LEGIBILITY NOTICE

A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although portions of this report are not reproducible, it is being made available in microfiche to facilitate the availability of those parts of the document which are legible.

3

CONF-8904107--5

LA-UR -89-2144

Received by OSTI

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36

LA-UR--89-2144

DE89 014290

TITLE RADIATIVE COLLAPSE OF A BENNETT-RELAXED Z-PINCH

AUTHOR(S) Leaf Turner, CTR-6

SUBMITTED TO AIP Conference Proceedings of the
2nd International Conference on High Density Pinches
April 26--28, 1989
Laguna Beach, CA

MASTER

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

FORM NO 836 R4
ST NO 2629 5/81

RADIATIVE COLLAPSE OF A BENNETT-RELAXED Z-PINCH

Leaf Turner
Los Alamos National Laboratory
University of California, Los Alamos, New Mexico 87545

ABSTRACT

The global evolution of a z-pinch has been studied with the assumption of a relaxed state consisting of ions and electrons, each in a rigidly drifting isothermal Maxwellian distribution. This speculative approach has the pragmatic feature of possessing phenomenologically useful global parameters such as drift velocity and temperature that vary in accordance with global physical quantities such as energy and entropy. The plasma gains energy from a time-dependent electric field by means of Poynting's vector. Coulomb collisions between electrons and ions is calculated with a Fokker-Planck treatment analogous to that used by Dreicer to calculate runaways. For a variety of initial conditions and time-independent applied electric fields, the pinch evolution always culminates in a time-independent (attractor) state whose current is the Pease-Braginskii current and whose final radius is proportional to (line density)$^{3/4}$/(electric field)$^{1/2}$. Before the final state is attained, the pinch may bounce toward and away from a highly collapsed state. For the case of a Bennett pinch, the classical limit of the resistivity is attained when the line density is much greater than $4\pi m_e/e^2\mu_0$; i.e., 3.55×10^{14} m^{-1}.

INTRODUCTION

We have modeled the radiative collapse of a z-pinch by adapting Dreicer's treatment of runaways[1] to the case of inhomogeneous geometry; namely, the cylindrically symmetric geometry of a straight z-pinch. We thus commence our study by representing the distribution of electrons and ions with separately drifting Maxwellians of the form

$$f_{e,i}(r,\underline{v}) = \exp - \left[\frac{\frac{1}{2}m_{e,i}v^2 + q_{e,i}\phi(r) - u_{e,i}p_{z_{e,i}}}{kT_{e,i}}\right] ,$$

in which $m_{e,i}$ are the electron and ion masses, $q_e = -e$, $q_i = e$, $\phi(r)$ is the quasineutral electrostatic potential, $T_{e,i}$ are the electron and ion temperatures (assumed to be spatially constant), $u_{e,i}$ are the electron and ion axial drift velocities (also spatially constant), and $p_{z_{e,i}}$ are the electron and ion canonical momenta:

$$p_{z_{e,i}} = m_{e,i}v_z + q_{e,i}A_z(r),$$

with $A_z(r)$ being the magnetic vector potential. This distribution is associated with a scalar pressure for each species.[2] Integrating over the velocity coordinates, we obtain the electron and ion number densities:

$$n_e(r) \propto \exp\left\{\frac{-e[u_e A_z(r) - \phi(r)]}{kT_e}\right\}, \quad n_i(r) \propto \exp\left\{\frac{e[u_i A_z(r) - \phi(r)]}{kT_i}\right\},$$

where k is Boltzmann's constant and which satisfy the equations of fluid equilibrium:

$$q_{e,i} n_{e,i}(r)[\underset{\sim}{E}(r) + \underset{\sim}{u}_{e,i} \times \underset{\sim}{B}(r)] = kT_{e,i} \nabla n_{e,i}(r)$$

where $\underset{\sim}{E}(r) = -\frac{d\phi(r)}{dr}\hat{r}$ and $\underset{\sim}{B}(r) = \nabla \times [A_z(r)\hat{z}]$.

We obtain the quasineutral potential $\phi(r)$ in terms of $A_z(r)$ by setting $n_e(r) = n_i(r)$. If we study the pinch in the center-of-mass frame in which $u_i << u_e$, we find that $\phi(r) = u_e A_z(r)/(1 + T_e/T_i)$. We shall simplify our analysis by letting the ions be cold, i.e., by setting $T_i = 0$ so that

$$n_e(r) = n_i(r) = n_o \exp - \left(\frac{eu_e A_z(r)}{kT_e}\right) ,$$

in which n_o is the on-axis number density of each species [because we are free to set $A_z(0)$ equal to zero]. The self-consistency condition on the vector potential is therefore

$$\nabla^2 A_z(r) = -\mu_o j_z(r) = \mu_o e n_o u_e \exp - \left[\frac{eu_e A_z(r)}{kT_e}\right] ,$$

whose solution yields the Bennett number density distribution[3],

$$n(r) = \frac{n_o}{\left(1 + \frac{r^2}{r_o^2}\right)^2} , \qquad (1)$$

where r_o is defined by $r_o^2 = 8kT_e/\mu_o e^2 u_e^2 n_o$ and which characterizes the plasma radius, and where μ_o is the magnetic permeability of vacuum.

We can define a line density N(r) by setting

$$N(r) = 2\pi \int_0^r r'n(r')dr' = \frac{\pi n_o r^2}{1 + \frac{r^2}{r_o^2}} ,$$

which prescribes the current within a surface of radius r by means of the relationship $I(r) = - eu_e N(r)$ and in turn the magnetic field by means of Ampere's Law:

$$B_\theta(r) = - \frac{\mu_o e u_e}{2\pi r} N(r) = - \frac{\mu_o e u_e n_o}{2} \left(\frac{r}{1 + \frac{r^2}{r_o^2}} \right) . \tag{2}$$

We shall choose the wall bounding the plasma to be located at $r = a$ and define the parameter β by

$$\beta = \frac{8\pi N_o k T_e}{\mu_o I_o^2} = 1 + \frac{r_o^2}{a^2} , \tag{3}$$

in which $N_o = N(a)$ and $I_o = I(a)$ specify the total line density and total current respectively. This clearly yields the usual Bennett relationship between total current and total line density in the limit of infinite wall radius.

THE BENNETT-RELAXED Z-PINCH MODEL

We shall assume that various high-frequency relaxation processes occur that, on the time scale of profile evolution, maintain the pinch in the Bennett profile. Instead of using the variables, $n_o(t)$, $u_e(t)$, and $T_e(t)$, to specify the mean plasma state, we shall use the global variables, $N_o(t)$, $\beta(t)$, and $T_e(t)$, to specify the state more conveniently. To determine the evolution of these three variables, we shall utilize three equations that in the absence of collisions between the electron and ions and in the absence of bremsstrahlung are expressible in conservation form. These equations, which can be derived by taking the appropriate moments of the Fokker-Planck equation and whose integrals express the evolution of the total line density, the total magnetohydrodynamic energy, and the total electron entropy, respectively, are

$$\frac{\partial n_e(\mathbf{r},t)}{\partial t} + \nabla \cdot [\mathbf{u}_e(\mathbf{r},t) n_e(\mathbf{r},t)] = \text{density source/sink terms,} \tag{4}$$

$$\frac{\partial}{\partial t}\left[\frac{m_e n_e(\mathbf{r},t) u_e^2(\mathbf{r},t)}{2} + \frac{B_\theta^2(\mathbf{r},t)}{2\mu_o} + \frac{3p_e(\mathbf{r},t)}{2}\right]$$

$$+ \nabla \cdot \left[\frac{m_e n_e(\underline{r},t) u_e^2(\underline{r},t) \underline{u}_e(\underline{r},t)}{2} + \frac{5}{2} p_e(\underline{r},t) \underline{u}_e(\underline{r},t) \right.$$

$$\left. + \underline{q}_e + \frac{\underline{E}(\underline{r},t) \times \underline{B}(\underline{r},t)}{\mu_o} \right] = \text{energy source/sink terms,} \tag{5}$$

$$\frac{\partial s_e(\underline{r},t)}{\partial t} + \nabla \cdot [\underline{u}_e(\underline{r},t) s_e(\underline{r},t)] = \text{entropy source/sink terms,} \tag{6}$$

in which $\underline{q}_e$ represents the electron kinetic energy flux density, which we shall assume to be negligible (since we are assuming no thermal gradients), p_e represents the electron pressure, $n_e(\underline{r},t)kT_e(t)$, and s_e represents the entropy density of the electrons which is equal to $n_e(\underline{r},t)\ln[p_e^{3/2}(\underline{r},t)/n_e^{5/2}(\underline{r},t)]$, or equivalently to $n_e(\underline{r},t)\ln[(kT_e(t))^{3/2}/n_e(\underline{r},t)]$. The equation for the energy evolution is accurate to leading order in m_e/m_i and is describing the situation with cold ions in the center-of-mass frame of the plasma, which is essentially the rest frame of the ions.

We shall assume that $\hat{r} \cdot \underline{u}_e$ vanishes at the cylindrical surface, $r = a$. Because the zeroth velocity moment of the Fokker-Planck collision term vanishes, we find from Eq. (4) that the line density is conserved:

$$\frac{dN_o}{dt} = 0. \tag{7}$$

Because the left-most bracket of Eq. (5) represents the energy of the total system, accurate to the order of interest, it varies only because of energy flow across the boundary at $r = a$. This flow consists of bremsstrahlung and a generally nonvanishing Poynting vector. From the Bennett profiles of Eqs. (1) and (2), we obtain,

$$2\pi \int_0^a r dr \, \frac{m_e n_e u_e^2}{2} = \frac{N_c k T_e}{\beta}, \qquad N_c = \frac{4\pi m_e}{\mu_o e^2} = 3.55 \times 10^{14} m^{-1} ,$$

$$2\pi \int_0^a r dr \, \frac{B^2}{2\mu_o} = \left[\beta \ln\left(\frac{\beta}{\beta - 1}\right) - 1\right] N_o k T_e ,$$

$$2\pi \int_0^a r dr \, \frac{3p_e}{2} = \frac{3}{2} N_o k T_e ,$$

$$2\pi \int_0^a r\,dr\; n_e^2 = \frac{N_o^2}{3\pi a^2} \left[\frac{\beta^3 - (\beta - 1)^3}{\beta(\beta - 1)}\right] ,$$

$$2\pi \int_0^a r\,dr\; n_e \ln(n_e) = \left[\ln\left(\frac{N_o}{\pi a^2}\right) + (2\beta - 1) \ln\left(\frac{\beta}{\beta - 1}\right) - 2\right] N_o .$$

Thus an integration of Eq. (5) within the boundary, $r = a$, yields

$$\frac{\partial}{\partial t}\left\{\left[\frac{N_c}{\beta N_o} + \beta \ln\left(\frac{\beta}{\beta - 1}\right) + \frac{1}{2}\right] N_o kT_e\right\}$$

$$= \left(\frac{8\pi N_o kT_e}{\mu_o \beta}\right)^{1/2} E_z - \frac{B_{br}(kT_e)^{1/2} N_o^2}{3\pi a^2} \left[\frac{\beta^3 - (\beta - 1)^3}{\beta(\beta - 1)}\right] , \qquad (8)$$

in which E_z is the time-dependent axial electric field at $r = a$. We have used the fact that the bremsstrahlung radiation rate is given by $B_{br}(kT_e)^{1/2} n_e^2$ watts/m^3 in which $B_{br} = 3.79 \times 10^{-29}$ in SI units.

We next integrate the evolution equation for the electron entropy, Eq. (6), observing that the divergence term vanishes by virtue of the boundary condition on $\underline{u}_e$, to obtain:

$$N_o \frac{\partial}{\partial t}\left[\frac{3}{2}\ln(kT_e) + (1 - 2\beta) \ln\left(\frac{\beta}{\beta - 1}\right)\right] = \left\{\frac{N_o^2}{3\pi a^2} \left[\frac{\beta^3 - (\beta - 1)^3}{\beta(\beta - 1)}\right]\right\}$$

$$\cdot \left\{\frac{4\pi}{ekT_e}\left(\frac{e^2}{4\pi\varepsilon_o}\right)^2 \ln(\Lambda)\left(\frac{2\pi}{\mu_o \beta N_o kT_e}\right)^{1/2} \Upsilon\left[\left(\frac{N_c}{\beta N_o}\right)^{1/2}\right] - B_{br}(kT_e)^{-1/2}\right\} . (9)$$

The bremsstrahlung term on the right-hand side is an electron entropy sink term obtained from the volume integral of the bremsstrahlung radiation rate divided by kT_e. The first term, on the right-hand side is an electron entropy source term, stemming from the Fokker-Planck collisions of the electrons with the cold ions, calculated to leading order in m_e/m_i. It is merely the scalar product of $- \underline{u}_e$ with the collisional force density divided by kT_e and integrated over the volume. The $\ln(\Lambda)$ term is the Coulomb logarithm and has been set equal to 10 in the numerical calculations discussed below; the function Υ was defined in Ref. 1. One can verify that the above argument of Υ is equal to $(m_e/2kT_e)^{1/2} u_e$. This Fokker-Planck treatment was shown to yield results that are in accord with the results of classical calculations of resistivity to within a few percent when the argument of Υ is much less than one[1], which is certainly satisfied whenever $N_o \gg N_c = 3.55 \times 10^{14}$ m^{-1}.

DISCUSSION OF RESULTS

If $N_c << N_o$, we observe from Eq. (9) that when $\beta I_o = 8.35 \times 10^5$ A, the rate of collisional generation of electron entropy is exactly cancelled by the loss rate through bremsstrahlung. For the case of high compression, $r_o << a$, $\beta = 1$ and we obtain

$$I = 8.35 \times 10^5 \text{ A}, \tag{10}$$

the Pease-Braginskii current[4] for the cold ion case, or equivalently, we obtain

$$kT_e = \frac{3.48 \times 10^4}{N_o}, \tag{11}$$

using the Bennett relation, Eq. (3). Similarly, we observe from Eq. (8) that the total energy of the plasma will be constant when the rate of electrical generation of energy is precisely dissipated by the rate of bremsstrahlung, which occurs when:

$$r_o = 3.00 \times 10^{-17} \frac{N_o^{3/4}}{E_z^{1/2}} . \tag{12}$$

Equations (10) - (12) are expressed entirely in SI units and are the conditions for steady-state operation at high compression.

Using the constancy of N_o, Eq. (7), we can numerically integrate Eqs. (8) and (9) to determine the time evolution of β and T_e, from which we also can extract the time evolution of the plasma radius, r_o, and the plasma current, I_o. In code comparison runs, we found good agreement between our code predictions and those of a one-dimensional transport code developed by Nebel using a two-fluid, quasistatic model.[5] A virtue of our Bennett-relaxed z-pinch model is its efficiency in tracking rapid radiative collapses, such as that shown in Fig. (1). We now wish to demonstrate some of our results.

Figure 1 demonstrates that for conditions of steady applied electric field strength, E_z, the pinch radiatively collapses down to a small but finite radius, in the neighborhood of which point the electron temperature is decreasing, and then rebounds, suffers some oscillations, and settles into a steady-state governed by Eqs. (10) - (12). No matter how strong the steady applied voltage, Eqs. (10) - (12) govern the final state. The effects of relativity, opacity, fusion, endloss, and inertial terms in the radial acceleration are, of course, absent in Eqs. (7) - (9) describing this collapse and must be accounted for in a more thorough treatment.

In Fig. (2), we compare the effect of two steady applied electric field strengths on the initial radial evolution of the pinch. Again, Eqs. (10) - (12) are observed to govern the final steady-states achieved at high compression. Indeed, we have found

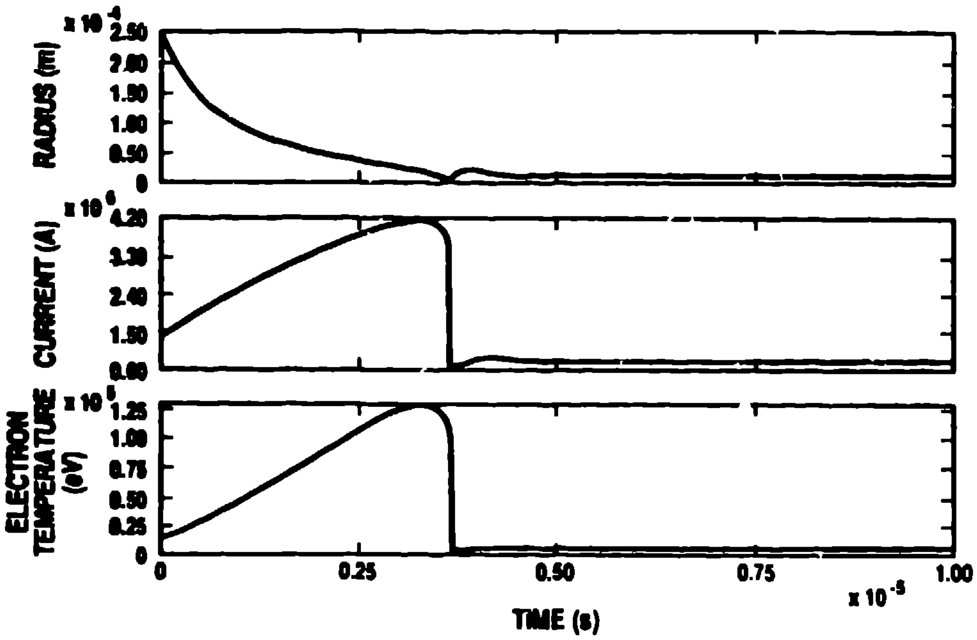

Fig. 1(a). The evolution of the radius, current, and electron temperature of a radiative collapse with a steady applied electric field of 1.19309 MV/m shown on a 10 μs time scale. The initial parameters are $r_0 = 2.5 \times 10^{-4}$ m, $a = 1.0 \times 10^{-2}$ m, $N_0 = 4.24 \times 10^{19}$ m^{-1}, and $T_e = 15$ keV.

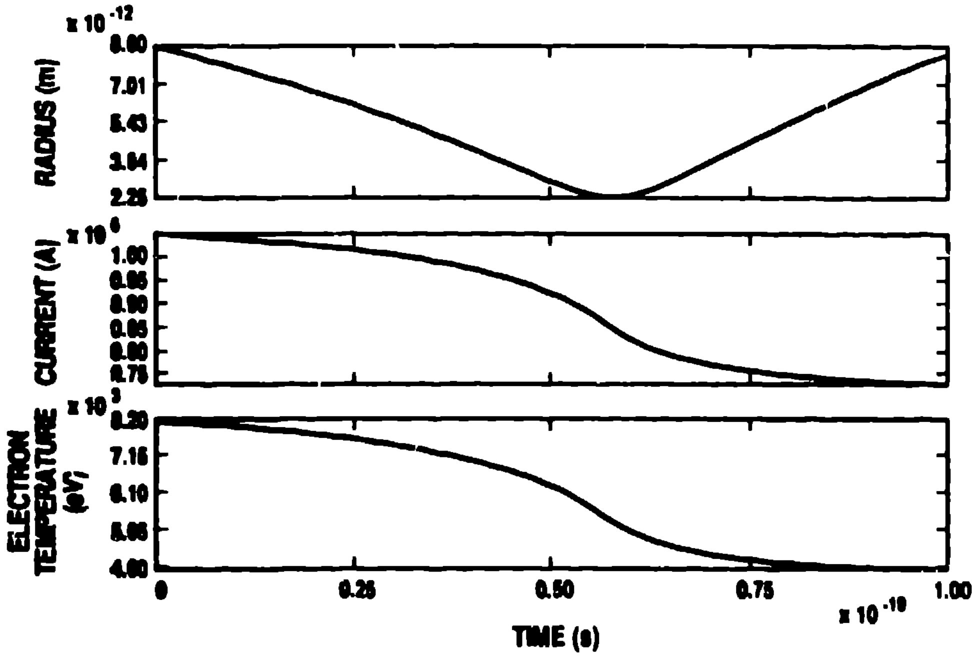

Fig. 1(b). The evolution of the radius, current, and electron temperature of the collapse shown in Fig. 1(a) exhibited here during a period of only 10^{-10} s approximately centered on the instant of collapse.

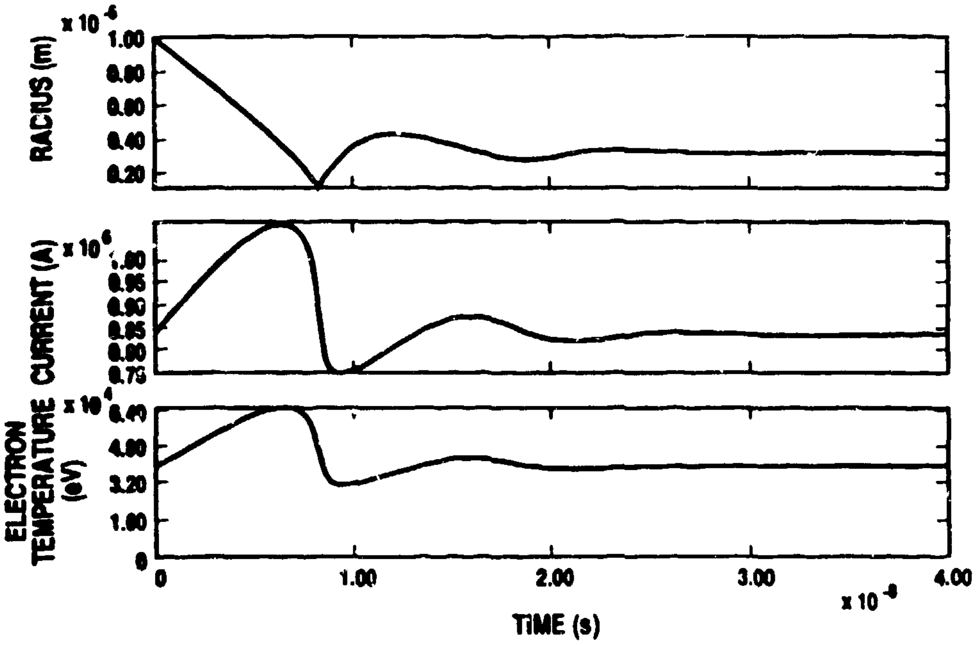

Fig. 2(a). The radius, current, and electron temperature of an evolving pinch with a steady applied electric field of 1.19309×10^2 MV/m. The initial parameters are $r_o = 1.0$ μm, $a = 0.01$ m, $N_o = 5.6 \times 10^{18}$ m^{-1}, and $T_e = 38.9$ keV.

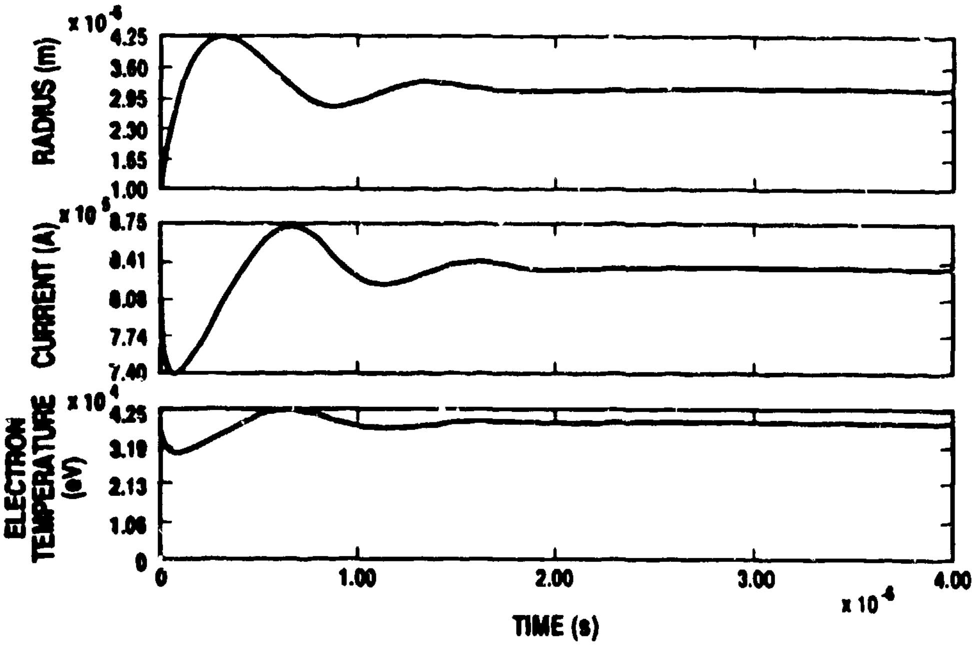

Fig. 2(b). The radius, current, and electron temperature of an evolving pinch with a steady applied electric field of 1.19309 MV/m. The initial parameters are $r_o = 1.0$ μm, $a = 0.01$ m, $N_o = 5.6 \times 10^{18}$ m^{-1}, and $T_e = 38.9$ keV.

computationally that at high compression for a variety of initial conditions and time-independent applied electric fields, the pinch evolution always culminates in a time-independent (attractor) state described by Eqs. (10) - (12).

Although absent in the above discussion, we have also included the effects of external inductance and resistance in our code. On the analytical side, we can explain certain features of the collapse to extremely small radius, such as the declining temperature at maximum collapse, as well as the final damped oscillatory approach to the steady-state configuration.

Finally, we wish to remark that isothermal models with a uniform current density[6], implying a parabolic number density, are pathological. As a result of $u_e \sim j_z/n_e$, one observes that $\int m_e n_e u_e^2 r dr \sim \int [1 - r^2/a^2]^{-1} r dr$ which implies a fluid kinetic energy that diverges logarithmically near the boundary.

ACKNOWLEDGMENTS

We wish to thank Jay Hammel, Jack Shlachter, and David Scudder for bringing this problem to my attention and for their support in this endeavor. We are also grateful to Richard A. Nebel for assisting with running code comparisons.

This work was performed under the auspices of the U. S. Department of Energy.

REFERENCES

1. H. Dreicer, Phys. Rev. 115, 238 (1959).
2. R. L. Morse and J. P. Freidberg, Phys. Fluids 13, 531 (1970).
3. W. H. Bennett, Phys. Rev. 45, 89 (1934).
4. R. S. Pease, Proc. Phys. Soc. B70, 11 (1957); S. I. Braginskii, Sov. Phys. JETP 6, 494 (1958).
5. P. Rosenau, R. A. Nebel, and H. R. Lewis, Phys. Fluids B 1, 1233, (1989).
6. M. G. Haines, "An Analytic Model of Radiative Collapse of a Z-Pinch," in Proceedings of the 1988 Z-Pinch and Plasma Focus Workshop, ed. by B. Etlicher (Nice, Ecole Polytechnique, 1988); A. E. Robson, Nucl. Fusion 28, 2171 (1988).