CONF-921206--1

LA-UR- 93-0289

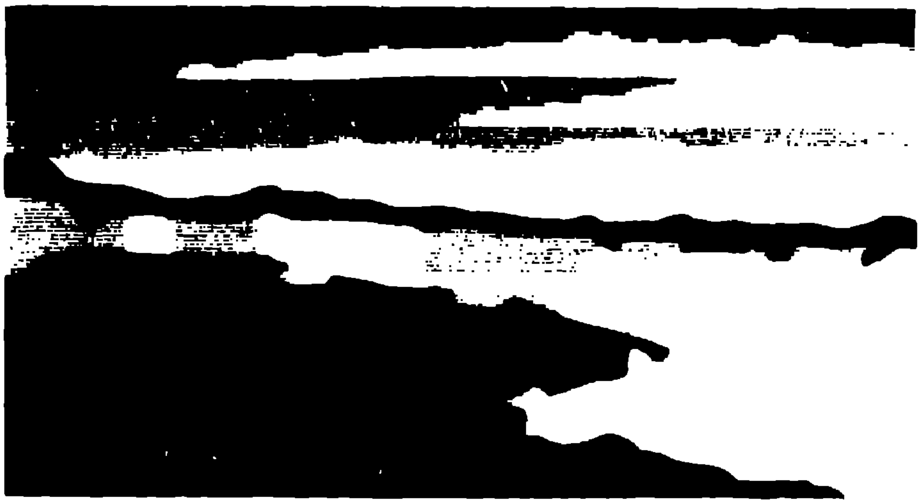
Title: SIMULTANEOUS DETERMINATION OF GAIN PARAMETERS IN STEADY STATE LASERS

LA-UR--93-289

DE93 007389

Author(s): C. C. Jensen, CLS-6
David A. Branning, CLS-6
Richard D. Marchbanks, CLS-6

Submitted to: Proceedings of the International Conference on Lasers '92, Houston, TX, 12/7/92



Los Alamos
NATIONAL LABORATORY

SIMULTANEOUS DETERMINATION OF GAIN PARAMETERS IN STEADY-STATE LASERS

Craig C. Jensen, David Branning & Richard Marchbanks

Advanced Laser and Systems Technology
MS J564
Los Alamos National Laboratory
Los Alamos, NM 87545

Abstract

The optical gain parameters of pulsed steady-state lasers, such as dye and excimer lasers, are determined from a single set of experiments from either a laser oscillator or an amplifier. The gain parameters determined by this technique are the small-signal gain coefficient, the saturation intensity for stimulated emission, the saturation fluence for stimulated emission, and the non-saturable absorption coefficient. The measured saturation properties of laser amplifiers and oscillators are used in a data fitting routine that finds all of the gain parameters simultaneously. The values of the gain parameters are determined with greater accuracy than by the conventional methods of measuring each of the parameters individually. The knowledge of the values of all the gain parameters allows for the determination of fluorescence lifetime of a gain medium.

Introduction

It is important to laser researchers to measure the gain parameters of lasers very accurately. Most methods for doing so are associated with questionable approximations. We present an accurate method for determining the gain parameters of pulsed steady-state lasers, such as dye lasers and excimer lasers. Recent work in dye lasers[1-3] demonstrated that accurate values for the spectroscopic constants of laser dyes can be obtained from measurements of optical saturation. A method was developed[1] for fitting the non-linear data, to the coupled differential equations that describe the optical saturation processes, to arrive at values for the spectroscopic constants with greater accuracy and simplicity than by conventional means. We have extended this method to determine all of the gain parameters of a laser gain medium from a single set of data. The data set can be from the measurement of output intensities from a laser amplifier as a function of input laser intensities to the amplifier, or from the measurement of output intensities from a laser oscillator as a function of the reflectivity of the output coupler. The data from either experiment can be fit, with our method, to the equation that describes the measurement whether the equation is analytic or differential. The gain parameters which can be determined are the small signal gain coefficient, the saturation intensity for stimulated emission, the saturation fluence for stimulated emission, and the non-saturable absorption coefficient. From the values of these gain parameters the fluorescence lifetime of the gain medium can be calculated. The fluorescence lifetime is difficult to measure in some lasers such as e beam pumped KrF excimer lasers. The method developed for laser amplifier measurements[1-3] was extended recently to laser oscillators[4].

Data Fitting Model

The gain parameters of a pulsed laser can be determined from: (a) the output intensity of an amplifier as a function of input intensity or, (b) measurements of output intensity from and oscillator as a function of output coupler transmission. The experimental data is fit to the equation that describes the experiment.

Amplifiers

We first consider the case of pulsed steady state amplifiers in which the temporal duration of the input laser pulse to the amplifier is *long* compared to the response time of the gain medium (gain fluorescence lifetime). The amplification of an input laser pulse through an amplifier is described by,

$$\frac{dI}{dx} = \frac{g_0 I}{1 + I / I_{sat}} - \alpha I\,, \tag{1}$$

where I is the intensity (photons/cm^2 sec) of the laser pulse passing through the amplifier, g_0 (cm^{-1}) is the small-signal gain coefficient, I_{sat} (photons/cm^2 sec) is the saturation intensity for stimulated emission, and α (cm^{-1}) is the non-saturable absorption coefficient. The saturation intensity for stimulated emission is defined by,

$$I_{sat} = \frac{1}{\sigma_e \, \tau}. \tag{2}$$

where σ_e (cm^2) is the cross section for stimulated emission and τ (sec) is fluorescence lifetime of the laser gain medium. The performance of KrF excimer laser amplifiers is described by (1). The performance of dye laser amplifiers are described by (1) with $\alpha = 0$, since most dye lasers exhibit no appreciable non-saturable absorption.

Next we consider the case of pulsed steady-state amplifiers in which the temporal duration of the input laser pulse to the amplifier is *short* compared to the response time of the gain medium. The amplification of an input laser pulse through an amplifier is described by[5],

$$\frac{dE}{dx} = g_0 \, E_S \left[1 + e^{-E/E_S} \right] - \alpha E\,. \tag{3}$$

where E is the fluence (photons/cm^2) of the laser pulse passing through the amplifier, and E_S is the saturation fluence for stimulated emission. The saturation fluence for stimulated emission is defined by,

$$E_S = \frac{1}{\sigma_e}. \tag{4}$$

By measuring the gain in a laser amplifier (i.e., KrF excimer) with both long and short temporal pulses (compared to the gain lifetime), the values of I_{sat} and E_S can be determined. The fluorescence lifetime of the gain medium can be calculated from the ratio of (2) and (4).

<u>Oscillators</u>

The light intensity inside the gain region of a one dimensional CW oscillator (Fig. 1) propagates according to the following differential equation:

$$\pm \frac{dI_\pm}{dx} = I_\pm (g - \alpha) \tag{5}$$

where g is the saturated gain of the medium and α is the non saturable absorption. With x as the coordinate for position in the oscillator, increasing toward the output coupler, then I_+ and I_- refer to intensities propagating in the positive and negative x direction, respectively. The saturated gain is related to the small signal gain g_0 via

$$g = \frac{g_0}{1 + I/I_{sat}} \tag{6}$$

where I is the total laser intensity and I_{sat} is the saturation intensity for the gain medium. Writing the total laser intensity I as the sum of I_+ and I_-, substitution of (6) into (5) yields the initial equation for the conventional Rigrod[6] formulation,

$$\frac{1}{I_+}\frac{dI_+}{dx} = -\frac{1}{I_-}\frac{dI_-}{dx} = \frac{g_0}{1 + I_+/I_{sat} + I_-/I_{sat}} - \alpha. \tag{7}$$

It is the integral solution of this propagation equation that ultimately yields the output intensity of the laser, given by

$$I_{out} = T_2(1 - R_2)I_+(L) = T_2(1 - R_2)\int_0^L I_+\left(\frac{g_0}{1 + I_+/I_{sat} + I_-/I_{sat}} - \alpha\right)dx \tag{8}$$

where L is the value of x at the end of the gain region and R_2 is the reflectance of the output coupler. T_2 is the combined transmission of the forward inter-cavity window and the space between this window and the output coupler (see Fig.1).

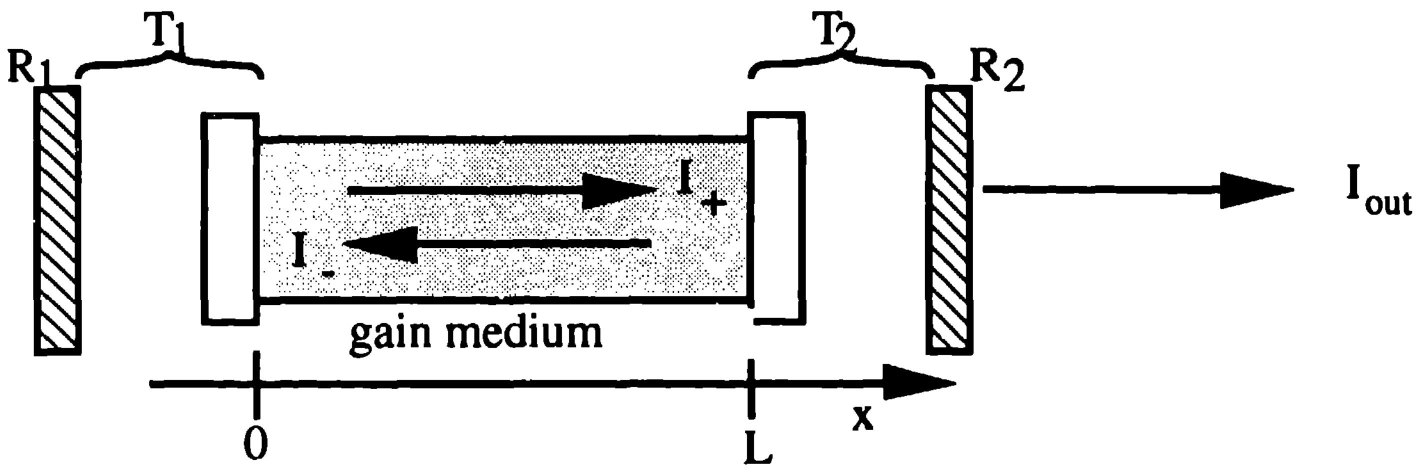


Figure 1. Diagram of a laser oscillator

Using Rigrod's catenary approximation[6] for the sum of the forward and backward propagating intensities in an oscillator, the output intensity is given by,

$$I_{out} \cong I_{sat}\frac{T_2(1 - R_2)\sqrt{R_a}}{(\sqrt{R_a} + \sqrt{R_b})(1 - \sqrt{R_aR_b})}\;\frac{(g_0 - \alpha)L + ln(\sqrt{R_aR_b})}{\left(1 - \frac{\alpha L}{ln(\sqrt{R_aR_b})}\right)}, \tag{9}$$

where R_a and R_b are effective reflectivities for the intensities as seen from the laser volume,

$$R_a = T_1^2R_1 \qquad \text{and} \qquad R_b = T_2^2R_2 \;. \tag{10}$$

For the two parameter oscillator, relating to dye lasers with no non saturable absorption, the output intensity is given by setting $\alpha = 0$,

$$I_{out} = \frac{I_{sat}T_2(1 - R_2)\sqrt{R_a}\;(g_0L + ln(\sqrt{R_aR_b}))}{(\sqrt{R_a} + \sqrt{R_b})(1 - \sqrt{R_aR_b})} \tag{11}$$

The output intensities of laser amplifiers are calculated simply by numerically integrating (1) or (3) depending upon the temporal pulse width. The output intensities of laser oscillators are calculated from (9) or (11) depending upon whether there is non-saturable absorption occurring in the laser gain medium. The gain parameters, g_0, I_{sat}, and α, of the gain medium can be determined from either amplifier experiments, that measure I_{out} vs. I_{in}, or oscillator experiments that measure I_{out} vs. R_2.

The χ^2 Minimization Fitting Technique

The data sets collected from amplifier and oscillator experiments are described by (1), (3), (9) or (11). The gain parameters, g_0, I_{sat}, and α, can be determined from such data sets by fitting the data directly to the relevant equations. The technique for fitting the data directly to the output intensity equations requires finding the values of the parameters that minimize the difference between the measured and calculated output intensities. An initial estimate of the values of the three parameters is used to calculate theoretical data points. The slope of the output intensity equation with respect to each parameter, at each data point, is calculated. The sum of the contributions from the slope and the difference between the measured and calculated output intensities for each data point is used to adjust the values of the parameters. The adjustments to the values of the parameters iterates through the procedure until the difference between the experimental data and the values calculated from the equation is minimized. The output intensity equations can be explicit, implicit or first-order differential equations. The procedure described below is an extension of the method developed by *Jensen*[1] for accurately determining the spectroscopic constants of laser dyes from optical saturation experiments.

We consider the general case of a differential equation that is numerically integrated to give output intensities commensurate with an experiment:

$$f = \int_0^L DE \, dx \tag{12}$$

is the equation used to calculate the data points f_i, that will be compared with corresponding measured values y_i. If the differential equation, DE, depends on the set of parameters, α, β, γ, etc., then we find values for those parameters such that the absolute difference between the theoretical points f_i and the measured points y_i is as small as possible. We can use χ^2 as a measure of this absolute error, as defined by

$$\chi^2 = \sum_i (y_i - f_i)^2. \tag{13}$$

As with many fitting routines, initial guesses must be made for the parameters α, β, etc. to generate the first set of f_i's for the theoretical curve. Then the theoretical curve is compared to the measured values and each of the parameters is changed by an analytically determined amount for the next iteration. In general, the dependence of f on the parameters need not be simple nor linear. We can write a Taylor series that approximates $f(\alpha, \beta, \gamma...)$ as a linear function at *each* data point, i:

$$f_i = f_{0_i} + \frac{\partial f_{0_i}}{\partial \alpha}\Delta\alpha + \frac{\partial f_{0_i}}{\partial \beta}\Delta\beta + \frac{\partial f_{0_i}}{\partial \gamma}\Delta\gamma \tag{14}$$

where the higher order terms in the expansion have been dropped. Here f_{0_i} is the initial value of f_i, obtained with the initial guesses for the parameters via (12), and $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ are small corrections to the values of these parameters. If we visualize the f_i's as points on a surface in a space spanned by α, β, and γ, then (14) is a way of approximating how the points move on this surface over small changes in the parameters. Equation (14) describes how the value f_i moves away from the initial guess f_{0_i} with the assumption that the changes are small enough that the surface nearly "flat" at a tangent point to the parameter surface.

The values for f_i are final when the total error is minimized. This happens when each parameter is adjusted by the correct amount to make χ^2 a minimum, i.e.

$$\frac{\partial\chi^2}{\partial\Delta\alpha} = 0, \quad \frac{\partial\chi^2}{\partial\Delta\beta} = 0, \quad \text{and} \quad \frac{\partial\chi^2}{\partial\Delta\gamma} = 0 \tag{15}$$

with like equations existing for any additional unique parameters determining f_i.

Substitution of (14) into (13) yields an approximate expression for χ^2. Its derivatives can be evaluated to rewrite (15) as

$$\sum_i (y_i - f_{0_i})\frac{\partial f_{0_i}}{\partial\alpha} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\alpha}\right)^2 \Delta\alpha + \sum_i \left(\frac{\partial f_{0_i}}{\partial\beta}\right)\left(\frac{\partial f_{0_i}}{\partial\alpha}\right)\Delta\beta + \sum_i \left(\frac{\partial f_{0_i}}{\partial\gamma}\right)\left(\frac{\partial f_{0_i}}{\partial\alpha}\right)\Delta\gamma \tag{16}$$

$$\sum_i (y_i - f_{0_i})\frac{\partial f_{0_i}}{\partial\beta} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\alpha}\right)\left(\frac{\partial f_{0_i}}{\partial\beta}\right)\Delta\alpha + \sum_i \left(\frac{\partial f_{0_i}}{\partial\beta}\right)^2 \Delta\beta + \sum_i \left(\frac{\partial f_{0_i}}{\partial\gamma}\right)\left(\frac{\partial f_{0_i}}{\partial\beta}\right)\Delta\gamma \tag{17}$$

$$\sum_i (y_i - f_{0_i})\frac{\partial f_{0_i}}{\partial\gamma} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\alpha}\right)\left(\frac{\partial f_{0_i}}{\partial\gamma}\right)\Delta\alpha + \sum_i \left(\frac{\partial f_{0_i}}{\partial\beta}\right)\left(\frac{\partial f_{0_i}}{\partial\gamma}\right)\Delta\beta + \sum_i \left(\frac{\partial f_{0_i}}{\partial\gamma}\right)^2 \Delta\gamma \tag{18}$$

This system of equations takes the form of a matrix equation when we let

$$X_1 = \sum_i (y_i - f_{0_i})\frac{\partial f_{0_i}}{\partial\alpha}, \qquad X_2 = \sum_i (y_i - f_{0_i})\frac{\partial f_{0_i}}{\partial\beta}, \qquad X_3 = \sum_i (y_i - f_{0_i})\frac{\partial f_{0_i}}{\partial\gamma}, \tag{19}$$

$$C_{11} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\alpha}\right)^2, \quad C_{12} = C_{21} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\beta}\right)\left(\frac{\partial f_{0_i}}{\partial\alpha}\right), \quad C_{13} = C_{31} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\gamma}\right)\left(\frac{\partial f_{0_i}}{\partial\alpha}\right), \tag{20}$$

$$C_{22} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\beta}\right)^2, \quad C_{23} = C_{32} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\gamma}\right)\left(\frac{\partial f_{0_i}}{\partial\beta}\right), \qquad C_{33} = \sum_i \left(\frac{\partial f_{0_i}}{\partial\gamma}\right)^2. \tag{21}$$

Now (16) – (18) become

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} \begin{vmatrix} \Delta\alpha \\ \Delta\beta \\ \Delta\gamma \end{vmatrix} \tag{22}$$

and the solutions for the parameter changes are given by

$$\Delta\alpha = \frac{\begin{vmatrix} X_1 & C_{12} & C_{13} \\ X_2 & C_{22} & C_{23} \\ X_3 & C_{32} & C_{33} \end{vmatrix}}{\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}}, \quad \Delta\beta = \frac{\begin{vmatrix} C_{11} & X_1 & C_{13} \\ C_{21} & X_2 & C_{23} \\ C_{31} & X_3 & C_{33} \end{vmatrix}}{\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}} \quad \text{and} \quad \Delta\gamma = \frac{\begin{vmatrix} C_{11} & C_{12} & X_1 \\ C_{21} & C_{22} & X_2 \\ C_{31} & C_{32} & X_3 \end{vmatrix}}{\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}}. \tag{23}$$

If the surface in parameter space were truly flat in all regions of the parameter-space, then changing the initial values for the parameters by the values given in (23) would make the next guess, f_j, the best possible guess (the one which reduces χ^2 to its lowest value). However, because (14) is only a linear approximation to a surface which is not likely to be flat, the process of changing estimates for the parameters will be an iterative one. After the initial guesses are made, the changes are solved for and the parameters are corrected for the next estimation by

$$\alpha_{n+1} = \alpha_n + \Delta\alpha, \quad \beta_{n+1} = \beta_n + \Delta\beta \quad \text{and} \quad \gamma_{n+1} = \gamma_n + \Delta\gamma \tag{24}$$

The new values for α, β, and γ are then inserted back into (19) - (21) and new changes in their values are arrived at via (23). This process continues until all three changes are within some specified margin of error, defined as

$$\frac{\Delta\alpha}{\alpha} < Er, \quad \frac{\Delta\beta}{\beta} < Er \quad \text{and} \quad \frac{\Delta\gamma}{\gamma} < Er. \tag{25}$$

At this point, the values of the parameters have been fixed to within their respective specified margins of error. The power of this fitting technique has been demonstrated in fitting non-linear data[1-4] and comes from the ability to fit all the parameters to *each* of the data points. The iterative approximations are rigorous and transcend the "eyeball" fitting, commonly found in fitting optical saturation data. The accuracy provided by this technique is greater than that provided by measuring the gain parameters "independently" or by making linear approximations to different large sections of the data set[1-3].

Data Generation and Data Fitting Results

To test the efficacy of the fitting technique simulated experimental data was generated with each of the photon propagation equations, (1), (3) and (7), and the data was fit to the output intensity equations, (1), (3), (9) and (11). The length of the gain medium was 10 cm. The initial estimates for the gain parameters were taken to be 200% to 500% of the true values. From these initial estimates the fitting routine program made systematic corrections to the values of all of the gain parameters until after several iterations the true values of the parameters were recovered. The error margin for fitting the data to the equations was set at Er = 1.0%.

Figure 2 displays the three parameter amplifier data generated with (1), the initial estimates for the gain parameter values, and the intermediate iterations made by the fitting program to arrive at a perfect fit. Similar results were obtained with generating and fitting three parameter amplifier data with (3).

Figure 3 displays the two parameter oscillator data generated with (7) and the fit of the data to (9). Figure 4 shows the three parameter oscillator data generated with (7) and the fit of the data to (11). In the examples shown in Figs 2-4 the initial estimates for the gain parameter values were set 200% away from the values used to generate the data. In each case the true values of the gain parameters were successfully recovered.

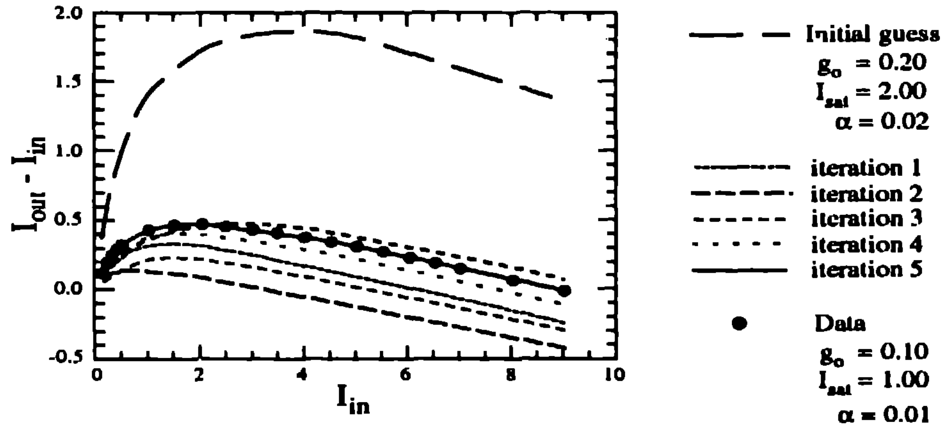


Figure 2. Fitting of three parameters to laser amplifier data. Plot is of extraction intensity (MW/cm^2) vs. input intensity (MW/cm^2).

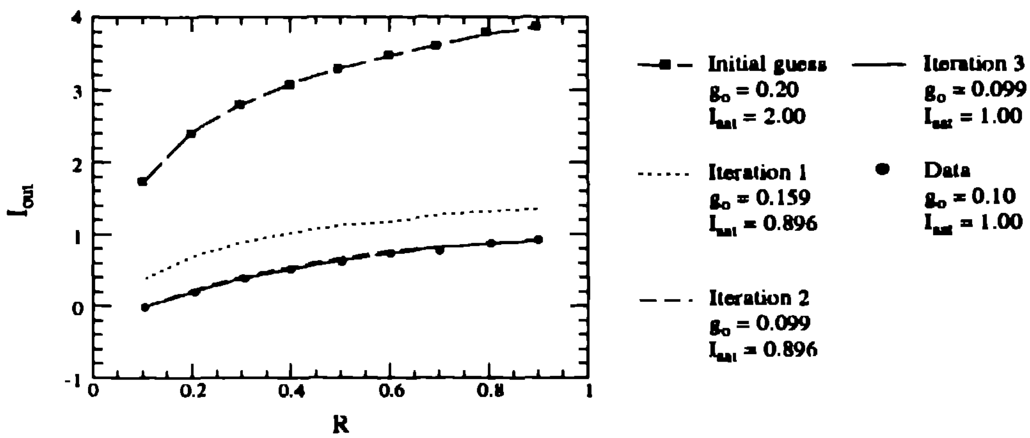


Figure 3. Fitting of two parameters to laser oscillator data. Plot is of output intensity (MW/cm^2) vs. output coupler reflectivity.

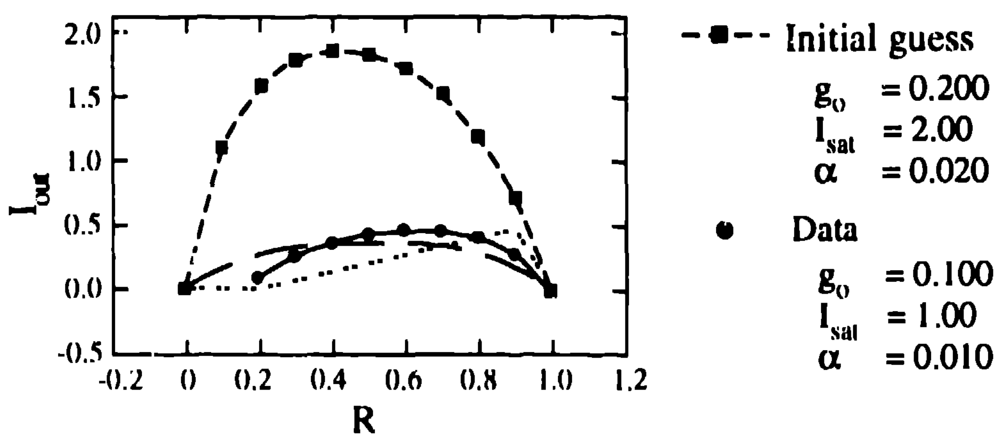


Figure 4. Fitting of three parameters to laser oscillator data. Plot is of output intensity (MW/cm^2) vs. output coupler reflectivity.

The data fitting technique is rigorous and systematic and requires no help from the experimenter. The three-parameter amplifier and two-parameter oscillator fitting results show that the systematic corrections of the gain parameter values follow a smooth surface in the parameter space. The three-parameter oscillator fitting results reveal a more complicated surface in the parameter space. There are regions in this parameter space that reject some initial estimates for the gain parameter values by driving the determinants in the denominators of (29)-(33) to zero. In this case the initial estimates are modified and the fitting routine proceeds. It is always found that the fitting proceeds to the correct values of the gain parameters or a rejection of the initial estimates, but not to an incorrect fit. The three-parameter fitting routines can be used for data that has only two non-zero valued parameters and the correct values for both parameters will be recovered. The only advantage of using two-parameter fitting is for oscillators with no non-saturable absorption.

Conclusions

We have described an extensively tested method for determining the gain parameters of a pulsed laser from measurements of the gain medium as an amplifier or an oscillator. The gain parameters, which include the small signal gain coefficient, the saturation intensity (or fluence) for stimulated emission and the non-saturable absorption coefficient, are all determined simultaneously from a single set of data. The data required in the case of amplifiers is the output laser intensity as a function of input laser intensity. In the oscillator experiments the data needed is the output laser intensity as a function of output coupler reflectance or transmission. We described a numerical method for providing the best fit of the data to the photon propagation equations in an oscillator or amplifier whether the equations are analytic or differential. The results provided by this method are simpler and more accurate than the common procedure of measuring the gain parameters independently.

References

1. Pulsed Dye Laser Gain Analysis and Amplifier Design; Craig C. Jensen, *High Power Dye Lasers*, F. J. Duarte, Ed. Chapter 3 (Springer Verlag, Heildleberg, 1991) PP. 45-91.

2. The Physics of Dye Laser Amplifiers; Craig C. Jensen, in Proceedings, International Conference on Lasers '90, F. J. Duarte, Ed. (STS Press McLean, VA, 1991), pp. 725-732.

3. Complete Optical Analysis of a UV Laser Dye Using Saturation Techniques; Craig. C. Jensen and Hartmut Schroeder, *Applied Optics*, 31 (33) 7012 7021 (1992).

4. Simultaneous Determination of Gain Parameters for Steady State Lasers with Distributed Non Saturable Loss; David A. Branning and Craig C. Jensen, Los Alamos National Laboratory Report #LA UR 93

5. Theory of Pulse Propagation in a Laser Amplifier; Lee M. Frantz and John S. Nodvik, J. Appl. Phys., 34 (8) 2346 2349 (1963).

6. Homogeneously Broadened CW Lasers with Uniform Distributed Loss; W. Rigrod, IEEE J. Quantum Electron., vol. QE 14, pp. 377 381, 1978.