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CHIRAL SYMMETRY AND CONFINEMENT

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Two principle features underlie the appearance of (approximate) chiral symmetry in hadronic systems. The first is that the conventional Dirac bispinor description of massless quarks hides the fact that this object is a direct sum of inequivalent representations of the Lorentz group. In a standard notation:

$$R[\psi] = (\tfrac{1}{2},0) + (0,\tfrac{1}{2}) \tag{1}$$

In general, there is an unrestricted phase between the two component representations which must be assigned physical meaning, if possible. For Majorana fermions, it is self-conjugacy (an operation outside the connected part of the Lorentz group) which fixes this meaning leaving only two independent degrees of freedom. For massive, charged fermions, the constraint arises (actually similarly) from the equality of the mass of a particle state of fixed spin (up, say) and the same spin antiparticle state.

In the charged, but massless, case, however, this last constraint vanishes, due (in one way of describing it,) to the absence of a rest frame in which to implement it. The phase loses physical meaning, and this fact is reflected in the chiral phase freedom (invariance) of the two (now unrelated within the proper orthochronous Lorentz group) components. Spontaneous chiral symmetry breaking (χSB) destroys this invariance by coupling the two representations by a scalar field vacuum expectation value (vev).

The second essential feature is that the dynamics must be chiral invariant as well. QCD satisfies this requirement by coupling equally to left and right chiral projections of the (nominally vector) color current (V±A) and so, to the separated components of the Dirac bispinor. (Conversely, the weak interactions explicitly violate chiral symmetry even without the formation of quark masses via the vev of the Higgs' scalar.) Chiral symmetry breaking, however, is now guaranteed: The attraction in the color singlet, Lorentz scalar channel between massless quark and antiquark must reduce the invariant mass-squared of the lowest state below the threshold value of zero. The resulting tachyonic values describe an unstable vacuum at zero vev, a conundrum cured by developing a quark vacuum condensate. The resulting effective Lagrangian of quark composite meson states is written as

$$\mathcal{L}_q = + m^2\left\{(\sigma)^2 + (\eta)^2 + \sum_j (a_j)^2 + \sum_j (\pi_j)^2\right\}$$

$$\lambda_1 \left\{(\sigma)^2 + (\eta)^2 + \sum_j (a_j)^2 + \sum_j (\pi_j)^2\right\}^2$$

$$- \lambda_2 \sum_j \left\{(\sigma\alpha_j + \eta\pi_j) - (f_{jk\ell}\alpha_k\pi_\ell)\right\}^2 + \text{kinetic} + \ldots \tag{2}$$

where terms with higher powers of momenta and fields have not been written out explicitly, and f_{ijk} are the structure constants of the flavor group. When the σ field is shifted by the vev,

$$\langle\sigma\rangle = 2\mu^2/\lambda_1 \tag{3a}$$

$$\tilde{\sigma} = \sigma - \langle\sigma\rangle \tag{3b}$$

L_q can be rewritten as

$$L_q = \ldots + 0(\sum_j \pi_j{}^2 + \eta^2) + \ldots + \text{kinetic} + \ldots. \tag{3c}$$

Where we have explicitly noted the masslessness of the pseudoscalar (composite) bosons which has developed in accordance with the Goldstone theorem. In addition instantons contribute a term of the form

$$L_I = + \lambda_3 \,[\sigma^2 + \sum_j \pi_j{}^2 - \eta^2 - \sum_j \alpha_j{}^2] \tag{4}$$

which breaks the $U(1)_{axial}$ symmetry. The effects of the term include a shift in $\langle\sigma\rangle$, and a non zero mass for the η, but the π_j remain massless.

We wish to point out here that a strong parallel exists in the sector of physical gluonic states, despite a markedly different initial appearance. (We ignore confinement, for the moment, just as was done above for quarks.) Although gluons appear in the (1/2,1/2) representation, the physical color electric (E_a) and magnetic (B_a) fields in the covariant stress tensor ($G_a^{\mu\nu}$) and its dual ($\tilde{G}_a^{\mu\nu}$) actually fill out a representation

$$R[G] = (1,0) \oplus (0,1) \tag{5}$$

with the same structure as $R[\psi]$ in Eq. (1).

The phase freedom between the two three-component representations has been implicitly recognized by the separation of solutions of the (covariant, source free) field equations into self-dual and anti-self-dual components. Thus, unless one refers to the underlying (1/2,1/2) representation, there is no Majorana-like constraint that may be imposed on Eq. (3). As in electrodynamics, the needed self-conjugacy can only be imposed by ruling out (independent) point magnetic (or electric) sources, with their opposing properties under parity. [Nor, obviously, is there any mass term to produce a Dirac-like constraint.]

Since the same dynamics is involved, it is immediate that pairs of (physical) gluons will bind to form tachyonic states, filling representations of a chiral symmetry entirely analogous to that for quarks, and differing only due to their being based on physically distinct degrees of freedom. In this case, however, the (color) octet channel is attractive as well as is the color singlet. [Any mixing with the repulsive (flavor singlet) color octet quark-antiquark channel will only serve to lower the lowest state, and so will not qualitatively affect the following argument.] Thus, (tachyonic) chiral $U(1) \times SU(3)$ color multiplets of gluonic mesons will be formed. The overall symmetry can not be a chiral $U(3)$ since the attraction in the color octet channel, while still stronger than for color singlet quarks and antiquarks, is weaker than for the color singlet combination of gluons.

As for quarks, this binding of massless objects must resolve its tachyonic tendencies by development of an SU(3) color singlet vev. Paralleling the quark notation, the effective Lagrangian of the (gluonic) composite meson states can be written as

$$\begin{aligned} L_G = &+\mu^2 \left\{ \left|S_o + iP_o\right|^2 + \zeta^2 \sum_{a=1}^{8} \left|S_a + iP_a\right|^2 \right\} \\ &- \chi_1 \left[\left|S_o + iP_o\right|^2 + \zeta^2 \sum_{a=1}^{8} \left|S_a + iP_a\right|^2 \right]^2 \\ &- \chi_2 \sum_a \left\{ (S_o S_a + P_o P_a) - \zeta^2 f_{abc} S_b P_c \right\}^2 \\ &+ \text{kinetic terms} + \ldots \end{aligned} \tag{6}$$

where ζ reflects the effect of the difference in U(1) and SU(3) strengths (and possible mixings with $q\bar{q}$ color octet components) referred to above.

As for the quark composite states, when S_o is shifted by the vev

$$\langle S_o \rangle = \beta \tag{7a}$$

$$\tilde{S}_o = S_o - \langle S_o \rangle \tag{7b}$$

L_G can be rewritten as

$$L_G = \ldots + 0(P_o^2 + \sum_a P_a{}^2) + \text{kinetic} + \ldots \tag{7c}$$

Note here that despite the separation of the U(1) factor, the pseudoscalar bosons all remain massless.

An addition corresponding to the instanton term now requires integrating over stationary (classical) quark configurations. These are not known to exist. If they did, however, their effects would be similar to that above: the value of $\langle S_o \rangle$ would be shifted, and P_o would become massive.

Th se two sectors (2) and (6) would remain isolated, for massless quarks, due to the properties of perturbative QCD. However, non-perturbative effects, such as those produced by instantons do couple the two chiral sectors. That is, while a massless quark may not perturbatively annihilate with its antiquark to produce two gluons, instantons couple a quark from $(\frac{1}{2},0)$ to an antiquark from $(0,\frac{1}{2})$. Thus, gluonic fluctuations coupled to the instanton are, in turn, coupled to a massless quark-antiquark pair in a J=0 state (among other). This allows spontaneous χSB from each sector to infect the other. To wit, we must add a term like

$$L_M = \psi\left[\left|\sigma + i\eta\right|^2 + \sum_j \left|\alpha_j + i\pi_j\right|^2\right]\left[\left|S_o + iP_o\right|^2 + \sum_a \left|S_a + iP_a\right|^2\right] + \ldots \tag{8}$$

It is now convenient to determine the vev's and mass terms by following the mixing of σ and S_o. Rather than carry the full complexity, we will keep only the singlet scalars and the (massless) multiplet of pseudoscalar states. In this minimal form, we consider

$$V_T = \lambda(\sigma^2 + \sum_j \pi_j^2 - v^2)^2 + \chi(S_o^2 + \sum_a P_a^2 - g^2)^2$$

$$+ \psi(\sigma^2 + \sum_j \pi_j^2 - w^2)^2(S_o^2 + \sum_a P_a^2 - h^2) \qquad (9),$$

the total effective potential in the σ, π_j, S_o, P_a field space. If the quark sector were isolated, $v = \langle\sigma\rangle$ would occur and similarly for an isolated gluon sector. In the mixing term, we allow $w \neq v$, $h \neq g$ for full generality, and $h = w = 0$ could also occur. Note that the λ and χ must be positive, although the sign of ψ is not determined in (9).

Our argument is that σ and S_o, since they have the same quantum numbers, and mix as argued above, form a system like the two real components of a complex Higg' scalar: The effective potential in σ, S_o space forms a trough surrounding the origin, and only a specific combination acquires a vev. Let $c = \cos\theta$, $s = \sin\theta$,

$$\sigma = cz + sy \qquad (10a)$$

$$S_o = - sz + cy \qquad (10b)$$

and choose

$$\langle z\rangle = \Lambda \ ; \ \langle y\rangle = 0 \qquad (11)$$

by definition. The combination is messy, but some algebra shows that $\partial V_T/\partial z = 0$ for $y=0$ when $z=0$ or when

$$\langle z^2\rangle = \frac{2\gamma c^2 v^2 + 2x s^2 g^2 + \psi(s^2 w^2 + c^2 h^2)}{2(\lambda c^4 + \chi s^4 + \psi s^2 c^2} \qquad (12),$$

which fixes $\Lambda^2 = -\langle z\rangle^2 - \langle z^2\rangle$. Of course, we must also have $\partial V_T/\partial y = 0$ at the same point. This fixes the angle θ via

$$2\lambda(c^2\Lambda^2 - v^2) + 2\chi(g^2 - s^2\Lambda^2) + \psi(w^2 - h^2 + [s^2 - c^2]\Lambda^2) = 0 \qquad (13)$$

It is then straightforward to show that

$$m_y^2 = 2\lambda s^2(3c^2\Lambda^2 - v^2) + 2\chi c^2(3s^2\Lambda^2 - g^2)$$

$$+ \psi\left\{[(c^2 - s^2)^2 - 2c^2 s^2]\Lambda^2 - (c^2 w^2 + s^2 h^2)\right\} \qquad (14)$$

Note that $m_{\pi_j}^2 \neq 0 = m_{Pa}^2$ in the general case. This occurs because the mixing is entirely in the U(1) sectors.

Returning to the σ and S_0, we recognize that both acquired mass in the usual way, but that the ψ-term induced off-diagonal terms in the mass matrix for the two fields. The case w=v, h=g is a little easier to follow, and the masses of the eigenstates are then:

$$m_z^2 = \frac{8(\lambda v4 + \psi g^2 v^2 + \chi g}{v^2 + g^2} \qquad (15a)$$

$$m_y^2 = \frac{8(\lambda - \psi + \chi)\ g^2 v^4}{v^2 + g^2} \qquad (15b)$$

Whether or not the lower mass state is composed mostly of glue or of quarks now depends on the detailed parameter values. It is apparent, however, that a 50%-50% mix is quite reasonable. We note in passing that this has significant consequences for the interpretation of the value of the σ-term of the purely quark model analysis.

We conclude with some comments regarding the P_a-states. Their masslessness is, of course, not a problem, since they are presumably confined by the usual color confinement mechanism. In this regard, they almost solve the confinement problem. For if it were only the S_a-scalars which were massless, the t-channel exchange of such a color octet between s-channel (say [heavy] quark and antiquark) sources of color would indeed provide a scalar potential. Since the effective coupling to the sources is $O(\alpha_s^2)$, it also grows at small (t-channel four-momentum-transfer-squared) q^2. Thus, similar conditions would apply to those Ball and Zachariasen suggest for t-channel gluon exchanges: a massless exchanged octet with diverging coupling. In this way, a linear scalar potential which confines color (including color octet and higher representation sources, not just quarks) could be obtained, with a color vertex structure parallel to that of single gluon exchange.

Unfortunately, it is the pseudoscalar which are massless, the Goldstone bosons of a chiral symmetry. It has never been suggested that the confining potential is a pseudoscalar! A vector O(9) symmetry of the states would be preferable, as the S_o (or mixture) vev would break it to O(8) and provide us with the desired (octet of) massless scalars. If nothing else, the SU(3)c invariance of the determinant of the octet of scalar and pseudoscalar glueball states explicitly violates such a symmetry. Thus, despite coming tantalizingly close, we are apparently still not able to link confinement to χSB in a transparent manner.

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