

LEGIBILITY NOTICE

A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

50-2786

SEP 07 1990

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-16

LA-UR--90-2776

DE90 016493

TITLE TRANSVERSE MOMENTUM DISTRIBUTIONS OF HADRONS

AUTHOR(S) Barbara Jacak

SUBMITTED TO Quark Matter 90, Menton, France, May 7-11, 1990

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

Los Alamos Los Alamos National Laboratory Los Alamos, New Mexico 87545

FORM NO. 100-104
MAY 1962 EDITION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED



TRANSVERSE MOMENTUM DISTRIBUTIONS OF HADRONS

BARBARA JACAK

Los Alamos National Laboratory

I. INTRODUCTION

The study of hadron production in heavy ion collisions is essential to the search for effects beyond independent nucleon-nucleon collisions, for example the predicted phase transition to quark matter.¹ Hadron distributions are known over a large range of transverse momenta for p-p collisions, so a careful study of the differences can be made.

The transverse momentum distributions of hadrons may provide global information about p-nucleus and nucleus-nucleus collisions, such as the degree of thermalization achieved,² and perhaps provide evidence for collective expansion of the highly excited central region.³⁻⁵ Comparison of the p_t and transverse mass, m_t , distributions of different hadronic species are crucial to extract this kind of information. Hadronic p_t spectra show effects of the collision dynamics, such as hard scattering processes,⁶ and possibly rescattering of partons⁷ as well as of the formed hadrons.⁸ Such modifications have been observed in p-nucleus collisions,⁹ and can be expected to be important in nucleus-nucleus reactions. The spectral shape changes arising in this manner cause a background in efforts to extract global information from hadronic p_t spectra.

Lastly, there is an excess of pions observed at low p_t in p-A and A-A collisions.^{10,11} The origin of these soft pions is not yet well understood. The phenomenon represents a major difference between p-p and nuclear collisions.

This paper is organized as follows: Section II shows measured hadron p_t and m_t distributions and compares to expectations from thermal emission. Section III introduces a search for collective flow from these distributions. Section IV reviews processes that affect high p_t hadron production. Section V shows the dependence of average p_t as a function of charged particle multiplicity or E_t . Section VI discusses the low p_t enhancement in pion cross sections. Section VII presents some conclusions.

II. THERMAL PICTURE OF HADRON DISTRIBUTIONS

Figure 1 shows the p_t spectra, $d\sigma/dp_t^2$, of negative particles measured by the HELIOS collaboration in 200 GeV/A p+W, O+W and S+W collisions. The events were selected via a cut on the transverse energy measured in the HELIOS calorimeters and represent central collisions (about 25 % of the geometrical cross section). The particles, predominantly pions, were measured in the External Spectrometer in the rapidity range $1 < y < 1.9$. The solid line is a parameterization of the pion p_t spectrum in minimum bias p-p collisions at the same \sqrt{s} .¹² The pion p_t spectrum is clearly quite different in p-p than in p-A and A-A, and cannot be described by a single exponential. The local inverse slope for S+W data is 210 MeV/c for $0.5 < p_t < 1.5$ GeV/c, and 85 MeV/c for $0.075 < p_t < 0.25$ GeV/c. p-Nucleus

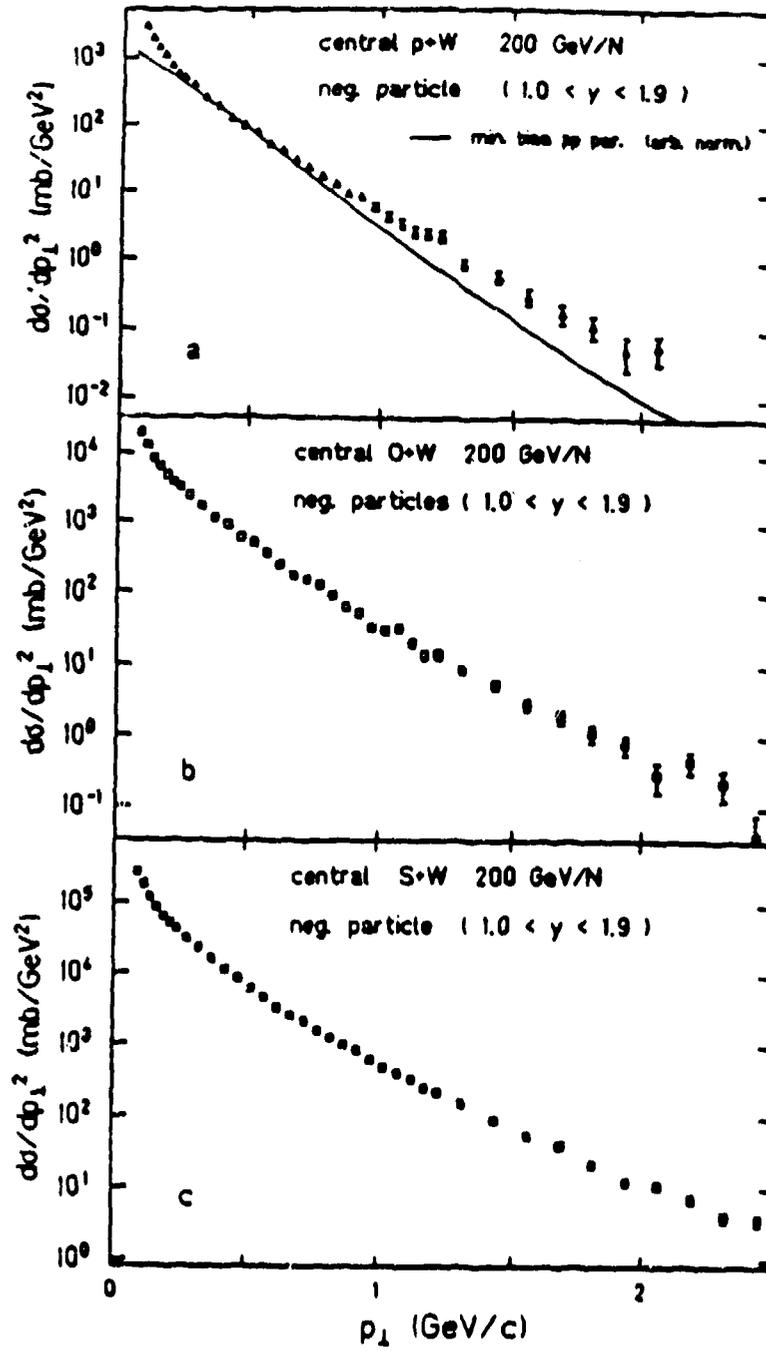


Fig. 1. $d\sigma/dp_1^2$ for central collisions of p, O and S + W at 200 GeV/A [11].

collisions result in excess pion production at $p_t > 1$ GeV/c, first observed by Cronin et al.⁹ From figures 1b and 1c, it is clear that the spectra from A-A collisions closely resemble those from p-A.

There is also a significant rise in the cross section at low p_t , visible in both p-A and A-A collisions. This has been previously reported at more central rapidities in heavy-ion collisions at CERN,¹⁰ in p-A collisions at Fermilab,¹³ for cosmic ray data,¹⁴ and in heavy ion collisions at Bevalac energies.¹⁵ This rise will be discussed in more detail below.

The particle distributions can also be plotted as a function of transverse mass,

$$m_t = \sqrt{m^2 + p_t^2} .$$

The invariant cross section is $dN/dp_t^2 = dN/dm_t^2$. Purely thermal radiation produces particle spectra which, if plotted logarithmically against m_t , fall almost on a straight line, with a very slight convex modulation. Emission of particles from a system varying in temperature, such as a system undergoing cooling, would be better described as a straight line in $1/m_t^{3/2} dN/dm_t$ vs m_t .⁵ These expectations of spectral shapes from purely thermal hadron emission have been compared to the distributions of π , K, and p in p-p collisions at $\sqrt{s} = 23$ GeV.¹² It is important to note that the p_t distributions look approximately thermal up to $p_t = 1 - 1.5$ GeV/c, above which a power law dependence is observed. This shift in spectral shape is expected from the onset of hard scattering processes, and limits the p_t or m_t range in which we may compare to expectations from thermal hadron emission. The distributions for the various hadrons have approximately the same slope in m_t , and have been described as obeying " m_t scaling." The data do not quite fall upon a universal curve; the cross sections vary by factors of 2-5 from each other. There has been some controversy on whether m_t scaling means that distributions form a universal curve or whether it merely implies that different mass hadrons have the same m_t slope.

Charged particle spectra have also been studied in $\bar{p} - p$ collisions at $\sqrt{s} = 200$ GeV - 1.8 TeV.^{16,17} Again, the hadrons of different masses show m_t spectra with very similar slopes.

Figure 2 shows $1/m_t^{3/2} dN/dm_t$ vs m_t for π , K, p and Δ measured by the NA35 collaboration in 200 GeV/A O + Au and S + S collisions.¹⁸ The extracted inverse slopes are in the range 180 to 220 MeV. The curves are linear, with the exception of the soft pions as shown above, and are very nearly parallel. For S+S the proton m_t spectrum is somewhat flatter than that of the other particles. Figure 3 shows the m_t spectra for π , K and p measured in 14.6 GeV/A Si + Au collisions by the E802 collaboration.¹⁹ At this energy there is an even more striking difference in the proton distribution from that of the pions: the proton inverse slope is 190 MeV, compared to 130 MeV for π . Though the pion and kaon distributions might be consistent with thermal emission, the protons clearly are not. This is not surprising as the data are measured in the rapidity range $y = 1.2-1.4$ for an asymmetric system. At this rapidity one would expect a significant contribution from protons emitted from the target remnant heated by rescattering of the produced particles. Even though there is complete stopping of the projectile by the target nucleus at these energies, the fireball does not include the entire target, so the observed spectra should include protons from the target spectator as well as from the fireball. This has been demonstrated in the

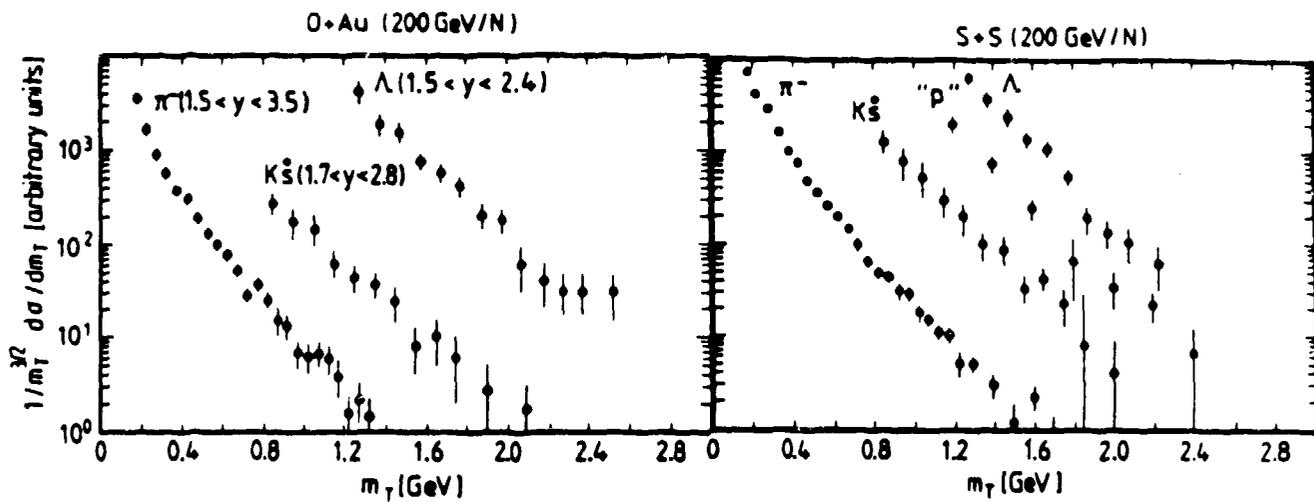


Fig. 2. Distributions of m_T for π , K^0 , p and λ in O+Au and S+S collisions at 200 GeV/A [18].

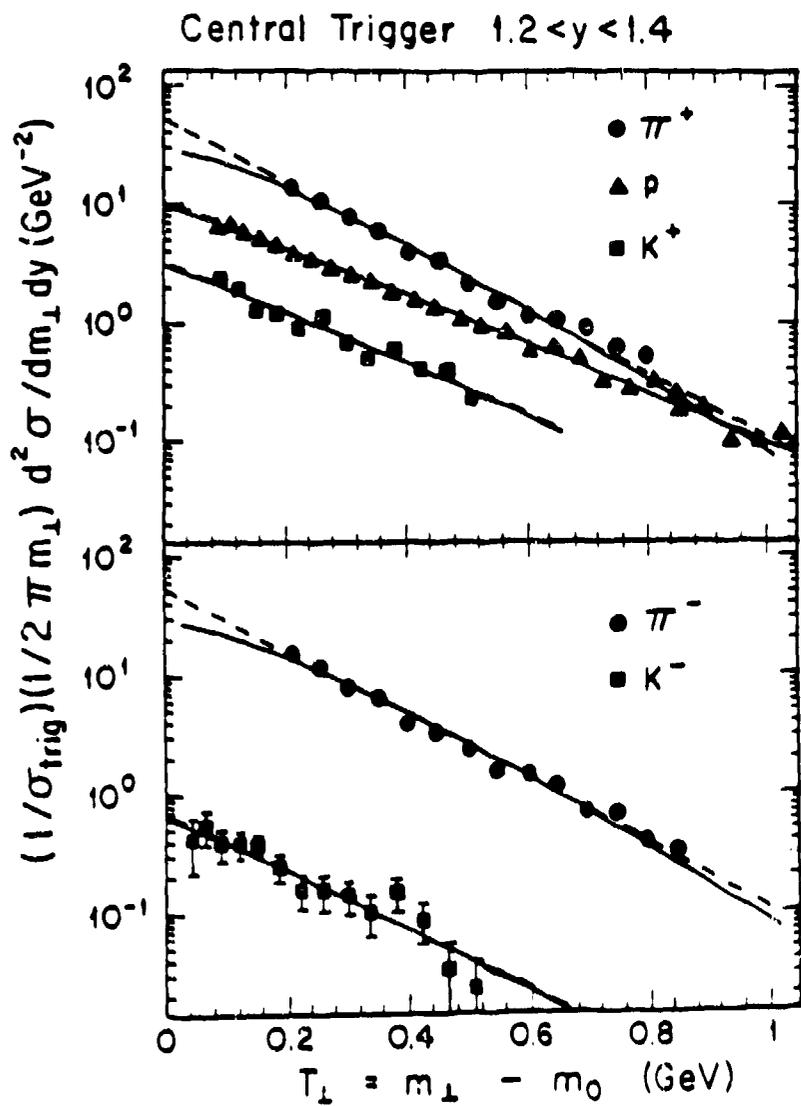


Fig. 3. Distributions of m_T for π , K , and p in central Si+Au at 14.6 GeV/A [19].

p-nucleus data from E802. They find that the proton m_t spectra are very similar to pions in p+Be collisions, but are flatter in p+Au.²¹ Kaon production is probably also influenced by rescattering with the baryons in the fireball and the target remnant²⁰⁻²² and should not be purely thermal.

Consequently, comparison to thermal models may only be made in an appropriate p_t or m_t and rapidity region, which is free of hadrons formed by other processes. At bombarding energies of 15 GeV/A this is difficult due to the small rapidity difference between the projectile and the target. At higher energies, the rapidity difference might be adequate to separate the central and target regions, but careful selection of the m_t range is necessary to exclude other processes. Rescattering effects should not be important in p-p collisions, but it is not clear that a thermal picture of hadron production in p-p collisions is sensible.

III. COLLECTIVE FLOW EFFECTS

If the hadrons are emitted from a fully thermalized system, deviations from exponential m_t distributions would indicate the presence of collective effects, such as transverse flow.^{3,5} Transverse flow would be evident upon comparing m_t distributions of different hadrons because a collective flow velocity increases the hadron momenta as a function of the hadron mass, while the average thermal momentum is independent of hadronic mass for a fixed temperature source.⁵

Lee, Heinz and Schnedermann have performed calculations of the hadron spectra expected in such a scenario, and compared with the data discussed above.⁵ Their model begins with a fireball in thermal and chemical equilibrium, with zero strangeness and initial energy and baryon densities from geometrical considerations. Using the equation of state of either a hadron resonance gas or a gas of free quarks and gluons, the isentropic expansion is calculated. The hadron p_t (or m_t) spectra arise from the local thermal distribution folded with the freezeout surface. The freezeout time is calculated by comparison of the scattering time and expansion time at each point in the system. A parabolic velocity profile of the surface expansion is used.

The resulting spectra of $1/m_t^{3/2} dN/dm_t$ vs m_t are compared with the data from S+S collisions in Figure 4. The m_t spectra are normalized to each other at m_0 of each particle. The curves then fall on top of one another, showing a universal slope vs m_t distribution. Pions show additional curvature at low m_t , where the other particles can be approximated by an exponential. Fitting the entire curve, including the curvature at low and high m_t , Lee, Heinz and Schnedermann infer a radial flow velocity of 0.43 c in a system with initial energy density of 1 GeV/fm³, temperature of 161 MeV and final temperature of 112 MeV. The flow velocity is a very sensitive function of the curvature in the spectrum, and if the curvature at low m_t is accounted for by different processes, the extracted flow velocity is much smaller.²³

IV. HIGH P_t HADRON PRODUCTION

The increase in pion production at high p_t is observed in both p-A and A-A collisions and can be associated with the 'anomalous enhancement' of high p_t hadron production in p-nucleus interactions, discovered by Cronin, et al.¹⁹ They found that the inclusive cross section can be parameterized by a power law dependence on the target mass number A,

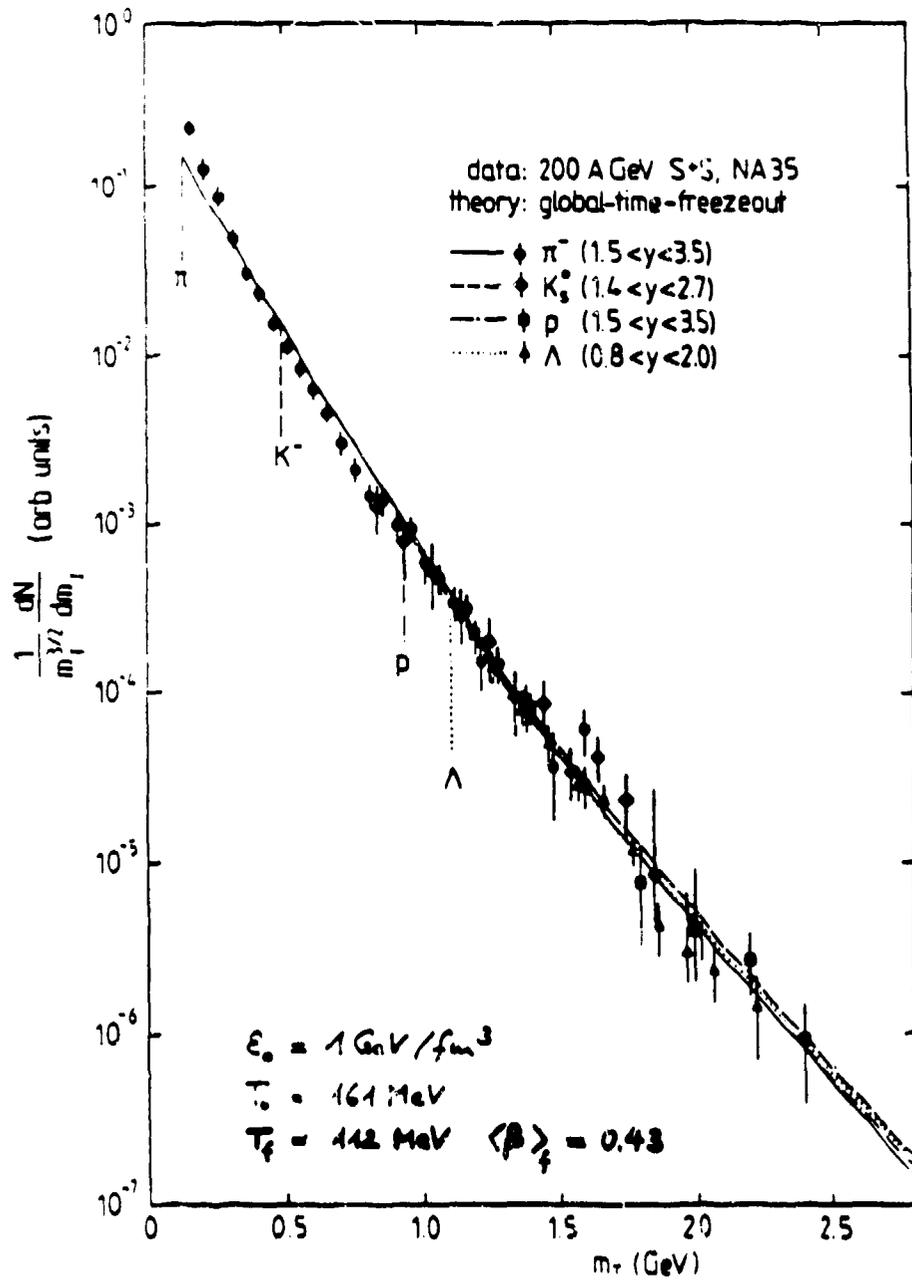


Fig. 4. Global fit [5] with a thermal model including radial collective flow to the distributions of π , K, p and λ in 200 GeV/A S+S data from ref. 18.

$d\sigma/dp_t^2 \sim A^{n(p_t)}$. The high p_t excess in nucleus-nucleus collisions was parameterized by HELIOS in terms of both the target mass, A and projectile mass B, as $(AB)^{n(p_t)}$. This was interpreted as a dependence of the Cronin effect on the projectile analogous to that on the target. It has been suggested that the Cronin effect is due to multiple scattering of the partons before hadronization.⁷ The effect can also be reproduced by rescattering of the formed pions with the target spectator nucleons.⁸

At $p_t > 1.5$ GeV/c, the hadron spectra continue to follow a power law distribution. This is shown in Figure 5 for $\bar{p} - p$ collisions at $\sqrt{s} = 1.8$ TeV, measured at FNAL by Alexopoulos, et al.¹⁷ The same kind of spectral shape was observed in nucleus-nucleus collisions by WA80 at CERN,²⁴ via a measurement of photons from the decay of π^0 's. The power law shape at large p_t , above the Cronin effect, is usually attributed to the onset of hard scattering processes.

It is clear that a search for global collision properties from the hadron p_t distributions can be very misleading if one looks in regions of p_t that are influenced by other factors. Such searches should be confined to the approximately exponential section of the spectra, corresponding to $p_t \sim .4 - 1.5$ GeV/c. The proton spectra are probably even more complicated due to rescattering effects.

V. $\langle P_t \rangle$ AS A FUNCTION OF ENERGY DENSITY

The dependence of the transverse momenta of hadrons on multiplicity or E_t in heavy ion collisions indicates the dependence of the temperature of the system, if it is thermalized, on the energy density. Naively, one would expect the temperature to rise linearly with the energy density in a pure hadron gas. If a phase transition were to take place, one would expect an initial rise, then a plateau at the region of phase coexistence, followed by a second rise due to the pure quark phase. Such an effect was predicted to be one of the first experimentally accessible signals of a phase transition.²⁵

Experimentally, such studies require the assumption that $\langle p_t \rangle$ of pions reflects the system temperature, and that increasing charged particle multiplicity, or transverse energy produced in the collision signifies increasing energy density. Figure 6 shows the distributions of π , K, and \bar{p} $\langle p_t \rangle$ vs charged particle multiplicity for $\bar{p} - p$ collisions at $\sqrt{s} = 1.8$ TeV.¹⁷ The calculation of $\langle p_t \rangle$ was made for particles with $p_t < 1.5$ GeV/c, so the values should not be influenced by hard scattering. For pions $\langle p_t \rangle$ initially increases with N_c then stays constant. No second rise is observed. Kaons behave very similarly to the pions; the plateau begins at the same N_c as for pions, though it appears to be somewhat less flat. $\langle P_t \rangle$ for \bar{p} rises linearly with N_c , over the entire range. It is interesting to note that the plateau values are about 375 MeV/c for pions, 525 for kaons and $\langle p_t \rangle \sim 625$ for \bar{p} . The hydrodynamical model, including a phase transition, of Kataja, et al. predicts values of 350-400 MeV/c for pions, 600-700 for kaons and 800-900 for protons.⁴ However, simple models for string fragmentation, which have nothing to do with a thermal system, also predict increasing values of $\langle p_t \rangle$ for heavier hadrons. In e^+e^- collisions, the $\langle p_t \rangle$ of pions inside jets above 10 GeV is also in the 300-400 MeV region, indicating that the value of $\langle p_t \rangle$ is not very dependent on the system giving rise to the pions.

Figure 7 shows the dependence of $\langle p_t \rangle$ on charged particle multiplicity for heavy ion collisions, measured by WA80²⁴ for 200 GeV/A O and S collisions. An initial rise of

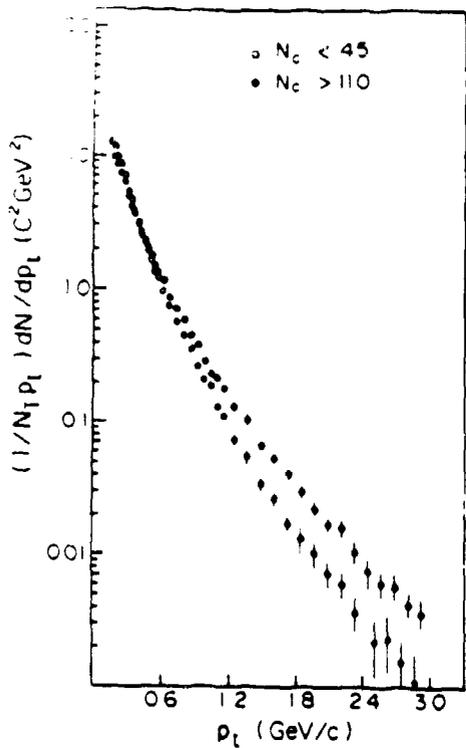
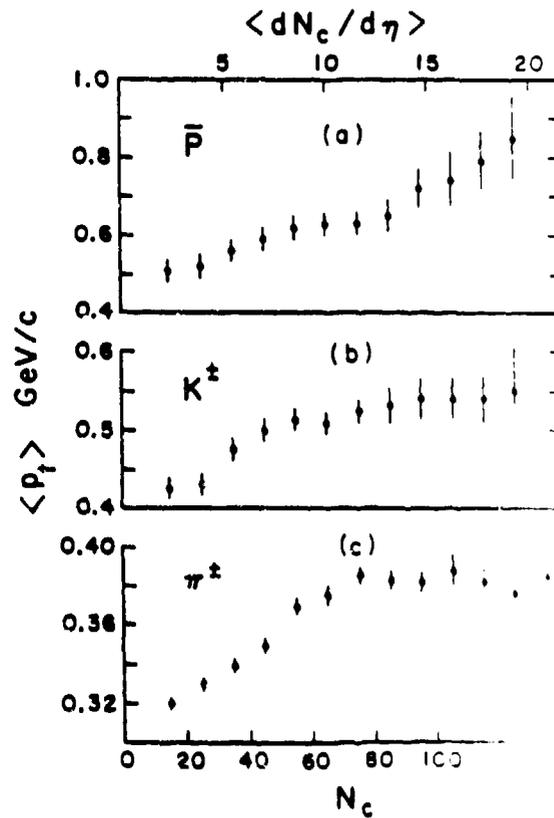


Fig. 5. The normalized single charged particle dN/dp_t^2 distribution for two different multiplicity regions in $\bar{p} - p$ collisions at $\sqrt{s} = 1.8$ TeV [17].

Fig. 6. Plots of $\langle p_t \rangle$ vs. $dN/d\eta$ for \bar{p} , K and π , for particles with $p_t < 1.5$ GeV/c [17].



$\langle p_t \rangle$, followed by a flattening is also seen. HELIOS has measured $\langle p_t \rangle$ as a function of E_t to large values of E_t ,¹¹ and observed a long plateau in pion $\langle p_t \rangle$. However, just as in $\bar{p} - p$ there is no second rise. In heavy ion collisions, it is not completely clear at which point increasing multiplicity or E_t indicates increasing energy density. Presumably this should set in once the impact parameter is small enough for the projectile to fully overlap the target, i.e. at the "knee" of the multiplicity or E_t distribution. The plateau definitely extends beyond this point.

The plateau in $\langle p_t \rangle$ may be interpreted either in hydrodynamic or in microscopic models. Hydrodynamic models including a phase transition produce a constant or relatively constant $\langle p_t \rangle$, as the hydrodynamical expansion effects are not extremely large at energy densities near the critical density.^{4,26} If only longitudinal expansion takes place, then even a pure hadron phase could have $\langle p_t \rangle$ independent of energy density. However, numerical calculations for a pure hadron phase show a transverse expansion,⁴ resulting in a $\langle p_t \rangle$ rise of at least 30% over the energy density range reached in these experiments. Microscopic models can give approximately constant $\langle p_t \rangle$, since the hadronization process masks the increased $\langle p_t \rangle$ of the partons arising from the many parton interactions that lead to high values of E_t . Consequently, the interpretation of the observed constant $\langle p_t \rangle$ remains somewhat ambiguous.

VI. LOW P_t ENHANCEMENT

An excess of soft pion production ($p_t < 300$ MeV/c), compared to p-p collisions is observed at $1 < y < 2$ in p-W, O-W and S-W collisions.¹¹ HELIOS has seen the low p_t excess in both positive and negative pion distributions. Approximately 40-50% of all pions are in this region. The low p_t enhancement is observed in both positive and negative pions. A similar, though somewhat smaller, excess is reported by NA35 in negative particle p_t spectra at $2 < y < 3$.¹⁰ Identification of pions via time-of-flight by HELIOS indicates that this excess is not due to electrons from conversion of π^0 's. It is important to note that this excess is visible in p_t distributions in fixed intervals of rapidity but not in fixed intervals of pseudorapidity. For soft pions, pseudorapidity is not equal to rapidity, but rather, $y < \eta$. A fixed pseudorapidity interval includes low p_t pions only at very low y . The number of soft pions observed is then dominated by the pion rapidity distribution and is very small due to the low pion dN/dy at small y .

An excess of soft pions was also observed in high multiplicity p - p and $\alpha - \alpha$ collisions at the ISR.²⁷ Figure 8 shows the normalized semi-inclusive p_t distributions, dN/dp_t , of charged particles produced in the central rapidity region. The distributions are shown for different bins of the charged particle multiplicity and are normalized to the full inclusive distribution. Both the p - p and $\alpha - \alpha$ results show an excess of low p_t particles in high multiplicity events; this excess is considerably smaller than that in p-A and A-A collisions, however.

A number of theoretical explanations have been proposed for the low p_t excess. Lee and Heinz,³ and Atwater et al.¹⁴ point out that such a modification of the spectral shape could occur due to collective transverse flow. However, one would expect different flow effects in central nucleus-nucleus collisions than in p-A or p-p, so a systematic study of the magnitude of the flow velocity as a function of projectile and impact parameter is

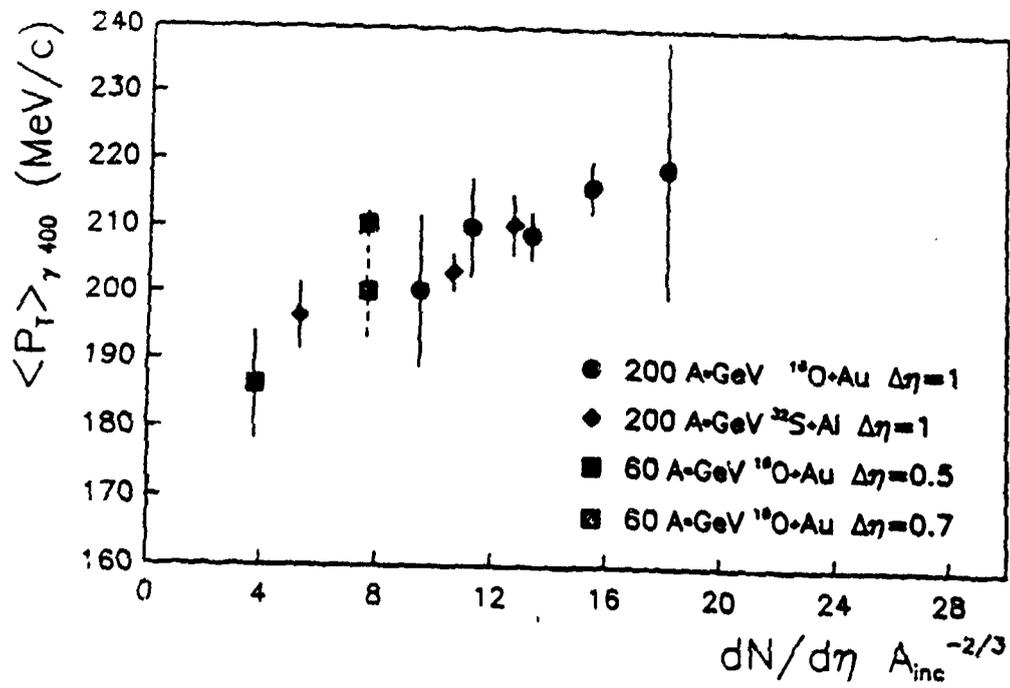


Fig. 7. $\langle p_t \rangle$ vs. multiplicity of photons from π^0 decay in nucleus-nucleus collisions at 200 GeV/A [24].

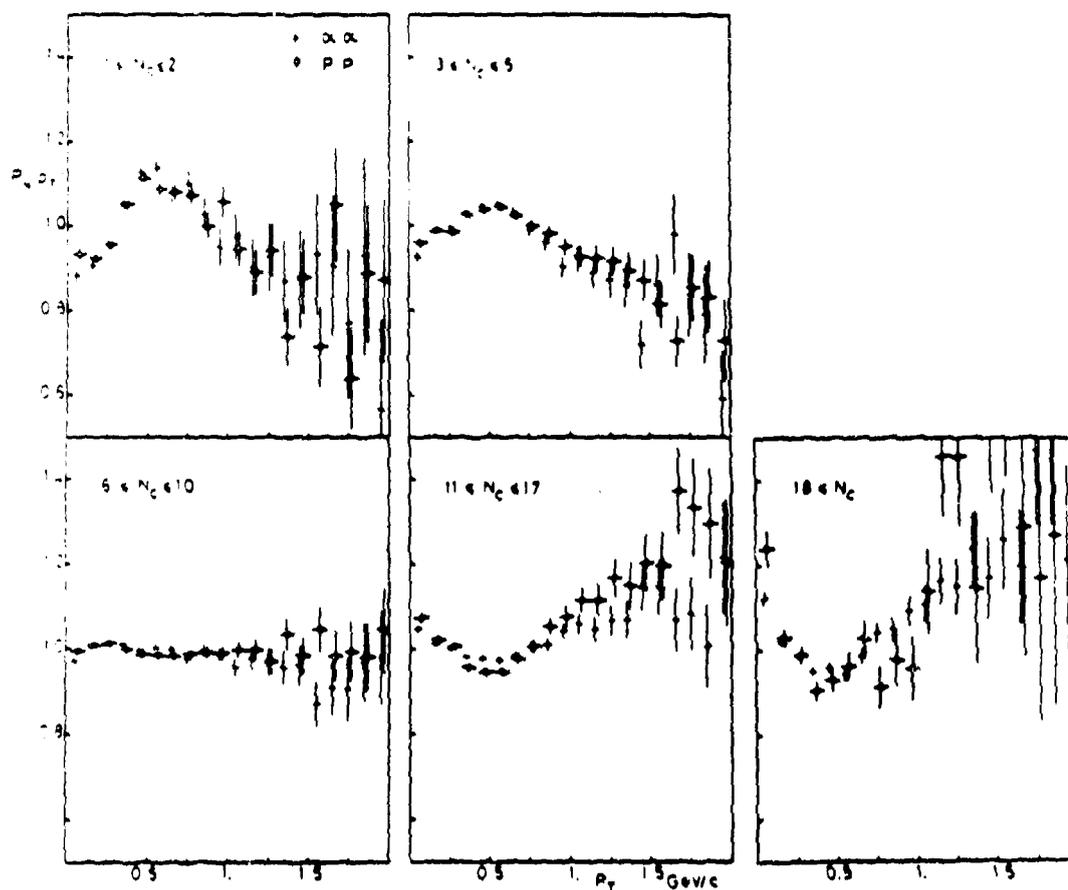


Fig. 8. Normalized ratio of charged particle p_T distributions for different multiplicity bins in p-p and $\alpha - \alpha$ collisions at the ISR [27].

necessary. Though the low p_t excess is smaller in p-p collisions than in nucleus-nucleus, p-W spectra are nearly identical to those in O-W and S-W.¹¹

Shuryak has proposed that the pion dispersion relation could be modified inside hadronic matter, making it more difficult for pions to leave the system.²⁸ The pion interaction becomes attractive, similar to the idea of pion condensation. Such an effect would explain observed soft photon emission in addition to the low p_t enhancement in pion distributions. The pion dispersion relation has been extensively tested for cold nuclear matter at normal density at LAMPF. For example, a search was made for an enhancement in polarization transfer in p-nucleus collisions at 500 MeV.²⁹ Such an enhancement would be expected if there were an attractive potential for pions in nuclear matter; no effect was seen in comparing H, Ca and Pb targets. Consequently, the pion dispersion relation would need to have a very strong temperature dependence to explain the low p_t excess. In addition, a hot and dense system would have to be formed in p-W collisions, where a large excess is observed.

Formation of small droplets of cold quark matter, which decay into many pions, has been proposed by Van Hove.³⁰ Such an effect would be largest at midrapidity, but the data imply that the excess is dominantly a target rapidity effect.^{11,31} Formation of pions by resonance decays such as ρ , N^* , and Δ could give rise to soft pions. The N^* and Δ effects would be largest at the target rapidity as that is the region with the most baryons. Though the Δ channel also absorbs pions, cascading in the target nucleus might produce enough N^* resonances to account for the observed pions. Clearly this mechanism would yield soft pions near the target rapidity, in agreement with the data. Detailed modeling needs to be done to test this hypothesis quantitatively.

VII. CONCLUSIONS

Hadron p_t spectra are influenced by various processes, which must be taken into account before drawing conclusions about global properties of nucleus-nucleus collisions. Hard scattering effects at $p_t > 2$ GeV/c and the Cronin effect above $p_t \sim 1.5$ GeV/c limit the p_t range which might be thermal. The excess of pions at low p_t , most likely from a non-thermal source, requires a low p_t threshold. This leaves a p_t range of .4 - 1.5 GeV/c where one might look for thermal and collective flow effects. M_t scaling holds approximately in this region, but this is true for minimum bias p-p as well as nucleus-nucleus collisions. Thermal interpretation of the spectra must explain formation of a thermalized system in this case. In the nucleus-nucleus collisions studied to date, baryon distributions show rescattering effects arising from target spectators. Only in central collisions of symmetric systems at large bombarding energies can the baryon distributions be used to look for evidence of thermalization. If we were convinced that thermalization were achieved in the central region, then the multiplicity (or E_t) dependence of $\langle p_t \rangle$ would indicate that a mixed phase has been reached. Unfortunately, the interpretation of the results at this point remains ambiguous.

The excess pion production at low p_t is not yet fully understood, but the data imply that it is an effect dominated by the target region. Formation of baryon resonances heavier than the Δ may provide the explanation for these pions.

ACKNOWLEDGEMENT

I would like to acknowledge many fruitful discussions with H. Van Hecke, A. Drees, M. Neubert, H. Specht, J. Moss, G. Baym and Y. Yariv.

REFERENCES

1. See Proceeding of previous Quark Matter conferences, i.e. Nordkirchen (1987) and Lenox (1988).
2. K. Redlich and H. Satz, Phys. Rev. D33, 3747 (1986).
3. K.S. Lee and U. Heinz, Univ. Regensburg preprint TPR-88-16 (1988).
4. M. Kataja, P.V. Ruuskanen, L. McLerran and H. von Gersdorff, Phys. Rev. D34, 2755 (1986).
5. K.S. Lee, U. Heinz, and E. Schnedermann, Univ. Regensburg preprint TPR-90-18 (1990).
K.S. Lee, these proceedings.
6. M.A. Faessler, Phys. Rep. 115, 1 (1984).
7. M. Lev and B. Petersson, Z. Phys. C38, 165 (1988).
8. M. Neubert, G. Baym, G. Friedman, B.V. Jacak and Y. Yariv, to be published.
9. J.W. Cronin et al., Phys. Rev. D11, 3105 (1975).
D. Antreasyan et al., Phys. Rev. D19, 764 (1979).
10. H. Stroebele, et al. (NA35 collaborations), Z. Phys. C38 89 (1988).
11. T. Akesson et al., Z. Physik C46, 361 (1990).
12. B. Alper, et al., Nucl. Phys. B100, 237 (1975).
13. D.A. Garbutt et al., Phys. Lett. 67B, 355 (1977).
14. T.W. Atwater, P.S. Freier and J.I. Kapusta, Phys. Lett. 199B,30 (1987).
15. R. Brockmann et al., Phys. Rev. Lett. 53, 2012 (1984).
16. G.J. Alner, et al. (UA5 collaboration), Nucl. Phys. B258, 505 (1985).
17. T. Alexopoulos, et al., Phys. Rev. Lett. 64, 991 (1990).
18. J.W. Harris, et al. (NA35 collaboration), Nucl. Phys. A498, 133c (1989).
H. Stroebele et al. (NA35 collaboration), in: "Hadronic Matter in Collision", ed. by P. Carruthers and J. Rafelski (World Scientific, Singapore, 1989), p. 357.
H. Stroebele et al. (NA35 collaboration), in: "The Nuclear Equation of State", NATO ASI Series B, ed. by W. Greiner (Plenum, New York, 1990), in press.
19. T. Abbott et al., Phys Rev. Lett. 64, 847 (1990).
20. J. Barrette, et al. (E814 collaboration), Phys. Rev. Lett 64, 1219 (1990).

21. Y. Miake, these proceedings.
22. R. Mattiello, H. Sorge, H. Stoecker and W. Greiner, Phys. Rev. Lett. 63, 1459 (1989).
23. U. Heinz, private communication.
24. R. Albrecht, et al. (WA80 collaboration), Z. Phys. C
25. L. van Hove, Z. Phys. C27 ,135 (1985).
26. E.V. Shuryak, Z. Phys. C38, 165 (1988).
27. W. Bell, et al., Z. Phys. C27, 191 (1985).
28. E. V. Shuryak, in L. Van Hove Festschrift (World Scientific Publishing, Singapore 1989).
and see also these proceedings.
29. L. Rees, et al., Phys. Rev. C34, 627 (1986).
30. L. Van Hove, CERN preprint, TH-5236/88 (1988).
31. H. Stroebele, private communication.