier quarks.

For equal coupling strength, the branching ratios of $\Delta S = 0$ neutral current decays are much smaller than those of the $\Delta S = 1$ neutral current decays. This is due to the large difference in the lifetimes of the decaying mesons. For example, the branching ratio of $K_L \to \mu e$ for a pseudoscalar coupling is [4] $B(K_L \to \mu e) \simeq 10^4 \mid h_{PP}^{(\mu e)} \mid^2$, and the branching ratio of $\pi^0 \to \mu e$ for the same type of coupling is $B(\pi^0 \to \mu e) \simeq 10^{-7} \mid g_{PP}^{(\mu e)} \mid^2$ (see Eq. 5.4), where $h_{PP}^{(\mu e)}$ and $g_{PP}^{(\mu e)}$ measure the strength of the couplings relative to $G/\sqrt{2}$. $K_L \to \mu e$ could not be, of course, a substitute for $\pi^0 \to \mu e$, since it is sensitive to a different interaction.

In the first and second part of the talk we shall discuss decays that have a bearing on the electron-quark and on the muon-quark interaction, respectively. Part 4 deals with π^0, η and η' decays involving neutrinos and some other light weakly interacting particles. In part 5 we discuss muon-number violating decays, and in part 6 decays that probe some aspects of the charged current interaction. The last part of the talk is devoted to decays sensitive to CP-violating interactions. We end the talk with our conclusions.

2. Electron - Quark Interactions

$\underline{\pi^0 \to e^+e^-}$

Before addressing the decay $\pi^0 \to e^+e^-$, let us consider some general features common to $\pi^0 \to e^+e^-, \eta \to e^+e^-$ and $\eta \to \mu^+\mu^-$. All these decays are suppressed in the minimal standard model, since the dominant amplitude is proportional to

$m_\ell \alpha^2$ ($m_\ell \equiv$ mass of the lepton). The general form of the P $\to \ell^+\ell^-$(P = π^0, η) amplitude is (see e.g. Ref. [5])

$$M(P \to \ell^+\ell^-) = a\bar{u}i\gamma_5 v + b\bar{u}v \tag{2.1}$$

and the decay rate is given by

$$\Gamma(P \to \ell^+\ell^-) = (m_\pi r/8\pi)(\mid a \mid^2 + r^2 \mid b \mid^2) \tag{2.2}$$

where r = $(1 - 4m_\ell^2/m_P^2)^{1/2}$. The constants a and b can be decomposed as

$$a = Rea^{(e)} + iIma^{(e)} + a^{(n)} \tag{2.3}$$

$$b = b^{(n)} \tag{2.4}$$

where $a^{(e)}$ is the electromagnetic contribution, and $a^{(n)}$, $b^{(n)}$ represent contributions from non-electromagnetic (effective neutral current) interactions [6]. We shall consider only tree-level contributions for the latter; consequently $a^{(n)}$ and $b^{(n)}$ are real.

The present experimental value of the $\pi^0 \to e^+e^-$ branching ratio B($\pi^0 \to e^+e^-$) $\equiv \Gamma(\pi^0 \to e^+e^-)/\Gamma(\pi^0 \to all)$ is [7].

$$B(\pi^0 \to e^+e^-)_{expt} = (1.8 \pm 0.7) \times 10^{-7} \tag{2.5}$$

One has $\mid Ima^{(e)} \mid \simeq 2.6 \times 10^{-7}$, yielding the unitarity bound B($\pi^0 \to e^+e^-$) $\geq 4.7 \times 10^{-8}$. Let us introduce the quantity

$$x_\pi^{(e)} \equiv [(Rea^{(e)} + a^{(n)})^2 + (b^{(n)})^2]^{1/2} / \mid Ima^{(e)} \mid \tag{2.6}$$

The experimental result (2.5) allows (at the 95% confidence level)

$$0 \leq x_\pi^{(e)} < 2.4 \tag{2.7}$$

A contribution to $x_\pi^{(e)}$ comes from $Rea^{(e)}$ [8]. The only other contribution in the minimal standard model is the one due to Z^0 – exchange, which is however negligible (see below). In general $\pi^0 \to e^+e^-$ can receive a contribution from an effective neutral-current interaction of the form

$$L^{(n)} = (G/\sqrt{2}) \ [g_{AA}^{(e)} \ \bar{e}\gamma_\lambda\gamma_5 e J_A + g_{PP}^{(e)} \ \bar{e}i\gamma_5 e \ J_P \ + g_{SP}^{(e)} \ \bar{e}e \ J_P] \ , \tag{2.8}$$

where G is the Fermi constant, and

$$J_{A\lambda} = \frac{1}{2}(\bar{u}\gamma_\lambda\gamma_5 u \quad \bar{d}\gamma_\lambda\gamma_5 d) \ , \tag{2.9}$$

$$J_P = \frac{1}{2}(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d) \quad . \tag{2.10}$$

If the interaction (2.8) is due to the exchange of a boson X of mass m_x, the quantities $g_{ij}^{(e)}$ (and also the analogous quantities in sections 3 - 5) are proportional (ignoring the width of X) to $1/(p_x^2 - m_x^2)$. For $\pi^0 \to e^+e^-$, for example in the case of a spin-zero boson exchanged in the s-channel, $(G/\sqrt{2})g_{ij}^{(e)} = f_i' f_j''/(m_\pi^2 - m_x^2)$ where f_i' and f_j'' are the couplings of X to the leptons and quarks, respectively.

The amplitudes $a^{(n)}, b^{(n)}$ are given by

$$a^{(n)} = (2Gm_\pi m_e/\sqrt{2})\ g_{AA}^{(e)}\rho_A + (Gm_\pi^2/\sqrt{2})g_{PP}^{(e)}\rho_P \tag{2.11}$$

$$b^{(n)} = (Gm_\pi^2/\sqrt{2})g_{SP}^{(e)}\rho_P \tag{2.12}$$

where ρ_A, ρ_P are defined as

$$< 0 \mid J_{A\lambda} \mid \pi^0 >= im_\pi \rho_A p_\lambda \tag{2.13}$$

$$< 0 \mid J_P \mid \pi^0 >= m_\pi^2 \rho_P \quad . \tag{2.14}$$

One has $\rho_A = -f_\pi/m_\pi\sqrt{2}$($f_\pi$ is the charged-pion decay constant defined by $< 0 \mid \bar{d}\gamma_\lambda\gamma_5 u \mid \pi^+ >= if_\pi p_\lambda$), and $\rho_P \simeq \rho_A m_\pi/(m_u + m_d) \simeq \pm\ 8$ (for $m_u = 4.2$ MeV, $m_d = 7.5$ MeV). Thus

$$a^{(n)} \simeq \pm(7.9 \times 10^{-10})g_{AA}^{(e)} \pm (1.2 \times 10^{-6})g_{PP}^{(e)} \quad , \tag{2.15}$$

$$b^{(n)} \simeq \pm(1.2 \times 10^{-6})g_{SP}^{(e)} \quad . \tag{2.16}$$

In the minimal standard model $g_{AA}^{(e)} = 1, g_{PP}^{(e)} = g_{SP}^{(e)} = 0$ and therefore $b^{(n)} = 0$, and $a^{(n)}$ is negligible ($\mid a^{(n)} \mid \simeq\ 3 \times 10^{-3} \mid Im a^{(e)} \mid$)

The experimental result (2.5) implies

$$\mid g_{SP}^{(e)} \mid \lesssim 0.5 \tag{2.17}$$

and, assuming $\mid Re a^{(e)} \mid \lesssim 2 \mid Im a^{(e)} \mid$,

$$\mid g_{PP}^{(e)} + 6.6 \times 10^{-4} g_{AA}^{(e)} \mid \lesssim 1 \quad . \tag{2.18}$$

Thus, barring cancellation,

$$\mid g_{PP}^{(e)} \mid \lesssim 1 \tag{2.19}$$

and

$$| g_{AA}^{(e)} | \lesssim 1.5 \times 10^3 \quad . \tag{2.20}$$

What information is available on quantities $g_{AA}^{(e)}, g_{PP}^{(e)}$ and $g_{SP}^{(e)}$ from other data? Constraints based on the uncertainty in calculations of the hyperfine splitting of the ground state of the hydrogen atom are $| g_{AA}^{(e)} | \lesssim 40$, $| g_{PP}^{(e)} | \lesssim 210$ [9] (for $p_x^2 \simeq 0$, if (2.8) is due to the exchange of a boson X). From a measurement [10] of the ratio $R \equiv \sigma(e^+e^- \to h)/\sigma(e^+e^- \to \mu^+\mu^-)$ for 14 GeV to 46.8 GeV center of mass energy we find $| g_{AA}^{(e)} |, | g_{PP}^{(e)} |, | g_{SP}^{(e)} | \lesssim 0\ (1)$ (for $m_x >> 46.8$ GeV). The $g_{SP}^{(e)}$–term contributes to the electric dipole moment of the neutron (D_n) and the electric dipole moment of the electron (D_e). The present experimental limits on D_n and D_e suggest $| g_{SP}^{(e)} | \lesssim 10^{-2} - 10^{-1}$.

A conclusion is that (2.19) is the best limit on $g_{PP}^{(e)}$ even for $m_x >> 46.8$ GeV (or for $g_{PP}^{(e)}$ independent of momentum transfer). More accurate experiments will yield better bounds on $x_\pi^{(e)}$. Given such bounds, the presence of $g_{ij}^{(e)}$–terms would be difficult if not impossible to discern, in view of the theoretical uncertainties in the value of $Rea^{(e)}$.

$\underline{\eta \to e^+e^-}$

The present experimental limit for $B(\eta \to e^+e^-)$ is [7]

$$B(\eta \to e^+e^-) < 3 \times 10^{-4} \qquad (90\% \text{ c.l.}). \tag{2.21}$$

The imaginary part of the electromagnetic amplitude is $| Ima^{(e)} | \simeq 2.9 \times 10^{-7}$ [11], implying the unitarity bound $B(\eta \to e^+e^-) \geq 1.75 \times 10^{-9}$. $| Rea^{(e)} |$ is expected to be of the same order of magnitude or smaller than $| Ima^{(e)} |$ [12]. Hence there is a large unexplored domain of branching ratios between the present limit and the electromagnetic contribution.

The limit (2.21) implies for the quantity $x_\eta^{(e)}$, defined in the same way as $x_\pi^{(e)}$ (Eq. 2.6),

$$0 \leq x_\eta^{(e)} < 414 \quad . \tag{2.22}$$

The decay $\eta \to e^+e^-$ is sensitive to an isoscalar effective neutral current interaction [13]. Neglecting contributions from heavy quarks (c,b,t, $\cdots$), the most general non-derivative four-fermion interaction that can contribute to $\eta \to e^+e^-$ is of the form

$$L^{(n)} = L_1^{(n)} + L_2^{(2)} \tag{2.23}$$

$$L_1^{(n)} = (G/\sqrt{2}) \left[f_{AA}^{(e)} \bar{e}\gamma^\lambda \gamma_5 e K_{A\lambda} + f_{PP}^{(e)} \bar{e} i \gamma_5 e K_P + f_{SP}^{(e)} \bar{e} e K_P \right] \quad , \tag{2.24}$$

$$L_2^{(n)} = (G/\sqrt{2})\left[\tilde{f}_{AA}^{(e)}\bar{e}\gamma^\lambda\gamma_5 e\tilde{K}_{A\lambda} + \tilde{f}_{PP}^{(e)}\bar{e}i\gamma_5 e\tilde{K}_P + \tilde{f}_{SP}^{(e)}\bar{e}e\tilde{K}_P\right] \quad , \tag{2.25}$$

where

$$K_{A\lambda} = \frac{1}{2}(\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d) \tag{2.26}$$

$$K_P = \frac{1}{2}(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d) \tag{2.27}$$

$$\tilde{K}_{A\lambda} = \bar{s}\gamma_\lambda\gamma_5 s \tag{2.28}$$

$$\tilde{K}_P = \bar{s}i\gamma_5 s \tag{2.29}$$

The amplitudes $a_1^{(n)}, a_2^{(n)}, b_1^{(n)}$ and $b_2^{(n)}(a^{(n)} = a_1^{(n)} + a_2^{(n)}, b^{(n)} = b_1^{(n)} + b_2^{(n)})$ are

$$a_1^{(n)} = (G/\sqrt{2})[2m_\eta m_e \kappa_A f_{AA}^{(e)} + m_\eta^2 \kappa_P f_{PP}^{(e)}] \tag{2.30}$$

$$b_1^{(n)} = (G/\sqrt{2})m_\eta^2 \kappa_P f_{SP}^{(e)} \quad , \tag{2.31}$$

$$a_2^{(n)} = (G/\sqrt{2})[2m_\eta m_e \tilde{\kappa}_A \tilde{f}_{AA}^{(e)} + m_\eta^2 \tilde{\kappa}_P \tilde{f}_{PP}^{(e)}] \tag{2.32}$$

$$b_2^{(n)} = (G/\sqrt{2})m_\eta^2 \tilde{\kappa}_P \tilde{f}_{SP}^{(e)} \quad , \tag{2.33}$$

where $\kappa_A, \kappa_P, \tilde{\kappa}_A$ and $\tilde{\kappa}_P$ are defined by

$$< 0 \mid K_{A\lambda} \mid \eta > = im_\eta \kappa_A p_\lambda \tag{2.34}$$

$$< 0 \mid K_p \mid \eta > = m_\eta^2 \kappa_P \tag{2.35}$$

$$< 0 \mid \tilde{K}_{A\lambda} \mid \eta > = im_\eta \tilde{\kappa}_A p_\lambda \tag{2.36}$$

$$< 0 \mid \tilde{K}_p \mid \eta > = m_\eta^2 \tilde{\kappa}_P \tag{2.37}$$

The quantities $\kappa_A, \tilde{\kappa}_A$ cannot be calculated as reliably as ρ_A (Eq. 2.13), which is related to a measured matrix element by isospin symmetry. The use of SU(3) symmetry does not solve the problem completely, since η is not a pure SU(3) octet state. To estimate κ_A and $\tilde{\kappa}_A$ we shall assume $\eta = \eta_8 cos\theta - \eta_0 sin\theta$ (and $\eta' = \eta_8 sin\theta + \eta_0 cos\theta$), i e. that η is a linear combination of the octet state η_8 and of the singlet state η_0 only. We shall take $f\eta_8 = -1.25 f\pi/\sqrt{2}$ in $< 0 \mid A_{8\lambda} \mid \eta_8 >$ $= if\eta_8 p_\lambda (A_{i\lambda} = \bar{q}\gamma_\lambda\gamma_5(\lambda_i/2)q, i = 0,...8)$ [14] and assume $< 0 \mid A_{8\lambda} \mid \eta_0 >$, $< 0 \mid A_{0\lambda} \mid \eta_8 > \simeq 0$. For θ and $f\eta_0$ in $< 0 \mid A_{0\lambda} \mid \eta_0 > = if\eta_0 p_\lambda$ we shall use $\theta \simeq -20°$ and $f\eta_0 \simeq -f_\pi/\sqrt{2}$ [14,15], deduced from the experimental values of $\Gamma(\pi^0 \to 2\gamma), \Gamma(\eta \to 2\gamma)$ and $\Gamma(\eta' \to 2\gamma)$.

We obtain then

$$\kappa_A \simeq \pm 0.16 \tag{2.38}$$

$$\tilde{\kappa}_A \simeq \mp 0.18 \tag{2.39}$$

(sign $\kappa_A = -$ sign f_π).

To estimate κ_P and $\tilde{\kappa}_P$ we use the equations for $\partial_\lambda A_{8\lambda}$ and $\partial_\lambda A_{0\lambda}$ and ignore the contribution of the gluon anomaly [16]. We find

$$\kappa_P \simeq \kappa_A m_\eta/(m_u + m_d) \simeq \pm 7.5 \tag{2.40}$$

$$\tilde{\kappa}_P \simeq \tilde{\kappa}_A m_\eta / 2m_s \simeq \mp 0.33 \tag{2.41}$$

(with $m_s \simeq 150$ MeV). Using (2.38 - 2.41) the $\eta \to e^+e^-$ amplitudes are

$$a^{(n)} \simeq \pm 7.4 \times 10^{-10} f^{(e)}_{AA} \mp 8.3 \times 10^{-10} \tilde{f}^{(e)}_{AA} \pm 1.9 \times 10^{-5} f^{(e)}_{PP} \mp 8.2 \times 10^{-7} \tilde{f}^{(e)}_{PP} \ , \tag{2.42}$$

$$b^{(n)} \simeq \pm 1.9 \times 10^{-5} f^{(e)}_{SP} \mp 8.2 \times 10^{-7} \tilde{f}^{(e)}_{SP} \ . \tag{2.43}$$

In the minimal standard model $\tilde{f}^{(e)}_{AA} = -\frac{1}{2}, f^{(e)}_{AA} = 0$, and $f^{(e)}_{ij} = \tilde{f}^{(e)}_{ij} = 0$ for ij = PP, SP; thus $b^{(n)} = 0$ and $a^{(n)}$ is negligible ($| a^{(n)} | \simeq 1.4 \times 10^{-3} \mid Im a^{(e)} |$). The limit (2.21) implies (barring cancellations).

$$| f^{(e)}_{PP} |, | f^{(e)}_{SP} | \lesssim 7 \tag{2.44}$$

$$| \tilde{f}^{(e)}_{SP} |, | \tilde{f}^{(e)}_{SP} | \lesssim 150 \tag{2.45}$$

Upper bounds on the axial-vector couplings are much weaker than (2.45).

Constraints from other data (valid under the conditions stated for $g^{(e)}_{ij}$ earlier) are $| f^{(e)}_{ij} |, | \tilde{f}^{(e)}_{ij} | \lesssim 0(1)$ (ij = AA,PP,SP) from $e^+e^- \to h$. The hydrogen ground state hyperfine splitting provides $| f^{(e)}_{AA} | < 70$, and much weaker limits for $\tilde{f}^{(e)}_{AA}, \tilde{f}^{(e)}_{PP}, \tilde{f}^{(e)}_{SP}$. $(D_n)_{expt}$ and $(D_e)_{expt}$ suggest $| f^{(e)}_{SP} | \lesssim 10^{-1} - 10^{-2}$ and $| \tilde{f}^{(e)}_{SP} | \lesssim 1$.

3. Muon-Quark Interactions

$\underline{\eta \to \mu^+\mu^-}$

The experimental value for the $\eta \to \mu^+\mu^-$ branching ratio is [7]

$$B(\eta \to \mu^+\mu^-) = (6.5 \pm 2.1) \times 10^{-6} \tag{3.1}$$

One has $| Im a^{(e)} | \simeq 1.5 \times 10^{-5}$, implying the unitarity bound $B(\eta \to \mu^+\mu^-) \geq 4.3 \times 10^{-6}$ [11,17,18]. For $x^{(\mu)}_\eta$ defined in analogy with (2.5), one obtains from the result (3.1) at the 95% confidence level

$$0 \leq x^{(\mu)}_\eta < 1.2 \tag{3.2}$$

The decay $\eta \to \mu^+\mu^-$ is sensitive to a neutral current interaction of the same form as (2.23), but with e replaced everywhere by μ [19]. The amplitudes $a^{(n)}, b^{(n)}$ are also the same as (2.30 - 2.33), except for $e \to \mu$. Using (2.38 - 2.41) we find

$$a^{(n)} \simeq \pm 1.5 \times 10^{-7} f_{AA}^{(\mu)} \pm 1.9 \times 10^{-5} f_{PP}^{(\mu)} \mp 1.7 \times 10^{-7}\ \tilde{f}_{AA}^{(\mu)} \mp 8.2 \times 10^{-7}\ \tilde{f}_{PP}^{(\mu)} \quad , \tag{3.3}$$

$$b^{(n)} \simeq \pm 1.9 \times 10^{-5}\ f_{SP}^{(\mu)} \mp 8.2 \times 10^{-7}\ \tilde{f}_{SP}^{(\mu)} \quad . \tag{3.4}$$

In the minimal standard model $\tilde{f}_{AA}^{(\mu)} = -\frac{1}{2}, f_{AA}^{(\mu)} = 0$, and $f_{ij}^{(\mu)} = \tilde{f}_{ij}^{(\mu)} = 0$ for $ij = PP, SP$. As a result $b^{(n)} = 0$, and $a^{(n)}$ is negligible ($\mid a^{(n)} \mid \simeq 5.7 \times 10^{-3} \mid Ima^{(e)} \mid$). The experimental result implies (barring cancellations)

$$\mid f_{SP}^{(\mu)} \mid \lesssim 1 \tag{3.5}$$

$$\mid \tilde{f}_{SP}^{(\mu)} \mid \lesssim 22 \tag{3.6}$$

and, assuming $\mid Rea^{(e)} \mid \lesssim 2 \mid Ima^{(e)} \mid$.

$$\mid f_{PP}^{(\mu)} \mid \lesssim 3 \tag{3.7}$$

$$\mid \tilde{f}_{PP}^{(\mu)} \mid \lesssim 58 \tag{3.8}$$

$$\mid f_{AA}^{(\mu)} \mid \lesssim 315 \tag{3.9}$$

$$\mid \tilde{f}_{AA}^{(\mu)} \mid \lesssim 280 \tag{3.10}$$

Information on $f_{AA}^{(\mu)}$ and $\tilde{f}_{AA}^{(\mu)}$ can be deduced from the results of a cross-section asymmetry measurement in deep inelastic muon-nucleus scattering [20]. The measured asymmetry is sensitive to interactions involving an axial-vector quark current. The result agrees with the prediction of the minimal standard model within the experimental error, which is about 30%. This implies roughly $\mid f_{AA}^{(\mu)} \mid \lesssim \frac{1}{3}$ (for $m_x >> 15$ GeV, if the new interaction is due to the exchange of a boson X). The contribution of a $\tilde{f}_{AA}^{(\mu)}$−term to the asymmetry would be suppressed by 1 - 2 orders of magnitude, since the corresponding current does not involve the valence quarks of the nucleon. Constraints from hyperfine splittings of muonic-atom levels are very weak, mainly because of the relatively large theoretical uncertainties [21]. For $f_{SP}^{(\mu)}$ and $\tilde{f}_{SP}^{(\mu)}$ limits of $\mid f_{SP}^{(\mu)} \mid \lesssim 10^{-1} - 10^{-2}$ and $\mid \tilde{f}_{SP}^{(\mu)} \mid \lesssim 1$ are suggested by D_n.

As in the case of $\pi^0 \to e^+e^-$, detection of the presence of a new interaction from a nonzero $x_\eta^{(\mu)}$ would be hampered by theoretical uncertainties in the value of $Rea^{(e)}$.

4. Neutrinos and Some Other Light Particles

$\pi^0, \eta, \eta' \to \nu\nu'$

These decays can proceed only if the neutrino states of both chiralities exist, or if lepton-number is not conserved. Consequently, they are forbidden in the minimal standard model. Below we shall discuss the decays $\pi^0 \to \nu\nu'$, $\eta \to \nu\nu'$ and $\eta' \to \nu\nu'$ assuming that all (additive) lepton numbers are conserved. Then $\nu' = \bar{\nu}$ where ν is any of the known neutrinos, or any possible new neutrino (with $m_\nu < m_p/2$).

The general form of the $\pi^0 \to \nu\bar{\nu}$ amplitude and the expression for the $\pi^0 \to \nu\bar{\nu}$ rate are those in Eqs.(2.1) and (2.2). Since the electromagnetic contribution is absent, the amplitudes a and b are $a = a^{(n)}$ and $b = b^{(n)}$.

The experimental limit [22]

$$B(\pi^0 \to X)_{expt} \equiv \sum_i \Gamma(\pi^0 \to X_i)/\Gamma(\pi^0 \to all) < 2.4 \times 10^{-5} \qquad (90\%\ \text{c.l.}) \quad (4.1)$$

holds for π^0-decay into all possible final states made up of unobserved weakly interacting particles of sufficiently long lifetime.

The decay $\pi^0 \to \nu\bar{\nu}$ is sensitive to an interaction that can be obtained from (2.8) with the replacement $e \to \nu$, $g_{ij}^{(e)} \to g_{ij}^{(\nu)}$ (ij = AA, PP, SP). In the minimal standard model (where $g_{AA}^{(\nu)} = 1$, $g_{SP}^{(\nu)} = g_{PP}^{(\nu)} = 0$), extended to incorporate massive neutrinos, the branching ratio for π^0-decay into a particular neutrino pair is [23]

$$B(\pi^0 \to \nu\bar{\nu}) \simeq (3 \times 10^{-8})(1 - 4m_\nu^2/m_\pi^2)^{1/2}(m_\nu/m_\pi)^2. \qquad (4.2)$$

The branching ratio (4.2) has a maximum $\simeq 3 \times 10^{-9}$ at $m_\nu = m_\pi/\sqrt{6} = 55$ MeV.

For the general interaction the branching ratio in the limit $m_\nu = 0$ is [22]

$$B(\pi^0 \to \nu\bar{\nu}) \simeq 10^{-6}[(g_{PP}^{(\nu)})^2 + (g_{SP}^{(\nu)})^2]. \qquad (4.3)$$

The experimental limit (4.1) implies

$$[(g_{PP}^{(\nu_i)})^2 + (g_{SP}^{(\nu_i)})^2]^{1/2} \leq 5 \qquad (i = e, \mu, \tau, ...) \quad (4.4)$$

For $\pi^0 \to \nu_e\bar{\nu}_e$, $\pi^0 \to \nu_\mu\bar{\nu}_\mu$ and $\pi^0 \to \nu_\tau\bar{\nu}_\tau$ one has the more stringent limits [24]

$$B(\pi^0 \to \nu_e\bar{\nu}_e)_{expt} < 3.1 \times 10^{-6} \qquad (90\%\ \text{c.l.}) \quad (4.5)$$

$$B(\pi^0 \to \nu_\mu\bar{\nu}_\mu)_{expt} < 3.1 \times 10^{-6} \qquad (90\%\ \text{c.l.}) \quad (4.6)$$

$$B(\pi^0 \rightarrow \nu_\tau \bar{\nu}_\tau)_{expt} < 2.1 \times 10^{-6} \qquad (90\% \text{ c.l.}) \tag{4.7}$$

implying

$$[(g_{SP}^{(\nu_i)})^2 + (g_{PP}^{(\nu_i)})^2]^{1/2} \leq 1.8 \qquad (i = e, \mu) \tag{4.8}$$

$$[(g_{SP}^{(\nu_\tau)})^2 + (g_{PP}^{(\nu_\tau)})^2]^{1/2} \leq 1.5 \tag{4.9}$$

$\eta \rightarrow \nu\bar{\nu}$

There is no direct experimental information on $\eta \rightarrow \nu\bar{\nu}$. A weak limit

$$B(\eta \rightarrow X) \equiv \sum_i \Gamma(\eta \rightarrow X_i)/\Gamma(\eta \rightarrow all) < 10^{-1} \qquad (90\% \text{ c.l.}) \tag{4.10}$$

for η-decay into final states composed of unobserved weakly interacting particles of long enough lifetimes follows from the total branching ratio for neutral η-decays and from the branching ratios of particular neutral η-decays [25].

The decay $\eta \rightarrow \nu\bar{\nu}$ is sensitive to an interaction that can be obtained from (2.23) with the replacements $e \rightarrow \nu$, $f_{ij}^{(e)} \rightarrow f_{ij}^{(\nu)}$, $\bar{f}_{ij}^{(e)} \rightarrow \bar{f}_{ij}^{(\nu)}$ (ij = AA, PP, SP). In the standard model ($\bar{f}_{AA}^{(\nu)} = -\frac{1}{2}$, $f_{AA}^{(\nu)} = 0$, $f_{ij}^{(\nu)} = \bar{f}_{ij}^{(\nu)} = 0$ (ij = PP, SP)) with massive neutrinos, the branching ratio for η-decay into a particular neutrino pair is

$$B(\eta \rightarrow \nu\bar{\nu}) \simeq (4 \times 10^{-9})(1 - 4m_\nu^2/m_\eta^2)^{1/2}(m_\nu/m_\eta)^2, \tag{4.11}$$

where we have used the value (2.39) for $\bar{\kappa}_A$. The branching ratio (4.11) has a maximum of $B(\eta \rightarrow \nu\bar{\nu}) \simeq 4 \times 10^{-10}$ at $m_\nu = m_\eta/\sqrt{6} = 224$ MeV. In the limit $m_\nu = 0$ only P-type couplings contribute. An experimental upper limit of 10^{-6}, for example, would set an upper bound of about 0.4 on $| f_{PP}^{(\nu)} |$ or $| f_{SP}^{(\nu)} |$, and an upper bound of about 8.5 on $| \bar{f}_{PP}^{(\nu)} |$ or $| \bar{f}_{SP}^{(\nu)} |$ [26].

$\eta' \rightarrow \nu\bar{\nu}$

This decay is sensitive to the same general neutral current interaction as $\eta \rightarrow \nu\bar{\nu}$. In the standard model with massive neutrinos we find

$$B(\eta' \rightarrow \nu\bar{\nu}) \simeq (1.3 \times 10^{-10})(1 - 4m_\nu^2/m_{\eta'}^2)^{1/2}(m_\nu/m_{\eta'})^2, \tag{4.12}$$

which has a maximum value of $B(\eta' \rightarrow \nu\bar{\nu}) \simeq 1.3 \times 10^{-11}$ at $m_\nu = 391$ MeV [26].

Experimental information on the couplings relevant for $\pi^0 \rightarrow \nu\bar{\nu}$ and $\eta, \eta' \rightarrow \nu\bar{\nu}$ is available, of course, only on those involving the ν_e and the ν_μ. The agreement of the experimentally deduced ν_μ-quark interaction with the one in the minimal standard model [27] indicates, roughly, that $| f_{AA}^{(\nu_\mu)} | \leq O(1/10)$ (for m_x heavier than about 10-20 GeV, if the underlying interaction is due to the exchange of a boson X). The constraint on $|\, \bar{f}_{AA}^{(\nu_\mu)} \,|$ is weaker, probably by an order of magnitude. The results of the experiment of Ref. [28] on the y-distributions in deep inelastic ν_μ, $\bar{\nu}_\mu$ scattering imply $|\, g_{SP}^{(\nu_\mu)} \,|$, $|\, g_{PP}^{(\nu_\mu)} \,|$, $|\, f_{SP}^{(\nu_\mu)} \,|$, $|\, f_{PP}^{(\nu_\mu)} \,| \;\lesssim\; 0.4$. The contribution of the $\bar{f}_{SP}^{(\nu_\mu)}$ and $\bar{f}_{SP}^{(\nu_\mu)}$ terms to the cross section is expected to be suppressed, and consequently the limits on $\bar{f}_{SP}^{(\nu_\mu)}$ and $\bar{f}_{SP}^{(\nu_\mu)}$ are weaker, probably by an order of magnitude. The low-energy reaction $\bar{\nu}_e d \;\rightarrow\; \bar{\nu}_e np$ [29] indicates, roughly, $|\, g_{PP}^{(\nu_e)} \,|$, $|\, g_{SP}^{(\nu_e)} \,| \;\lesssim\; O(1)$.

The final states in $\pi^0, \eta, \eta' \rightarrow X$ may include other particles besides neutrinos. An example is the decay into photinos, if they are sufficiently light. A recent estimate [30] of the branching ratios for $\pi^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$ yields $B(\pi^0 \;\rightarrow\; \tilde{\gamma}\tilde{\gamma}) \;\leq\; 5 \times 10^{-8}$.

While the decays $P \rightarrow \nu\bar{\nu}$ are sensitive to pseudoscalar-type couplings, other types of neutrino-quark interactions could be probed through the decays $\eta, \eta' \rightarrow M\nu\bar{\nu}$, where M is a meson (or mesons) with appropriate quantum numbers [31]. An interesting case is $\eta \rightarrow \pi^0\nu\bar{\nu}$. This decay can occur in the minimal standard model only via isospin invariance breaking effects (for a discussion of the analogous decay $\eta \rightarrow \pi e\nu$ see Section 6). One expects therefore the branching ratio to be around $10^{-13} - 10^{-14}$. Beyond the standard model $\eta \rightarrow \pi^0\nu\bar{\nu}$ could proceed via neutral second-class currents [32], or via an interaction which couples neutrinos to a scalar quark current. For the known neutrinos we would expect the upper limits for branching ratios of $\eta \rightarrow \pi^0\nu\bar{\nu}$ generated by second-class or scalar couplings to be not larger than those in Eqs. (6.4), (6.6), (6.9) and (6.10). But for decays into a new neutrino the branching ratio could be larger. An upper limit on $B(\eta \rightarrow \pi^0$ "X") would, for example, set constraints on the scalar couplings of new neutrinos.

$\underline{\pi^0, \eta, \eta' \rightarrow \nu\bar{\nu}\gamma}$

Unlike $\pi^0, \eta, \eta' \rightarrow \nu\nu'$, these decays are allowed in the minimal standard model. The differential rate for $\pi^0 \;\rightarrow\; \nu\bar{\nu}\gamma$ decay into a specific neutrino pair is given by [33]

$$\frac{d\Gamma}{dx} \;=\; \Gamma_0(1 - 4sin^2\theta_W)^2 \frac{G^2 m_\pi^4}{96\alpha\pi^3}\; x\;(1-x)^3(1-b/4x)(1-b/x)^{1/2}, \qquad (4.13)$$

where $b \;=\; 4m_\nu^2/m_\pi^2$, $x \;=\; 1 - 2E/m_\pi$, $E \;=$ photon energy in the π^0-rest frame and $\Gamma_0 \equiv \Gamma(\pi^0 \rightarrow 2\gamma)$. For massless neutrinos the resulting branching ratio is [34]

$$\frac{\Gamma(\pi^0 \;\rightarrow\; \nu\bar{\nu}\gamma)}{\Gamma(\pi^0 \;\rightarrow\; 2\gamma)} \;=\; \frac{G^2 m_\pi^4}{1920\alpha\pi^3}(1 - 4sin^2\theta_W)^2. \qquad (4.14)$$

Thus, to the extent that the effects of the neutrino masses can be neglected, and if there is no appreciable contribution from other final states, $\pi^0 \rightarrow \gamma$ "X" would be a reliable tool for determining the number of neutrino families. Unfortunately, the branching ratio (4.14) is very small: for $sin^2\theta_W = 0.23$ [27] one obtains

$$\frac{\Gamma(\pi^0 \rightarrow \nu\bar{\nu}\gamma)}{\Gamma(\pi^0 \rightarrow 2\gamma)} \simeq 6.6 \times 10^{-19}. \tag{4.15}$$

The rates of $\eta \rightarrow \nu\bar{\nu}\gamma$ and $\eta' \rightarrow \nu\bar{\nu}\gamma$ cannot be calculated as reliably as the rate of $\pi^0 \rightarrow \nu\bar{\nu}\gamma$. An estimate [34] yields $B(\eta \rightarrow \nu\bar{\nu}\gamma) \simeq 2 \times 10^{-15}$ and $B(\eta' \rightarrow \nu\bar{\nu}\gamma) \simeq 2 \times 10^{-14}$.

In addition to $\pi^0, \eta, \eta' \rightarrow \nu\bar{\nu}\gamma$ the decays $\pi^0, \eta, \eta' \rightarrow$ "X"γ may include other decay modes, for example the decays $\pi^0, \eta, \eta' \rightarrow \tilde{\gamma}\tilde{\gamma}\gamma$. The branching ratio for $\pi^0 \rightarrow \tilde{\gamma}\tilde{\gamma}\gamma$ was estimated in Ref. [30] to be $B(\pi^0 \rightarrow \tilde{\gamma}\tilde{\gamma}\gamma) \lesssim 10^{-12}$.

<u>Higgs Bosons, Axions</u>

Some decay modes of the η and η' could aid in the search for the standard Higgs boson, or for nonstandard Higgs particles. Such are the decays $\eta' \rightarrow \eta\mu^+\mu^-, \eta \rightarrow \pi^0\mu^+\mu^-$ and $\eta' \rightarrow \pi^0\mu^+\mu^-$. The present experimental limits for the branching ratios are [35] $1.5 \times 10^{-5}, 5 \times 10^{-6}$ and 6×10^{-5} respectively. Assuming the absence of a CP-violating electromagnetic interaction, one-photon-exchange is forbidden for these decays; two-photon exchange leads to branching ratios much smaller than 10^{-6}. Another decay mechanism could be $P' \rightarrow PH \rightarrow P\mu^+\mu^-$ ($H \equiv$ Higgs boson). The above experimental limits for the branching ratios may already exclude some mass range for the Higgs boson [36], depending on the theoretical uncertainties in the calculations.

Axion-like particles could be searched for, e.g. in the decay $\eta \rightarrow \pi^+\pi^-a$ [37]. Assuming that the axion-like particle mixes with the light pseudoscalar mesons with a strength $\sim 4 \times 10^{-4}$, the branching ratio is expected to be of the order of $\sim 10^{-5}$ [37]. Another possibility at light-meson facilities is to look for the production process $p + d \rightarrow He^3 + a$ [37]. The cross section is expected to be of the order of $(10^{-7} - 10^{-8})\sigma(p + d \rightarrow He^3\pi^0)$ [37].

5. Muon - Number-Violation

In the minimal standard model muon-number (as well as electron number and tau-number) is conserved, and consequently processes such as $\mu \rightarrow e\gamma, K_L \rightarrow \mu e, \pi^0 \rightarrow \mu e$, etc. are forbidden. The underlying reason is that the neutrinos are massless and that the Z^0 and the Higgs boson have only flavor-diagonal couplings to the fermions. If the minimal standard model is extended to include massive neutrinos, muon-number violation is expected, but from the existing limits on the masses of the neutrinos one can conclude that the rates of muon-number violating processes would be too small to be observable. However in some other extensions of the minimal standard model some muon-number violating processes may have rates near the present experimental limits. Here we shall consider the decays $\pi^0 \to \mu e, \eta \to \mu e$ and $\eta' \to \mu e$.

$\underline{\pi^0 \to \mu e}$

This decay has been considered in Refs. [38] and [39]. The experimental limit for the branching ratio is [7]

$$B(\pi^0 \to \mu^+ e^-) + B(\pi^0 \to \mu^- e^+) < 7 \times 10^{-8} \qquad (90\% \text{ c.l.}) \qquad (5.1)$$

The $\pi^0 \to \mu e$ amplitudes have the same general form as the amplitude in Eq. (2.1). The decay rate is given by [40]

$$\Gamma(\pi^0 \to \mu e) \simeq (m_\pi/8\pi)\,(1 - r_\mu^2)^2(|\,a\,|^2 + |\,b\,|^2) \ , \qquad (5.2)$$

where $r_\mu = m_\mu/m_\pi$. $\pi^0 \to \mu e$ is sensitive to a quark-lepton interaction of the form

$$\begin{aligned} L = (G/\sqrt{2})\ \ &[(g_{VA}^{(\mu e)}\ \bar{e}\gamma_\lambda\mu + g_{AA}^{(\mu e)}\ \bar{e}\gamma_\lambda\gamma_5\mu)J_{A\lambda} \\ &\qquad + (g_{SP}^{(\mu e)}\ \bar{e}\mu + g_{PP}^{(\mu e)}\ \bar{e}i\gamma_5\mu)J_P + H.c. \end{aligned} \qquad (5.3)$$

where $J_{A\lambda}$ and J_P are given by Eqs. (2.9) and (2.10).

Let us consider the case when only pseudoscalar-type couplings are present. The $\pi^0 \to \mu e$ branching ratio is then

$$B(\pi^0 \to \mu e) \simeq 1.1 \times 10^{-9}\omega^2 \left(|\,g_{SP}^{(\mu e)}\,|^2 + |\,g_{PP}^{(\mu e)}\,|^2\right) \ , \qquad (5.4)$$

where $\omega = m_\pi/(m_u + m_d)$.

Constraints on the couplings in (5.3) come from experiments searching for $\mu^- \to e^-$ conversion in nuclei. Stringent experimental limits exist for coherent $\mu^- \to e^-$ conversion [41,42]. The couplings in (5.3) do not lead however to coherent conversion. The conversion strength is expected to be spread, with an average electron energy of about 80 MeV, as the average neutrino energy in ordinary muon capture. In Ref. [41] the electron spectrum was measured for electron momenta above 80 MeV/c. A comparison of this spectrum with the one expected from μ-decay in orbit and from radiative μ-capture leads to a limit [39] on the sum of the $\mu^- \to e^-$ conversion rates corresponding to electron momenta between 80 MeV/c and the maximum electron momentum (104.7 MeV/c). Assuming that this sum accounts for a half or more of all $\mu^- \to e^-$ transitions implies for the total $\mu^- \to e^-$ conversion rate the bound

$$\Gamma^{tot}_{\mu e}(^{32}\mathrm{S}) \ / \ \Gamma^{tot}_{\mu\nu}(^{32}\mathrm{S}) \lesssim 8 \times 10^{-9} \tag{5.5}$$

($\Gamma^{tot}_{\mu\nu}$ = total $\mu^- + ^{32}\mathrm{S} \ \to \nu_\mu + ^{32}P^*$ rate $\simeq 1.352 \times 10^6 sec^{-1}$). $\Gamma^{tot}_{\mu e}$ can be estimated using the closure approximation. The result [39] and the limit (5.5) imply

$$| \, g^{(\mu e)}_{SP} \, |^2 + | \, g^{(\mu e)}_{PP} \, |^2 \ \lesssim \ (5 \times 10^{-7})\omega^{-2} \tag{5.6}$$

and consequently [39]

$$B(\pi^0 \to \mu e) \lesssim 6 \times 10^{-16} \ . \tag{5.7}$$

For $\pi^0 \to \mu e$ due to axial-vector type couplings the upper bound on $B(\pi^0 \to \mu e)$ is smaller by a factor of 20.

The bound on $B(\pi^0 \to \mu e)$ would be weaker than (5.7) if the sum of $\mu^- \to e^-$ rates in the region of electron momenta above 80 MeV/c would represent a smaller fraction of $\Gamma^{tot}_{\mu e}(^{32}\mathrm{S})$ than assumed, and/or if the average electron energy was smaller than $\sim$ 80 MeV. However, the values of these quantities are not likely to differ appreciably from those taken. The bound (5.7) could be violated considerably in the unlikely event that the $q\bar{q} \to \mu e$ transition is mediated by a light boson of mass near that of the pion, exchanged in the s-channel. This would produce an enhancement factor in the ratio $B(\pi^0 \to \mu e)/\Gamma^{tot}_{\mu e}$ due to the boson propagator (see Ref. 39).

Hence the conclusion [39] for $\pi^0 \to \mu e$ is that the upper bound on $B(\pi^0 \to \mu e)$ is not likely to be larger than about 10^{-15}.

<u>$\eta \to \mu e$</u>

There is no direct experimental information on this decay. A limit [43]

$$B(\eta \to \mu e) < 8 \times 10^{-2} \qquad (90\% \text{ c.l.}) \tag{5.8}$$

can be established from the total branching ratio for charged η-decays and from branching ratios of particular charged η-decays. The decay $\eta \to \mu e$ is sensitive to a quark-lepton interaction of the form

$$L^{(\mu e)}_{I=0} = L^{(\mu e)}_1 + L^{(\mu e)}_2 \ , \tag{5.9}$$

with

$$L_1^{(\mu e)} = \left(G/\sqrt{2}\right)\left[\left(f_{VA}^{(\mu e)}\bar{e}\gamma_\lambda\mu + f_{AA}^{(\mu e)}\bar{e}\gamma_\lambda\gamma_5\mu\right)K_{A\lambda} + \left(f_{SP}^{(\mu e)}\bar{e}\mu + f_{PP}^{(\mu e)}\bar{e}i\gamma_5\mu\right)K_P\right] + H.c. \ , \tag{5.10}$$

$$L_2^{(\mu e)} = \left(G/\sqrt{2}\right)\left[\left(\tilde{f}_{VA}^{(\mu e)}\bar{e}\gamma_\lambda\mu + \tilde{f}_{AA}^{(\mu e)}\bar{e}\gamma_\lambda\gamma_5\mu\right)\tilde{K}_{A\lambda} + \left(\tilde{f}_{SP}^{(\mu e)}\bar{e}\mu + \tilde{f}_{PP}^{(\mu e)}\bar{e}i\gamma_5\mu\right)\tilde{K}_P\right] + H.c. \ , \tag{5.11}$$

where $K_{A\lambda}, K_P, \tilde{K}_{A\lambda}$ and $\tilde{K}_P$ are given by Eqs. (2.26), (2.27), (2.28) and (2.29).

Let us consider the case when only the $\tilde{f}_{PP}^{(\mu e)}$-and $\tilde{f}_{SP}^{(\mu e)}$-terms contribute (the constraints on the branching ratios of $\eta \to \mu e$ due to the other terms in (5.9) turn out to be more stringent [44]). The $\eta \to \mu e$ branching ratio is then

$$B(\eta \to \mu e) \simeq (1.2 \times 10^{-7})\tilde{\kappa}_P^2(|\ \tilde{f}_{SP}^{(\mu e)}\ |^2 + |\ \tilde{f}_{PP}^{(\mu e)}\ |^2) \ , \tag{5.12}$$

where $\tilde{\kappa}_P$ is defined in (2.37). The limit (5.5) constrains also the couplings in (5.9). For the contribution of (5.9) to the total $\mu^- \to e^-$ conversion rate on a nucleus of charge Z and atomic number A we obtain

$$\Gamma_{\mu e}^{tot} = (G^2 m_\mu^3 \bar{E}^2/4\pi^2)\alpha^3 Z^3 A\ |\ \phi_\mu\ |_{av}^2\ \lambda_{I=0}\left(1 - \frac{1}{4}\delta_{I=0}^{(n)}\right) \tag{5.13}$$

where $\bar{E}$ is the average electron energy, $|\ \phi_\mu\ |_{av}^2$ is an average of $|\ \phi_\mu\ |^2$ (ϕ_μ = muon wave function; $\phi_\mu(0) = 1$), and $\delta_{I=0}^{(n)}$ is a nucleon-nucleon correlation parameter. For the case under discussion

$$\lambda_{I=0} = \tilde{H}_P^2(\bar{E}/2m_N)^2(|\ \tilde{f}_{SP}^{(\mu e)}\ |^2 + |\ \tilde{f}_{PP}^{(\mu e)}\ |^2) \ , \tag{5.14}$$

where $\tilde{H}_P$ is defined by $< N'\ |\ \tilde{K}_P\ |\ N >= \tilde{H}_P\bar{N}i\gamma_5 N$. From (5.5), (5.12) and (5.14) it follows that

$$B(\eta \to \mu e) \lesssim (2.7 \times 10^{-12})(\tilde{\kappa}_P/\tilde{H}_P)^2 \ . \tag{5.15}$$

$\tilde{H}_P$ is related to the nucleon matrix element of $\partial_\lambda \tilde{K}_{A\lambda}$. Ignoring the contribution of the gluon anomaly, we have $\tilde{H}_P \simeq (m_N/m_s)\tilde{F}_A$, where $\tilde{F}_A$ is defined by $< N'\ |\ \tilde{K}_{A\lambda}\ |\ N >\simeq \tilde{F}_A\bar{N}\gamma_\lambda\gamma_5 N$. The value of $\tilde{F}_A$ is uncertain [45]. As a guess we shall assume $\tilde{F}_A \simeq (10^{-1} - 10^{-2})F_A$, where F_A is given by $< N' \mid K_{A\lambda} \mid N > \simeq F_A \bar{N}\gamma_\lambda\gamma_5 N$. Then also $< N' \mid K_A \mid N > \simeq < N' \mid K_A - \tilde{K}_A \mid N >$. The latter matrix element can be evaluated using SU(3) symmetry. We obtain (with $F = 0.477, D = 0.756$ [46]) $F_A \simeq 0.37$ and consequently $\tilde{H}_P \simeq 0.21 - 0.021$. It follows (using the value (2.41) for $\tilde{\kappa}_P$) that

$$B(\eta \to \mu e) \lesssim 7 \times 10^{-10} \quad . \tag{5.16}$$

This upper bound corresponds to $\tilde{F}_A = 10^{-2}F_A$. For $\tilde{F}_A = 10^{-1}F_A$ one would have $B(\eta \to \mu e) \lesssim 7 \times 10^{-12}$. Given a value of $\tilde{F}_A$, values of $B(\eta \to \mu e)$ larger than those allowed by (5.16) cannot be completely ruled out, already because of the possibilities mentioned for $\pi^0 \to \mu e$ in connection with the bound (5.7). In addition, the value of $\mid \tilde{\kappa}_P \mid$ could be larger than that in Eq. 2.41 (although probably not by more than a factor of two [16]). Finally, $B(\eta \to \mu e)$ could be larger than the upper limit in (5.16) if μe couples to a heavy quark (c, b, t, ...) current for which the ratio of $\eta \to$ vacuum and $N \to N$ matrix elements is larger than $\mid \tilde{\kappa}_P/\tilde{H}_P \mid$.

$\underline{\eta' \to \mu e}$

The same general neutral current interaction which contributes to $\eta \to \mu e$ would contribute also to $\eta' \to \mu e$. In the same framework as the one we used above for $\eta \to \mu e$, we find for $\tilde{F}_A = 10^{-2}F_A$ the upper bound on $B(\eta' \to \mu e)$ to be smaller than the upper bound (5.16) for $\eta \to \mu e$ by an order of magnitude [44].

6. Charged Current Interactions

Second-Class Currents

The term "second-class currents" is used for hadronic currents which have opposite G-parity than the usual quark currents $\bar{q}\Gamma_k q'$ [47]. Second-class currents constructed from quark fields must contain derivatives of the quark fields, and are not allowed therefore in a renormalizable gauge theory with elementary quarks.

It was pointed out by Singer [48] that the decays $\eta \to \pi e \nu_e$ and $\eta \to \pi\mu\nu_\mu$ have the special feature that they are forbidden for a first-class vector current, but allowed for a second-class vector, or a second-class tensor current.

Let us consider $\eta \to \pi e \nu_e$ assuming an interaction

$$H_e^{(2)} = (G_e^{(2)}/\sqrt{2})V_\lambda^{(2)}\bar{e}\gamma^\lambda(1-\gamma_5)\nu_e + H.c. \tag{6.1}$$

where $V_\lambda^{(2)}$ is a primitive second-class vector current chosen so that (6.1) is CP-invariant. Using the results of Ref. [48], the $\eta \to \pi e \nu_e$ branching ratio is given by

$$B(\eta \to \pi e \nu_e) \simeq (1.7 \times 10^{-10})(G_e^{(2)} f_+/G)^2 \quad , \tag{6.2}$$

where f_+ is a form factor in $< \pi \mid V^{(2)} \mid \eta >$. The best limit on $G_e^{(2)}$ comes from a bound on the induced scalar interaction in β-decay [49]. This implies $\mid G_e^{(2)} f_S \mid \lesssim 2.4 G f_M$, where f_S and f_M are the induced scalar and the weak magnetism form factors. It follows that

$$B(\eta \to \pi e \nu_e) \sim 10^{-9}(f_M f_+/f_S)^2 \quad . \tag{6.3}$$

Assuming $\mid f_M/f_S \mid \lesssim 1, \mid f_+ \mid \lesssim 1$ one has

$$B(\eta \to \pi e \nu_e) \lesssim 10^{-9} \quad . \tag{6.4}$$

Second-class interactions involving a muonic current would be governed in general by a different coupling constant, and might even involve a different second-class current (similarly, second-class current contributions to the decays $\tau \to \eta \pi \nu_\tau$ would be in general unrelated to those in $\eta \to \pi e \nu_e$ or $\eta \to \pi \mu \nu_\mu$). We shall assume here for $\eta \to \pi \mu \nu_\mu$ a coupling of the form (6.1) with $G_e^{(2)}$ replaced by $G_\mu^{(2)}$. Using the results of Ref. [48], and neglecting the contribution of the f_- form factor in $< \pi \mid V^{(2)} \mid \eta >$ we find

$$B(\eta \to \pi \mu \nu_\mu) \simeq 1.2 \times 10^{-10}(G_\mu^{(2)} f_+/G)^2 \quad . \tag{6.5}$$

Second-class interactions involving muonic currents are constrained by μ-capture rates. The best limits [50] imply $\mid G_\mu^{(2)} f_S/G \mid \lesssim 3$. Thus

$$B(\eta \to \pi \mu \nu_\mu) \lesssim 10^{-10} \quad , \tag{6.6}$$

assuming $\mid f_S \mid \simeq \mid f_M \mid$ and $\mid f_+ \mid \simeq 1$.

The decays $\eta \to \pi \ell \nu$ can proceed also via a second-class tensor coupling [48]. The upper bounds on $\eta \to \pi \ell \nu$ due to such an interaction would not be probably very different from (6.3) and (6.6).

For CP-violating second-class interactions the limits from β-decay and μ-capture rates are weaker, because there is no interference in the rates between such second-class current couplings and the first-class ones. But the upper limits on the branching ratios of $\eta \to \pi e \nu_e$ and $\eta \to \pi \mu \nu_\mu$ are smaller than those in (6.4) and (6.6), due to constraints imposed by $(D_n)_{expt}$ (see Eq. 7.4).

Scalar Currents

The decays $\eta \rightarrow \pi e \nu_e$ and $\eta \rightarrow \pi \mu \nu_\mu$ can proceed also via first-class scalar currents [48]. Scalar couplings are far less exotic than second-class currents. They can be generated for example by the exchange of charged Higgs bosons, which are present e.g. in the standard model extended to contain more than one Higgs doublet. Let us consider the coupling

$$H_S = (G/\sqrt{2}) h_{SS}^{(e)} \bar{e} \nu \bar{u} d + H.c. \tag{6.7}$$

$B(\eta \rightarrow \pi e \nu_e)$ due to (6.7) is given by [51]

$$B(\eta \rightarrow \pi e \nu_e) \simeq (4.7 \times 10^{-11}) \mid h_{SS}^{(e)} \mid^2 g_S^2 \tag{6.8}$$

where $g_S(t)$ is given by $< \pi^+ \mid \bar{u} d \mid \eta >= \sqrt{2} g_S(t) m_\eta$. g_S is related to the matrix element of the divergence of the vector current, and can be experessed in terms of the quark masses as [52] $g_S(0) = m_\pi^2 / m_\eta (m_u + m_d) \sqrt{3} \simeq 1.75$. The best limit on $h_{SS}^{(e)}$ comes from the experimental value of the ratio of the rates for ${}^{14}0 \rightarrow {}^{14} Ne^+$. and $\pi^+ \rightarrow \pi^0 e^+ \nu$ decays, implying $\mid h_{SS}^{(e)} \mid \lesssim 0.4$ [53]. It follows that

$$B(\eta \rightarrow \pi e \nu_e) \simeq 3 \times 10^{-11} \quad . \tag{6.9}$$

For a coupling $h_{SS}^{(\mu)}$ defined as $h_{SS}^{(e)}$ in (6.7) but with e replaced by μ, we deduce from the results of Ref. [50], roughly $\mid h_{SS}^{(\mu)} \mid \lesssim 0.6$. We find then

$$B(\eta \rightarrow \pi \mu \nu_\mu) \lesssim 4 \times 10^{-11} \quad . \tag{6.10}$$

We note yet that the decays $\eta \rightarrow \pi e \nu_e$ and $\eta \rightarrow \pi \mu \nu_\mu$ occur also in the minimal standard model due to isospin invariance violating effects. The corresponding rates are expected to be smaller than the upper limits (6.4) and (6.6) by about four orders of magnitude.

7. CP-Violation

The interaction responsible for the observed CP-violation in the neutral kaon system has not been identified as yet. It could be the usual weak interaction, or it resides outside of the minimal standard model. In extensions of the minimal standard model there are many possible sources of CP-violation, which can give rise to a variety of CP-violating effects. Here we shall consider the effects of CP-violating interactions in the decays of π^0, η and η'.

$\pi^0, \eta, \eta' \to 3\gamma$

The decays $P \to 3\gamma$, where P is a $C = +1$ pseudoscalar meson, are forbidden in the limit of C-invariance. They can be induced by a P-violating CP-conserving interaction, or by an interaction which violates C and CP. Since the first mechanism is provided by the usual weak interaction, the main interest in $P \to 3\gamma$ is in connection with the CP-violating mode.

$\pi^0 \to 3\gamma$

The present experimental limit for the branching ratio $B(\pi^0 \to 3\gamma) \equiv \Gamma(\pi^0 \to 3\gamma)/\Gamma(\pi^0 \to all)$ is [7]

$$B(\pi^0 \to 3\gamma) < 3.8 \times 10^{-7} \qquad (90\% \text{ c.l.}) \tag{7.1}$$

Before considering $\pi^0 \to 3\gamma$ due to some new interaction, it is important to have an estimate of $B(\pi^0 \to 3\gamma)$ due to the usual CP-conserving weak interaction. We shall denote the latter contribution $B(\pi^0 \to 3\gamma)_w$. An estimate of $B(\pi^0 \to 3\gamma)_w$ was given by Dicus [54] based on a quark-loop model. He finds

$$\begin{aligned} B(\pi^0 \to 3\gamma)_w &\simeq \Gamma(\pi^0 \to 3\gamma)_w/\Gamma(\pi^0 \to 2\gamma) \\ &\simeq (1.2 \times 10^{-5})\alpha(2\pi)^{-5} G^2 m_\pi^4 (m_\pi/m)^8 \end{aligned} \tag{7.2}$$

where G is the Fermi constant, and m is an effective quark mass. The high power of m_π/m in Eq.(7.2) is due to the fact that the simplest effective Hamiltonian for $\pi^0 \to 3\gamma$ contains seven derivatives [55]. This renders $\pi^0 \to 3\gamma$ extremely sensitive to the value of m. In Ref. [54] $m = m_N$ ($m_N \equiv$ nucleon mass) was chosen, yielding $B(\pi^0 \to 3\gamma)_w \simeq 10^{-31\pm6}$, where $10^{\pm6}$ is a guess of the error involved in the estimate. For $m = (1/3)m_N$ one would obtain $B(\pi^0 \to 3\gamma)_w \simeq 7 \times (10^{-28\pm6})$. There is some indication that m in quark-loop calculations may have to be taken as light as 120 - 150 MeV [56]. With $m = (1/7)m_N$ Eq.(7.2) would yield $B(\pi^0 \to 3\gamma)_w \simeq 6 \times (10^{-25\pm6})$. As can be seen from here, the size of $B(\pi^0 \to 3\gamma)_w$ is quite uncertain, but it appears that it is not likely to be larger than $\sim 10^{-18}$.

The CP-violating contribution to $B(\pi^0 \to 3\gamma)$ in the minimal standard model is negligible relative to $B(\pi^0 \to 3\gamma)_w$. There are two sources of CP-violation in the minimal standard model: the θ-term in the effective QCD Lagrangian and the Kobayashi-Maskawa (KM) phase δ in the quark mixing matrix.

The θ-term does not contribute, as it is C-conserving, and the KM CP-violation contributes to $\pi^0 \to 3\gamma$ (and to any flavor-conserving nonleptonic process) only in second-order in the weak interaction [57].

Considering CP-violating contributions from possible new interactions, it can be shown [58] that in renormalizable gauge models with elementary quarks the flavor conserving nonleptonic interactions of the quarks do not contain in first order a P-conserving CP-violating component. The CP-violating contributions to $B(\pi^0 \to 3\gamma)$ in such models are therefore also negligible relative to $B(\pi^0 \to 3\gamma)_w$. In models with composite quarks P-conserving CP-violating effective interactions may conceivably be present at the quark level, but their strength is most likely weaker than that of the weak interaction.

The conclusion of the above discussion is that in the current theoretical framework $\pi^0 \to 3\gamma$ is not expected to occur at an observable level. But let us free ourselves now of all theoretical prejudices and allow for the existence of a flavor-conserving C- and CP-violating interaction ($\overline{H}$), or for the existence of a C- and CP-violating photon-hadron interaction ($\overline{H}_{em}$) with strength ($\bar{f}$ and $\bar{f}_{em}$ respectively) constrained only by experiment. Such interactions have been invoked in the sixties to account for the observed CP-violation [59]. The most stringent bound on $\bar{f}$ and $\bar{f}_{em}$ comes from the experimental limit [60]

$$D_n \; < \; 2.6 \times 10^{-25} ecm \qquad (95\% \text{ c.l.}) \qquad (7.4)$$

for the electric dipole moment of the neutron D_n. A rough estimate of D_n due to $\overline{H}$ is [61]

$$D_n \; \simeq \; (e/m_N)\,(Gm_N^2/4\pi)\bar{f} \quad , \qquad (7.5)$$

where m_N is the nucleon mass. Thus $|\,\bar{f}\,| \;\lesssim\; 1.3 \times 10^{-5}$. The same bound is obtained for $\bar{f}_{em}$. Allowing an order of magnitude error in this estimate, we take the limits for $\bar{f}$ and $\bar{f}_{em}$ to be $|\,\bar{f}\,| \;\lesssim\; 10^{-4}$ and $|\,\bar{f}_{em}\,| \;\lesssim\; 10^{-4}$. $\pi^0 \to 3\gamma$ due to $\overline{H}$ and $\overline{H}_{em}$ was investigated by Tarasov [62]. The ratio $\Gamma(\pi^0 \to 3\gamma)/\Gamma(\pi^0 \to 2\gamma)$ is proportional to $(m_\pi/\overline{m})^{12}$ rather than to $(m_\pi/\overline{m})^8$, since now $\Gamma(\pi^0 \to 3\gamma)$ does not involve the Fermi constant; $\overline{m}$ is an effective mass, taken in Ref. [62] to be of the order of the hadron masses into which π^0 dissociates. Tarasov argues that for values of his parameters, which correspond here to $|\,\bar{f}\,| \;\lesssim\; 1$ or $|\,\bar{f}_{em}\,| \;\lesssim\; 1$, the $\pi^0 \to 3\gamma$ branching ratio can be $B(\pi^0 \to 3\gamma) \;\lesssim\; 10^{-9}$. Then for $|\,\bar{f}\,| \;\lesssim\; 10^{-4}$ (or $|\,\bar{f}_{em}\,| \;\lesssim\; 10^{-4}$) one would have

$$B(\pi^0 \to 3\gamma)_{\overline{H},\overline{H}_{em}} \;\lesssim\; 10^{-17} \qquad (7.6)$$

- a branching ratio again much too small to be observable.

$\underline{\eta \to 3\gamma}$

$B(\eta \to 3\gamma)$ is expected to be enhanced relative to $B(\pi^0 \to 3\gamma)$ because of the high power of (m_P/m_{eff}) ($m_P \equiv$ mass of the decaying meson) involved in the $P \to 3\gamma$ rate [63]. An estiamte of $B(\eta \to 3\gamma)_w$ can be obtained from the estimate of $B(\pi^0 \to 3\gamma)_w$ of Dicus by replacing m_π by m_η, and m by another effective mass $\tilde{m}$. If we take $\tilde{m} = (1/5)m_N$ we obtain [64]

$$B(\eta \to 3\gamma)_w \simeq 3 \times 10^{-19\pm 6} \quad . \tag{7.7}$$

For $\eta \to 3\gamma$ due to $\overline{H}$ or $\overline{H}_{em}$ the guess is

$$B(\eta \to 3\gamma)_{\overline{H},\overline{H}_{em}} \simeq 0.39 B(\pi^0 \to 3\gamma)_{\overline{H},\overline{H}_{em}} (m_\eta/m_\pi)^{12} \lesssim 10^{-10} \tag{7.8}$$

The present experimental limit for $B(\eta \to 3\gamma)$ is [7]

$$B(\eta \to 3\gamma)_{expt} < 5 \times 10^{-4} \qquad (90\% \text{ c.l.}). \tag{7.9}$$

$\underline{\eta' \to 3\gamma}$

Using the estimate of Dicus, and taking $\frac{1}{3}m_N$ for the effective quark mass – a slightly larger value than for $\eta \to 3\gamma$ in (7.8) – we obtain (using $\Gamma(\eta' \to all) \simeq 240$ KeV [7]) $B(\eta' \to 3\gamma)_w \lesssim 2 \times 10^{-19\pm 6}$.

For $\eta' \to 3\gamma$ generated by $\overline{H}$ or $\overline{H}_{em}$ we have from (7.8) $B(\eta' \to 3\gamma) \lesssim 3 \times 10^{-9}$ if $\Gamma(\eta' \to 3\gamma)/\Gamma(\eta' \to 2\gamma) \simeq (m_{\eta'}/m_\eta)^{12}\Gamma(\eta \to 3\gamma)/\Gamma(\eta \to 2\gamma)$. However this is probably an overestimate, since the effective intermediate hadron mass for $\eta' \to 3\gamma$ is presumably larger than that for $\eta \to 3\gamma$.

The present experimental limit for the $\eta' \to 3\gamma$ branching ratio is $B(\eta' \to 3\gamma) < 10^{-4}$ (90% c.l.) [65].

$\underline{\eta \to \pi^+\pi^-}$

This decay violates simulataneously P- and CP-invariance [66]. It can be induced by a P-, CP-violating interaction ($H_{P,CP}$), or by a P-conserving CP-violating interaction (H_{CP}) through interference with the usual weak interaction.

The present experimental limit for $B(\eta \to \pi^+\pi^-) \equiv \Gamma(\eta \to \pi^+\pi^-)\Gamma(\eta \to all)$ is [7]

$$B(\eta \to \pi^+\pi^-) < 1.5 \times 10^{-3} \qquad (90\% \text{c.l.}). \tag{7.10}$$

A rough estimate of the $\eta \to \pi^+\pi^-$ rate can be obtained by comparing $\eta \to 2\pi$ to $K_S \to 2\pi$. We expect

$$R_{\eta,K} \equiv \Gamma(\eta \to 2\pi)/\Gamma(K_S \to 2\pi) \simeq (G^2 m_\eta^4/G^2 m_K^4 s_1^2) \mid f \mid^2 \quad , \tag{7.11}$$

where $s_1 \equiv sin\theta_1$, (θ_i are the quark mixing angles), and f is the strength of $H_{P,CP}$ relative to the weak interaction, or the strength of H_{CP} relative to the strong interaction. If H_{CP} is due to $\overline{H}_{em}$ then $f \simeq e^2 \bar{f}_{em}$; otherwise $f \equiv \bar{f}$ (see the discussion of $\pi^0 \to 3\gamma$ earlier). From $(D_n)_{expt}$ (Eq. 7.4) we have $| f | \lesssim 10^{-4}$($| f | \lesssim 10^{-4}e^2$ in the case of $\overline{H}_{em}$), so that $R_{\eta,K} \lesssim 3 \times 10^{-7}$ and therefore

$$B(\eta \to \pi^+\pi^-) \lesssim 1.5 \times 10^{-15} \quad . \tag{7.12}$$

Let us consider $\eta \to \pi^+\pi^-$ in the minimal standard model. The contribution due to the KM phase δ is negligible, since it appears only in second order (one expects $(R_{\eta,K})_\delta \simeq (Gm_\eta^2/s_1\sqrt{2})^2(s_1^2 s_2 s_3 s_\delta)^2$ implying $B(\eta \to \pi^+\pi^-) \lesssim 10^{-26}$ [67]). Calculations [68,69] of $\eta \to \pi^+\pi^-$ due to the θ-term yield $B(\eta \to \pi^+\pi^-)_\theta \simeq 90\theta^2$. The contribution of the θ-term to D_n can be estimated in the same framework. The result is [68] $D_n \simeq (3.6 \times 10^{-16})\theta ecm$, so that $\theta \lesssim 7 \times 10^{-10}$. It follows that

$$B(\eta \to \pi^+\pi^-)_\theta \lesssim 5 \times 10^{-17} \quad . \tag{7.13}$$

A different estimate (A. Soni, Ref. 2) using the QCD rules of Ref. [70] yields a five times larger upper bound.

Examples of models where CP-violation in the flavor-conserving nonleptonic sector occurs already in first order are left-right symmetric models [71] and Weinberg's Higgs model [72]. $B(\eta \to 2\pi)$ in these models have not been, to my knowledge, estimated. The strength of the relevant CP-violating interactions in these models is constrained to be less than $\sim 10^{-4}$ relative to G (in left-right symmetric models by $(D_n)_{expt}$ and $(\epsilon'/\epsilon)_{expt}$ [73], in the Higgs model by $(D_n)_{expt}$ [74]). $B(\eta \to 2\pi)$ should obey therefore the bound (7.12). In view of the surrounding uncertainties the possibility of somewhat larger values for $B(\eta \to \pi^+\pi^-)$ cannot be ruled out.

<u>$\eta' \to 2\pi$</u>

The $\eta' \to 2\pi$ rate has not yet been estimated, to my knowledge, in any model. A rough estimate, as for $\eta \to 2\pi$ in (7.11), comparing $\eta' \to 2\pi$ to $K_S \to 2\pi$ yields $B(\eta' \to 2\pi) \lesssim 6 \times 10^{-17}$

An experimental limit of $B(\eta' \to \pi^0\pi^0)_{expt} < 10^{-3}$ (90% c.l.) has been established [65] for the branching ratio of $\eta' \to \pi^0\pi^0$. For $B(\eta' \to \pi^+\pi^-)$ no experimental limit is available as yet.

$\eta \to \pi^0 e^+ e^-$, $\eta \to \pi^0 \pi^+ \pi^-$, $\eta, \eta' \to \pi^+ \pi^- \gamma$

The decay $\eta \to \pi^0 e^+ e^-$ and the C-violating asymmetries in $\eta \to \pi^0 \pi^- \pi^+$ and $\eta, \eta' \to \pi^+ \pi^- \gamma$ have been investigated in the past to set limits of $\bar{f}$ and $\bar{f}_{em}$ [75].

Let us consider $\eta \to \pi^0 e^+ e^-$ with the interaction $\overline{H}$. Using the results of Ref. 76 and the present experimental limit $B(\eta \to \pi^0 e^+ e^-)_{expt} < 5 \times 10^{-5}$ [7] we find (identifying $\bar{f}$ with the ratio of the $\rho\eta\pi$ and $\rho\pi\pi$ coupling constants) $|\bar{f}| \lesssim 3 \times 10^{-2}$. About the same limit follows for $\bar{f}_{em}$ for an isovector $\overline{H}_{em}$. For $|\bar{f}| \lesssim 10^{-4}$ (the rough limit from D_n) we would have $B(\eta \to \pi^0 e^+ e^-) \lesssim 4 \times 10^{-10}$. The CP-invariant contribution from two-photon exchange is expected at the level of $10^{-8} - 10^{-9}$ [77]. An experimental limit on $B(\eta \to \pi^0 e^+ e^-)$ at the 10^{-8} level would set an upper bound of $\sim 5 \times 10^{-4}$ on $|\bar{f}|$ and $|\bar{f}_{em}|$.

The asymmetries in $\eta \to \pi\pi\pi$ and $\eta \to \pi\pi\gamma$ (see Refs. 76 and 78) are (unlike $B(\eta \to \pi^0 e^+ e^-)$) linear in f (or $\bar{f}_{em}$). The asymmetries in $\eta \to \pi^0 \pi^+ \pi^-$ have been measured with a sensitivity of $\sim 10^{-3}$ [7], implying $|f| \lesssim 10^{-2}$ and a comparable upper limit for $|\bar{f}_{em}|$. The upper limit on the C-violating asymmetry in $\eta \to \pi^+ \pi^- \gamma$ is of the order of 10^{-2} [7], which does not provide a significant limit. The asymmetry in $\eta' \to \pi\pi\gamma$ is about an order of magnitude more sensitive than in $\eta \to \pi\pi\gamma$ [78] but it has been searched for so far only with a sensitivity of $\sim 10^{-1}$ [75].

Lepton Polarization in $P \to \ell^+ \ell^-$

An interesting observable in $P \to \ell^+ \ell^-$ decays (P = pseudoscalar meson) is the degree of longitudinal polarization P of the ℓ^+ (or ℓ^-). The amplitude is given by Eq. (2.1). If the meson is an eigenstate of CP, the parity-conserving amplitude is CP-conserving and the parity-violating amplitude is CP-violating [79]. As a consequence, the leptons can have a longitudinal polarization only in the presence of a CP-violating quark-lepton neutral current interaction [80]. In the minimal standard model P therefore vanishes. The polarization (defined as $\mathrm{P} = (N_R - N_L)/(N_R + N_L)$, where $N_R (N_L)$ is the number of μ^-'s (or μ^+'s) emerging with positive (negative) helicity) is given by

$$\mathrm{P} = \frac{m_p r^2 Im(ba^*)}{4\pi\Gamma} \tag{7.14}$$

where Γ is given by Eq. (2.2). Since $b = b^{(n)}$ is real, we can write (7.14) as

$$\mathrm{P} = -\frac{m_p r^2}{4\pi\Gamma} b^{(n)} Im a^{(c)} \tag{7.15}$$

The experimental $P \to \ell^+\ell^-$ rate sets bounds on $x_P^{(\ell)}$ (defined as $x_\pi^{(e)}$ in Eq. 2.6). Since $r \mid b^{(n)} \mid \leq x_P^{(\ell)} \mid Ima^{(e)} \mid$, we have

$$\mid \mathrm{P} \mid \leq \frac{2x_P^{(\ell)}}{1+(x_P^{(\ell)})^2} \tag{7.16}$$

The largest upper bound corresponds to $x_P^{(\ell)} = 1$. Note that $\mathrm{P} = 1$ for $Rea^{(e)} + a^{(n)} = 0, x_P^{(\ell)} = 1$.

The actual size of P depends on the strength of the SP-couplings in the effective neutral current interaction.

Let us consider P in $\pi^0 \to e^+e^-$ [5]. The amplitude $b^{(n)}$ is given by Eq. (2.12). The interactions that give rise to the $g_{SP}^{(e)}$-term contribute also to the electric dipole moment of the electron and to the electric dipole moment of the neutron (in orders $Gg_{SP}^{(e)}$ and $Gg_{SP}^{(e)}\alpha$ for leptoquark exchange and Higgs exchange, respectively). We find that $(D_n)_{expt}$ (Eq. 7.4) indicates, roughly, $\mid g_{SP}^{(e)} \mid \lesssim 10^{-1} - 10^{-2}$. It follows that

$$\mathrm{P}_{\pi^0 \to e^+e^-} \lesssim 0.8 \quad . \tag{7.17}$$

The upper limit in (7.17) corresponds to $\mid g_{SP}^{(e)} \mid \simeq 10^{-1}$ and $Rea^{(e)} + a^{(n)} \simeq 0$ (and therefore to $B(\pi^0 \to e^+e^-) \simeq 5.7 \times 10^{-8}$).

The b-amplitude for $\eta \to e^+e^-$ is given by Eqs. (2.31) and (2.33). The rough limits for $f_{SP}^{(e)}$ and $\bar{f}_{SP}^{(e)}$ are $\mid f_{SP}^{(e)} \mid \lesssim 10^{-2} - 10^{-1}$ and $\mid \bar{f}_{SP}^{(e)} \mid \lesssim 0(1)$. It follows that

$$\mid \mathrm{P}_{\eta \to e^+e^-} \mid \lesssim 1 \tag{7.18}$$

The upper limit in (7.18) corresponds to $Rea^{(e)} + a^{(n)} = 0, rb^{(n)} = Ima^{(e)}$ (i.e. to $x_\eta^{(e)} = 1$). The branching ratio would be then $B(\eta \to e^+e^-) \simeq 3.5 \times 10^{-9}$.

Even at the level of $\mathrm{P} \simeq 1$ a search for lepton polarization in $\pi^0 \to e^+e^-$ or in $\eta \to e^+e^-$ (even for the anticipated higher η-fluxes) would be forbiddingly difficult because of the small branching ratios and the small analyzing power for $e^\pm$-polarization measurements.

A search for muon polarization in $\eta \to \mu^+\mu^-$ might be less demanding, because of the possibility to measure muon-polarization through muon-decay [81], and also because of the relatively large branching ratio. P in $\eta \to \mu^+\mu^-$ is sensitive to the $f_{SP}^{(\mu)}$ and $\bar{f}_{SP}^{(\mu)}$-type couplings (cf. Eq. 3.4). $(D_n)_{expt}$ indicates $\mid f_{SP}^{(\mu)} \mid \lesssim 10^{-2} - 10^{-1}$ and $\mid \bar{f}_{SP}^{(\mu)} \mid \lesssim 0(1)$, implying

$$\mid \mathrm{P}_{\eta \to \mu\mu} \mid \lesssim 0.2 \tag{7.19}$$

The limit $\mid \bar{f}_{SP}^{(\mu)} \mid \lesssim 0(1)$ involves the assumption that the contribution of a coupling involving the s-quark to the neutron dipole moment is suppressed (for equal coupling strength) by a factor of 10 relative to the contribution of an interaction involving the u – and d-quarks. If the suppression is stronger, $P_{\eta\to\mu\mu}$ could be larger than (7.19).

The vanishing of P for $\eta \to \mu^+\mu^-$ (and $\pi^0 \to e^+e^-, \eta \to e^+e^-$) in the minimal standard model is in contrast with P in $K_L \to \mu^+\mu^-$. $P_{K_L\to\mu\mu}$ is nonzero in the minimal standard model due to the fact that K_L is not a CP-eigenstate. The ϵ-term in $K_L = K_2 + \epsilon K_1$ gives rise to $| P_{K_L\to\mu\mu} | \simeq 10^{-3}$ [82], which sets the level below which one could not search for new physics. $P_{K_L\to\mu\mu}$ provides, of course, information on other types of CP-violating interactions than $P_{\pi^0\to ee}$ or $P_{\eta\to\ell\ell}$.

8. Conclusions

The main purpose of the analysis contained in this talk was to prepare ourselves to give an answer to the question of which experiments in the field of the rare and forbidden decays of the π^0, η and η', excluding decays with forbiddingly small branching ratios, could give us new information on possible physics beyond the minimal standard model. We conclude with the following, most likely incomplete list:

- *Searches for $\eta \to e^+e^-$ below the present experimental upper limit of 3×10^{-4} for the branching ratio.*

The electromagnetic contribution to $B(\eta \to e^+e^-)$ is expected at the level of $\sim 2 \times 10^{-9} - 10^{-8}$. An experimental upper limit on $B(\eta \to e^+e^-)$ of the order of 10^{-8} would yield upper bounds of $\sim 4 \times 10^{-2}$ on I=0 pseudoscalar $e - u, d$ couplings and upper bounds of the order of one on I=0 pseudoscalar $e - s$ couplings.

- *Improving the limits on $B(\pi^0 \to X)$, $B(\eta \to X)$, $B(\pi^0 \to \nu_i\bar{\nu}_i)$, and setting limits on $B(\eta' \to X)$ and $B(\eta, \eta' \to \nu_i\bar{\nu}_i)$. Also, setting limits on $B(\eta, \eta' \to MX)(M \equiv$ meson or mesons).*

This would constrain neutrino-quark couplings for both the known neutrinos and also for possible new neutrinos. The $P \to X$ limits would in addition set limits on the branching ratios into other possible weakly interacting light particles.

- *Improving the limits on $B(\eta \to \pi^0\mu^+\mu^-), B(\eta' \to \pi^0\mu^+\mu^-)$ and $B(\eta' \to \eta\mu^+\mu^-)$.*

This would extend the searches for light Higgs bosons.

- *Searches for $\eta \to \mu e$ with a sensitivity to branching ratios of $\sim 10^{-9} - 10^{-10}$ and smaller.*

These would provide new information on possible muon-number violating interactions.

- *Searches for $\eta \to \pi e \nu_e$ and $\eta \to \pi \mu \nu_\mu$ at the branching ratio level of* 10^{-9} – 10^{-10} *and below.*

These would set new limits on second-class vector current couplings. Searches with a sensitivity $\sim 10^{-11}$ and better would provide also new limits on scalar charged current quark-lepton couplings. In renormalizable gauge theories with elementary quarks second class current must be absent. Charged scalar current couplings can arise for example in the standard model with an extended Higgs sector.

- *Searches for $\eta \to 3\gamma$ and $\eta' \to 3\gamma$ at the branching ratio level of* 10^{-10} *and below.*

In current theories a CP-violating contribution to $B(\eta \to 3\gamma)$ is expected to be smaller than the contribution from the CP-conserving weak interactions. A guess for the latter is $B(\eta \to 3\gamma) \simeq 3 \times 10^{-19\pm6}$. However, phenomenologically $B(\eta \to 3\gamma) \simeq 10^{-10}$ cannot be ruled out. It should be noted that the interpretation of a null result in terms of a limit on a CP-violating coupling constant would be difficult because of the large uncertainties in estimates of $B(\eta \to 3\gamma)$.

- *Improving the limit on $B(\eta \to \pi^0 e^+ e^-)$ and on the C-violating asymmetries in $\eta \to \pi^0 \pi^+ \pi^-$ and $\eta' \to \pi\pi\gamma$.*

In current theories the CP-violating contribution to $\eta \to \pi^0 e^+ e^-$ as well as the C-violating asymmetries are expected to be negligibly small. Phenomenologically, the experimental limit on the electric dipole moment of the neutron indicates that the CP-violating contribution to $B(\eta \to \pi^0 e^+ e^-)$ should be below the contribution of 2γ-exchange, and that the C-violating asymmetry in $\eta \to \pi^0 \pi^+ \pi^-$ should be less than 10^{-5}. Nevertheless, limits on $\bar{f}$ and $\bar{f}_{em}$ (for the definitions see the discussion of $\pi^0 \to 3\gamma$) independent of the dipole moment would be useful.

- *Searches for muon polarization in $\eta \to \mu^+ \mu^-$ at the $\sim 10^{-1}$ level and below.*

Such studies would provide information on CP-violating muon-quark neutral current interactions.

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