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TITLE NEUTRINO OSCILLATIONS: AN ESSAY IN HONOR OF FELIX BOEHM

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1 ABSTRACT

We briefly review the theory of neutrino oscillations and the MSW effect and report on new calculations by Rosen and Gelb for solar neutrino-electron scattering. The aim of these calculations is to try to use the scattering process as a means of choosing between the three types of MSW solutions for the ^{37}Cl experiment. Both the efficiency and the resolution of the Kamiokande II detector are taken into account and the ratio R of the MSW prediction to the standard solar model prediction is calculated for different cuts on the minimum electron energy. We find that the adiabatic solution requires R to be less than 1/3, the large angle one requires it to be less than 2/3, and the nonadiabatic one restricts it to a value close to 1/2. The central value of the published KII data is close to 1/2, but the errors are too large to exclude the other solutions.

2 INTRODUCTION

I would like to wish Felix 'Many Happy Returns' on the occasion of his 65th birthday and to express the hope that he will have many more birthdays to come! There is still much work to be done in the field of neutrino oscillations and we shall need all of his insight and skill to complete it.

It is entirely fitting to begin a discussion of neutrino oscillations, one which may ultimately turn out to be nothing more than wishful thinking on the part of this particular theorist, with a quotation attributed to Felix at a recently held international conference, and with the principal result of his work.

The quotation was made by Martin Perl in his review of neutrino properties at the 1989 International Symposium on Lepton and Photon Interactions at High Energies, held at SLAC in August. 'Neutrino Oscillations?' asked Perl rhetorically, 'There is *NO* confirmed evidence for neutrino oscillations!' and he cited a talk by

Felix at the Yamada Conference held at Osaka in June of this year[1].

In a similar vein, we can refer to the beautiful series of reactor experiments led by Felix. These experiments[2], carried out at the Goesgen reactor over a period of several years, provide the most sensitive laboratory limits on the disappearance of ν_e in the parameter domain of small Δm^2 and large $\sin^2\theta$. Other experiments at accelerators[3] provide rather tight limits on the disappearance and appearance of different types of neutrino in the domain of small mixing angle and large mass differences.

Given the weight of negative evidence, why should we pursue the idea of neutrino oscillations? Let me give you my own reasons for doing so:

1. Because the idea[4] is a beautiful one, especially when coupled with the elegant matter enhancement mechanism of Mikheyev and Smirnov[5], and Wolfenstein[6]. Invoking a Diracian principle, I would like to suggest that whatever is beautiful should be true.

2. Because the observation of oscillations is a definitive signal for the existence of neutrino mass[4], and it is the most sensitive method of searching for masses well below $1 eV$.

3. Because there are significant regions of parameter space remaining to be explored, especially with low energy neutrinos over very long distances[2,3].

4. Because there exists the *possibility* that we are seeing oscillations with solar neutrinos. Indeed the Davis[7] and Kamiokande II[8] experiments provide the best (remember that 'best' does not necessarily imply 'good') evidence for neutrino oscillations but they are not definitive.

We appear to have a conflict between theoretical desires on the one hand and hard experimental facts on the other. Now it happens that on my journey to Pasadena I was reading a book by an expert negotiator who believes in looking for 'win-win' resolutions to conflicts between people. So here I am going to look for a 'win-win' resolution of the physics.

3 QUICK REVIEW OF THE MSW EFFECT

As you might gather from the first point, I have a strong predilection for the Mikheyev-Smirnov-Wolfenstein effect (MSW)[5,6,9]; indeed my hope and my prejudice are that it will provide us with the solution to the solar neutrino problem. The basis for this effect, and also for the Kamiokande II experiment, is to be found in the two lowest order diagrams for neutrino-electron scattering. One diagram involves the exchange of a neutral Z^0 boson between the electron and the neutrino, and in the standard GWS model, it has the same strength for all types of neutrino; the other is a 'charge-exchange' process involving the exchange of a charged W^+ boson, and it comes into play only for the electron-type neutrino ν_e. The existence of this charged-current diagram means that ν_e has a different refractive index, or effective mass, in matter as compared with the other types of neutrino, ν_μ and ν_τ, and that it has a much larger cross-section for scattering from electrons than do the other neutrinos. Both of these effects will play important parts in this discussion.

The phenomenon of neutrino oscillations is a purely quantum mechanical effect involving two almost degenerate neutrino mass eigenstates[4,10], ν_1 and ν_2, with masses m_1 and m_2 respectively, and with a common momentum p which is much greater than both masses. The energies of the two states are then given by the approximate formula

$$E_i = p + m_i^2/2p \quad (i = 1,2) \tag{1}$$

As these mass eigenstates evolve in time, they acquire the appropriate phase factors

$$exp(-iE_jt)(j = 1,2) \tag{2}$$

and hence the phase difference between them oscillates with time. Whenever we have new states defined as coherent combinations of these two states with definite phase relations between them, the character of the new states will oscillate in time along with the phase difference.

Let us define the electron neutrino ν_e and the muon neutrino ν_m as the orthogonal combinations:

$$\nu_e = \cos\theta\nu_1 + \sin\theta\nu_2$$

$$\nu_m = \cos\theta\nu_2 - \sin\theta\nu_1. \tag{3}$$

By electron neutrino and muon neutrino we mean those flavor eigenstates that take part in weak interactions with the electron and muon respectively. As these states evolve in time, the relative phase between ν_1 and ν_2 will change, and what was initially a pure electron neutrino or a pure muon neutrino will become an admixture of the two flavor states.

To explore the MSW effect, we write down the Schroedinger-like time development equation for the oscillation amplitudes[5]:

$$i\frac{dA}{dt} = HA \tag{4}$$

where A represents a column vector of the probability amplitudes for ν_e to remain ν_e and for ν_e to turn into another neutrino type ν_m, a_e and a_m respectively,

$$A = \begin{pmatrix} a_e \\ a_m \end{pmatrix}. \tag{5}$$

The Hamiltonian H is given by:

$$H = \begin{pmatrix} X Y \\ Y Z \end{pmatrix} \tag{6}$$

with

$$\begin{aligned} X &= \frac{m_1^2c^2 + m_2^2s^2}{2p} + 2^{1/2}G_F N_e \\ Z &= \frac{m_1^2s^2 + m_2^2c^2}{2p} \\ Y &= \frac{m_2^2 - m_1^2}{2p}cs = \frac{\Delta m^2}{2p}cs \\ c &= \cos\theta \\ s &= \sin\theta \end{aligned} \tag{7}$$

For neutrinos propagating through the sun, matter oscillations are incorporated through the last term in the expression for X, which depends upon the Fermi constant G_F and the density of electrons, N_e. All the other terms in the expressions for X, Y, Z in eq (7) correspond to neutrinos propagating in vacuo. Adding the matter term can have powerful consequences: for, under the right conditions, it gives us a Hamiltonian matrix which is not only symmetric, but also one which has equal elements down the diagonal. The eigenvectors of such a matrix are equal admixtures of electron- and muon- neutrino, and for a given off-diagonal element, the separation between the eigenvalues is minimal. In other words the extra term in X gives us a chance to progress from non-maximal mixing in vacuo to maximal mixing in matter.

The condition for equal diagonal elements, $X = Y$ can be written as:

$$2^{1/2}G_F N_e = \frac{\Delta m^2 \cos 2\theta}{2p} \qquad (8)$$

Now the electron density N_e is inherently positive, and the Fermi constant G_F, since it arises from the exchange of a gauge boson, is also positive: therefore the product of Δm^2 and $\cos 2\theta$ must also be positive if the condition (8) is to be satisfied. It is not difficult to show that this requires the electron-neutrino to be dominantly composed of the lighter of the two mass eigenstates, and the muon-neutrino of the heavier.

We take the neutrino masses to be so small that the particles travel with the speed of light; in units where c=1 we can then equate the time t to the distance R travelled. For oscillations which occur in vacuo rather than in a material medium, the probabilities for the survival of the original flavor and the appearance of a new flavor can be written in the standard forms:

$$P(\nu_e, \nu_e; R) = 1 - \sin^2 2\theta \sin^2(\pi R/L) \qquad (9)$$

$$P(\nu_m, \nu_e; R) = \sin^2 2\theta \sin^2(\pi RL). \qquad (10)$$

where the oscillation length L is:

$$\begin{aligned} L &= 2\pi/(E_2 - E_1) = 4\pi p/(m_2^2 - m_1^2) = 4\pi p/\Delta m^2 \\ &= 2.5(p/MeV)/(\Delta m^2/eV^2) meters. \qquad (11) \end{aligned}$$

For oscillations in a medium with a constant density of electrons, we can describe the oscillation problem in the same way as in the in vacuo case except that the mixing angle and the oscillation length are both modified. The survival probability for an electron neutrino is now:

$$P(\nu_e, \nu_e; R) = 1 - \sin^2 2\theta_M \sin^2(\pi R/L_M). \tag{12}$$

The new oscillation parameters are obtained by diagonalising the equations of motion in eq.(4-7) above:

$$\begin{aligned} \sin^2 2\theta_M &= \sin^2 2\theta/[\sin^2 2\theta + (L/L_0 - \cos 2\theta)^2] \\ L_M &= L/[sin^2 2\theta + (L/L_0 - \cos 2\theta)^2]^{1/2} \\ L &= 4\pi p/\Delta m^2 \\ L_0 &= 2\pi/2^{1/2} G_F N_e. \end{aligned} \tag{13}$$

The formula for the new mixing angle in matter has some important properties: no matter how small the in vacuo angle θ may be, the matter angle θ_M reaches its maximal value when

$$L/L_0 = \cos 2\theta \tag{14}$$

which is just another way of writing the equal diagonal element condition of eq(8). Therefore, as long as θ is different from zero, there is always a density for which the neutrino will oscillate with maximal mixing.

For a medium of varying density such as the sun, the enhancement condition of eq. (8) can be satisfied at different locations for neutrinos of different oscillation lengths. Thus all neutrinos in an energy band which is determined by the density profile of the medium and the value of Δm^2 will pass through an enhancement point somewhere in the medium.

To gain some insight into this situation, we express the electron density in units of Avogadro's Number and L_0 in meters. The condition (8) becomes

$$\frac{p}{\Delta m^2} = \frac{7 * 10^6 \cos 2\theta}{\rho_e} \tag{15}$$

The values of ρ_e encountered in the sun vary from about 150 in the core to close to zero at the edge and so the band of oscillation lengths covers several orders of magnitude:

$$10^4 \leq \frac{p}{\Delta m^2} \leq 10^8. \tag{16}$$

For neutrino momenta in the range of 1-10 MeV, typical of the ^8B neutrinos to which the Davis experiment is sensitive, and a small mixing angle, condition (15) corresponds to Δm^2 in the range of 10^{-7} to 10^{-3} eV^2.

In the earth, ρ_e varies from about 3 at the surface to 13 at the center, and so the range of Δm^2 which will give rise to enhanced oscillations for ^8B neutrinos is 10^{-8} to 10^{-6} eV^2. We may therefore anticipate that for certain sets of parameters, neutrinos can undergo enhanced oscillations in the earth as well as the sun. This in turn can give rise to a "day-night" effect in which electron neutrinos are converted to another type in the sun and rejuvenated in their passage through the earth[11].

When we apply the MSW effect to the ^{37}Cl experiment of Ray Davis and his collaborators, we find that there are three types of solution in the parameter space of Δm^2 and $\sin^2 2\theta$: the ***adiabatic*** solution[5,12], in which 'low' energy solar neutrinos remain as electron neutrinos while 'high' energy ones are almost completely converted to muon or other neutrino type; the ***nonadiabatic*** solution[13], in which the 'low' energy neutrinos are completely converted to another neutrino type while the 'high' energy ones have about a 50% of remaining as electron neutrinos; and the *large angle* solution[14], in which the probability for the solar neutrinos to remain as electron neutrinos is independent of energy. The dividing line between 'low' and 'high' is in the neighborhood of 6-8 MeV.

These solutions arise from the different ways in which the spectrum of solar neutrinos can overlap the curve of probability for ν_e to remain ν_e as a function of the oscillation length parameter $l = p/\Delta m^2 (MeV/ev^2)$. The characteristic behavior of the probability is that it is close to unity for $l \leq 3 \times 10^4$ and then falls rapidly to the value of $\sin^2\theta$ in the neighborhood of $l = 10^5$; this is the adiabatic part of the curve. It remains at this value for a decade or more, depending on the magnitude of the mixing angle, and then begins an exponential, nonadiabatic rise back to unity as l approaches 10^8.

For small mixing angles, the probability in the central region is close to zero and so for values of Δm^2 in the range 10^{-4} to 10^{-7}, we find that the probability curve divides the spectrum of ^8B neutrinos into two parts, one with a large probability and the other with a small one. In the adiabatic solution the large probability occurs for low energies, and in the nonadiabatic one, it occurs for high energies.

For large mixing angles with $\sin^2\theta = 1/4 - 1/2$, the central region is large enough to contain the entire solar neutrino spectrum. Thus we obtain a solution in which the probability for ν_e to remain ν_e is independent of energy. This solution spans the entire range of Δm^2 for which MSW comes into play, and it includes the values for which the day-night effect occurs.

It is clear that in order to distinguish between these solutions, we must find a way of measuring the probability as a function of energy. One way is to look at the low energy pp neutrinos: for the adiabatic solution they remain as electron neutrinos, whereas for the nonadiabatic one they can be almost entirely converted to another type[13]. Another way of making the distinction is to look at the scattering of solar neutrinos by electrons.

4 SOLAR NEUTRINO-ELECTRON SCATTERING AND THE KII EXPERIMENT

Solar neutrino-electron scattering is sensitive to all types of neutrino except sterile ones[9]. Electron neutrinos have the largest cross-section, by a factor between 6 and 7, because they interact with electrons through both charged- and neutral-currents; the muon- or tau-type neutrinos into which they might oscillate interact only through the neutral-current. Should solar neutrinos oscillate into sterile types, then the sterile neutrinos will not scatter from electrons at all.

Total cross-sections for solar neutrino-electron scattering in the context of the standard particle physics model and the MSW mechanism have been calculated by Bahcall, Gelb, and Rosen[15]. Subsequently, Gelb and Rosen[16] have examined the implications of the different MSW solutions of the ^{37}Cl experiment for electron scattering. They found that there is a definite relationship between each solution and the ratio R of the observed signal to the signal expected on the basis of the standard solar model in the absence of neutrino oscillations. They found that the adiabatic solution predicts values of R less than, or equal to 1/3, while the large angle solution allows values up to approximately 2/3; by contrast the non-adiabatic solutions predicts that R should be in a narrow range around 50%. In addition R can never fall below 1/6 unless solar neutrinos oscillate into sterile types. This summer Gelb and I have re- examined our calculations taking into account certain properties of the KII detector that were ignored in earlier work. I would like to describe the results here.

To use solar neutrino-electron scattering as a tool for distinguishing between different MSW solutions of the Davis experiment we follow Bahcall, Gelb, and Rosen[15] and write the differential cross-section for producing a recoil electron with kinetic energy T as:

$$\langle\frac{d\sigma}{dT}\rangle = \frac{1}{\phi_{\rm total}}\int dq\phi(q)\left[\frac{d\sigma}{dT}(\nu_e e)P_{ee}(q) + \frac{d\sigma}{dT}(\nu_\mu e)(1 - P_{ee}(q))\right] \qquad (17)$$

where $\phi(q)$ is the spectrum of neutrinos of energy q produced in the sun, $\frac{d\sigma}{dT}$ is the differential cross-section for neutrino electron scattering, and $P_{ee}(q)$ is the probability for an electron-type neutrino of energy q to remain an electron neutrino. The basic differential cross- section $\frac{d\sigma}{dT}(\nu e)$ depends upon q, T, and the flavor of the incident neutrino.

Since we actually measure the product of the basic differential cross-section times the flux, we must assume that the neutrino flux $\phi(q)$ is given by the standard solar model[9)] in order to extract information about MSW from the data. In addition we must take into account the detector efficiency for recording electrons of an apparent energy T_a and its resolution function, which gives the probability that the true electron energy is T_t when the apparent energy is T_a. In our new calculations, Gelb and I use the prescriptions for efficiency and resolution given in various publications of the Kamiokande collaboration[17)] and described by Bahcall and Haxton[18)] in their recent study of MSW and the standard solar model.

The efficiency function for the Kamioka detector is:

$$\begin{aligned} f(T_a) &= 1 - e^{-[(\frac{T_a - T_0}{4})]^{2.03}} \quad T_a \geq T_0 \\ f(T_a) &= 0 \quad T_a \leq T_0 \\ T_0 &= 4.2MeV. \end{aligned} \qquad (18)$$

The detector resolution is approximately given by a Gaussian:

$$\begin{aligned} \rho(T_a, T_t) &= N \exp^{-(T_a - T_t)^2/2\sigma^2(T_a)} \\ \sigma(T_a) &= 0.22T_a\sqrt{\frac{10MeV}{T_a}}, \end{aligned} \qquad (19)$$

where N is chosen so that the integral of $\rho(T_a, T_t)$ over apparent energies from the electron mass to infinity is equal to unity.

To obtain the total cross-section, we perform a double integral over both the apparent and true electron energies of the differential cross-section in eq. (17)

weighted by the product of efficiency and resolution functions. The integration runs from the experimentally required minimum energy T_{min} to the kinetically allowed maximum T_{max}:

$$\langle\sigma(\nu e)\rangle = \int_{T_{min}}^{T_{max}}\int_{T_{min}}^{T_{max}} dT_a dT_t f(T_a)\rho(T_a,T_t)\langle\frac{d\sigma(T_t)}{dT_t}\rangle \ . \qquad (20)$$

It follows from eqs. (17) and (20) that $\langle\sigma(\nu e)\rangle$ is bounded from above by the cross-section for pure electron-neutrino scattering, and from below by that for pure muon-neutrino scattering:

$$\sigma(\nu_\mu e) \le \langle\sigma(\nu e)\rangle \le \sigma(\nu_e e) \ . \qquad (21)$$

A signal below this lower bound would indicate that the oscillation must take place into a sterile neutrino.

The effect of detector efficiency and resolution upon the magnitude of the cross-section in the standard solar model varies with the minimum energy cut T_{min}: they tend to increase the cross-section when the cut is large, and decrease it when the cut is relatively low. For example, when $T_{min} = 9.3MeV$, the cross-section is 0.014 SNU with perfect efficiency and resolution and 0.02 SNU with the actual properties given in eqs. (18,19) above. By contrast, for $T_{min} = 7.5MeV$, the cross-section is 0.042 SNU in the perfect case and 0.037 SNU in the actual one. Here SNU stands for one solar neutrino-electron scattering event per 10^{36} electrons per second. Given a fiducial volume of approximately 680 tons for the KII detector, these cross- sections correspond to event-rates of 0.27, 0.38, 0.82, and 0.72 events per day respectively.

We can understand the proportionately large effect for the higher energy cut on the basis of the behavior of the basic cross-section and the energy spectrum of the ^8B solar neutrinos[19]. The cross-section increases linearly with energy while the spectrum behaves approximately quadratically, rising to a maximum in the vicinity of 6.5 MeV and then declining to zero near the end-point of 14 MeV. Now the resolution function spreads an apparent energy over a range of roughly 3-4 MeV and, at the higher cut it tends to bring in many more neutrinos with slightly lower energies and cross-sections; the net effect is to give a significantly larger signal. As the cut moves back towards the peak of the spectrum, the reverse tends to happen: the resolution spreading tends to bring in fewer neutrinos, but with higher energies and cross-sections, and the overall effect is a slightly smaller signal.

The results of our new calculations are shown in Figure (1) where we plot the MSW predictions for the cross-section as a function of Δm^2 for a series of values of $\sin^2 2\theta$. We express the cross-section as a fraction of the cross-section predicted by the standard solar model for the same minimum energy cut $T_{\min}$ and the same efficiency and resolution functions as given above, but with no oscillations. Two energy cuts are considered: $T_{\min} = 9.3$ and 7.5 MeV.

Imposing the restrictions corresponding to the MSW solutions of the ^{37}Cl experiment. we find that the results for the *fraction* of MSW to standard model cross-section are very little changed from the case when efficiency and resolution were not taken into account. The adiabatic solution still implies that the fraction R must be less than 1/3 and the large angle one restricts it to 2/3 or less. For the nonadiabatic solution, however, R tends to be slightly larger than before: whereas in the original calculation R hovered between 0.5 and about 10% *below* it, R now hovers between 0.5 and 10% *above* it.

The value R = 0.5 in the case of the nonadiabatic solution and its relative constancy are not difficult to understand. Solar neutrino-electron scattering in the KII experiment is sensitive to neutrinos in the higher energy half of the spectrum, a region for which the nonadiabatic solution gives about a 40% survival probability to electron-type neutrinos[13]; thus 40% of the neutrinos will scatter with the maximal electron-type neutrino cross-section, while the other 60% will scatter with the neutral-current cross-section which is a factor of 6 smaller. Simple arithmetic then yields a value of 0.5 for R.

At the time of writing, the KII collaboration has published the results of 450 days of observation between January, 1987 and May, 1988, an interval which overlaps with the period during which the ^{37}Cl experiment has been yielding a signal[7] of about 4 SNU rather than the overall average of 2.1 SNU. The value of R that they obtain, namely[8]

$$R = 0.46 \pm 0.13(stat) \pm 0.08(syst), \tag{22}$$

tends to favor the nonadiabatic solution, but the errors are sufficiently large that it does not exclude the adiabatic one. In addition, the result is also consistent with the large angle solution. As more data is accumulated, one may hope that the central value will remain unchanged, while the errors shrink down to a level where one can make a more definite statement about the adiabatic solution; however to distinguish between the other solutions one must turn to other experiments, especially the ^{71}Ga

one.

In so far as the large angle solution is concerned, the conversion of electron-type neutrinos into other types is independent of energy and it predicts that the signal for the ^{71}Ga experiment, which is predominantly sensitive to the low energy pp solar neutrinos, will be suppressed by the same factor of 1/4 - 1/2 as in the ^{37}Cl one. The nonadiabatic solution, on the other hand, yields a much stronger conversion for low energy neutrinos than it does for high energy ones and so it can yield a much greater suppression of the ^{71}Ga; in fact, the signal could be as low as 10 or 20 SNU[13] instead of the 130 SNU predicted by the standard model. This is illustrated in Figures (2a, 2b and 4e) of a recent paper of Bahcall and Haxton[18]. We look forward to the results of this experiment with great anticipation.

5 CONCLUSION: THE WIN-WIN SOLUTION

We also look forward to a new *reactor* experiment recently proposed by Felix and his colleagues[20]. It will be performed with a kiloton detector to be located in a railway tunnel 13 kilometers from the Goesgen reactor, and it will probe the region of parameter space corresponding to large mixing angles and Δm^2 of $10^{-4}eV^2$.

If Felix and colleagues find an effect, then we shall be blessed with a major discovery of profound importance for physics beyond the standard model. If they find no effect, then perhaps they will be confirming, indirectly, the scenario preferred by this author.

Happy Birthday Felix, and please give us the answer when we come to celebrate your 70th !

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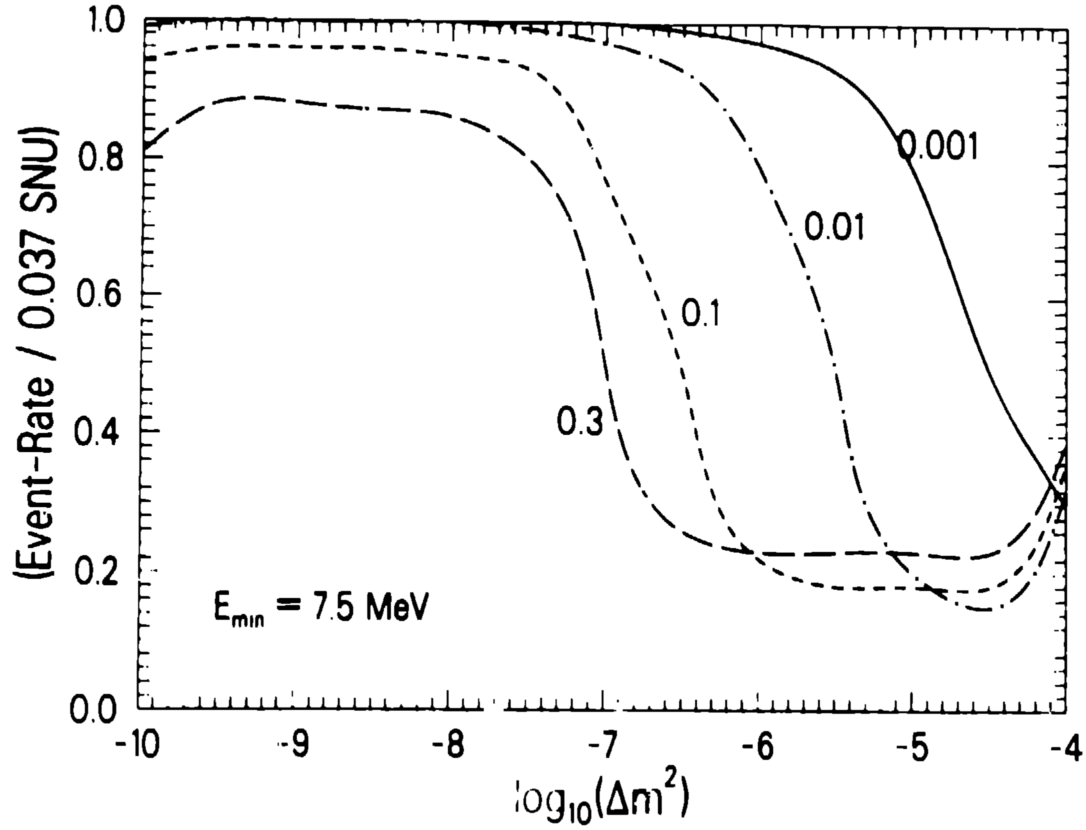
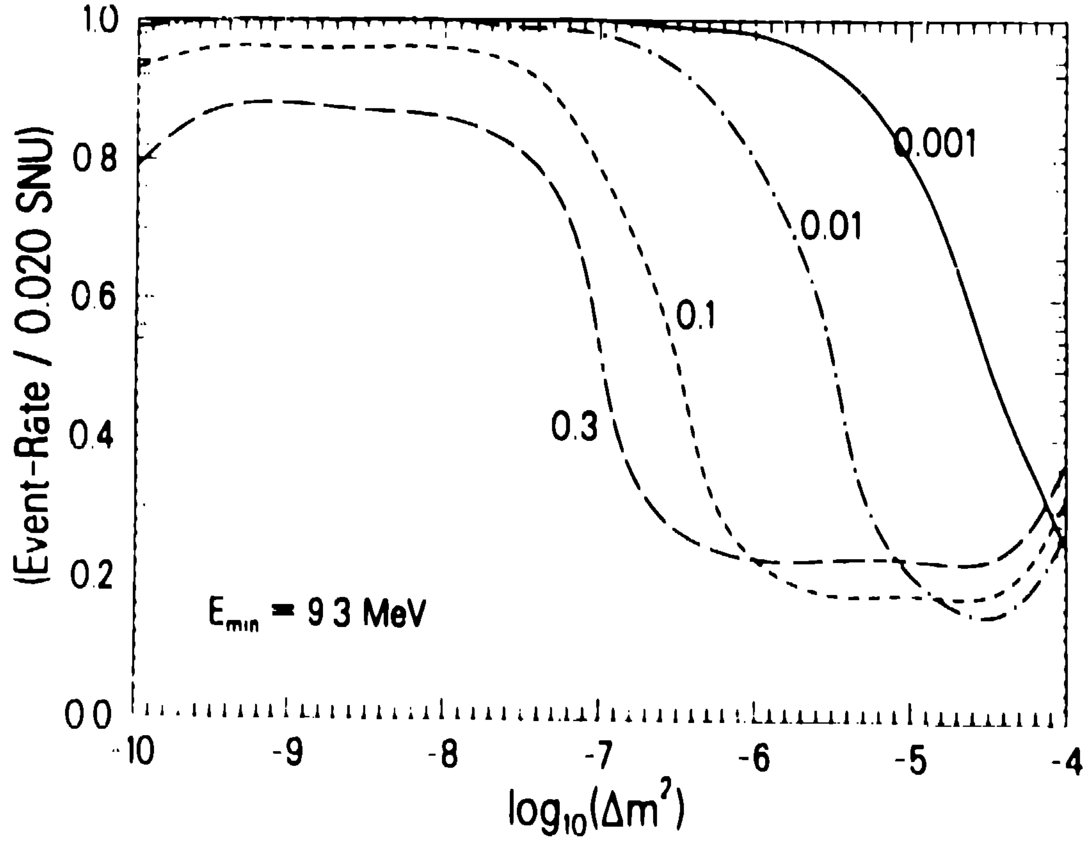
7 FIGURE CAPTIONS

Fig. 1 The ratio of MSW to standard model predictions for solar neutrino electron scattering taking into account detector efficiency and resolution for two different minimum electron energy cuts. Each curve corresponds to the value of $\sin^2 2\theta$ indicated beside it.


1.0
0.8
0.6
0.4
0.2
0.0
(Event-Rate / 0.037 SNU)
0.001
0.01
0.1
0.3
E_{min} = 7.5 MeV
-10
-9
-8
-7
-6
-5
-4
$\log_{10}(\Delta m^2)$


1.0
0.8
0.6
0.4
0.2
0.0
(Event-Rate / 0.020 SNU)
0.001
0.01
0.1
0.3
E_{min} = 9.3 MeV
-10
-9
-8
-7
-6
-5
-4
$\log_{10}(\Delta m^2)$