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BEAM HALO IN HIGH-INTENSITY BEAMS*

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ABSTRACT

In space-charge dominated beams the nonlinear space-charge forces produce a filamentation pattern, which in projection to the 2-D phase spaces results in a 2-component beam consisting of an inner core and a diffuse outer halo. The beam-halo is of concern for a next generation of cw, high-power proton linacs that could be applied to intense neutron generators for nuclear materials processing. We describe what has been learned about beam halo and the evolution of space-charge dominated beams using numerical simulations of initial laminar beams in uniform linear focusing channels. We present initial results from a study of beam entropy for an intense space-charge dominated beam.

INTRODUCTION

For beams with high average intensity, one may be concerned not with the rms or average phase-space areas, but with the outer part of the distribution, often called the beam halo, which affects particle losses in an accelerator. Relatively small losses in a high-energy accelerator may produce enough radioactivation of the accelerator structure or radiation damage of components to create practical difficulties in maintenance and operation[1]. A major cause of beam-halo growth in low-velocity intense beams is the Coulomb self force. In most accelerator beams this is predominantly a collective force; small-impact-parameter binary collisions usually have little effect on the dynamics. This smoothed or average Coulomb force is called the space-charge force and is described by a repulsive self-electric field and an attractive self-magnetic field. The magnetic term is only important for relativistic beams and its contribution reduces the total space-charge force.

The space-charge force is complicated because the field depends upon the time-varying charge density of the beam, is nonlinear, time dependent, and coupled between the three planes. In the presence of external focusing forces, one observes phenomena that are common in plasma physics, such as plasma oscillations and Debye shielding. The plasma period determines a basic time scale for these phenomena, and the Debye length determines a basic length scale for the particle distribution. The net force, consisting of the external focusing plus the time-dependent space-charge force, may be either attractive or repulsive, and the sign of the net force may even vary across the beam. These conditions can lead to very nonlinear behavior, and one must rely on numerical simulation codes to study the detailed dynamics.

PHASE-SPACE DYNAMICS

Numerical Simulation

We use numerical simulation to look at the multiparticle dynamics to see what changes occur in phase space. We examine the case of a round continuous beam in a uniform linear focusing channel with purely radial focusing. This system represents a smooth approximation for beams in quadrupole focusing channels, and therefore we expect that phenomena observed in the uniform channel will also be observed in the

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quadrupole channel. We use a numerical simulation code[2] for these studies in which the radial space-charge forces are calculated from Gauss's Law. Consequently, we are studying the effect of the collective forces acting on each particle and ignoring the small-impact-parameter binary Coulomb collisions. With radial symmetry this is a 1-D (strictly speaking a single variable) problem. Our computer code has been run with 2000 simulation particles through 56 steps per plasma period, choices that are adequate to represent the main features of the space-charge forces. We have chosen to study the dynamics of an initial rms-mismatched laminar (zero emittance) beam. Laminar beams are idealizations because all real beams have finite emittance. Nevertheless, the laminar beam represents the extreme space-charge limit and allows us to emphasize the effects of the space charge.

In Figs. 1 and 2 we show the distributions of a) the radial or r - r' phase space, b) the projected or x - x' (and y - y') phase space, and c) the x - y beam cross section.

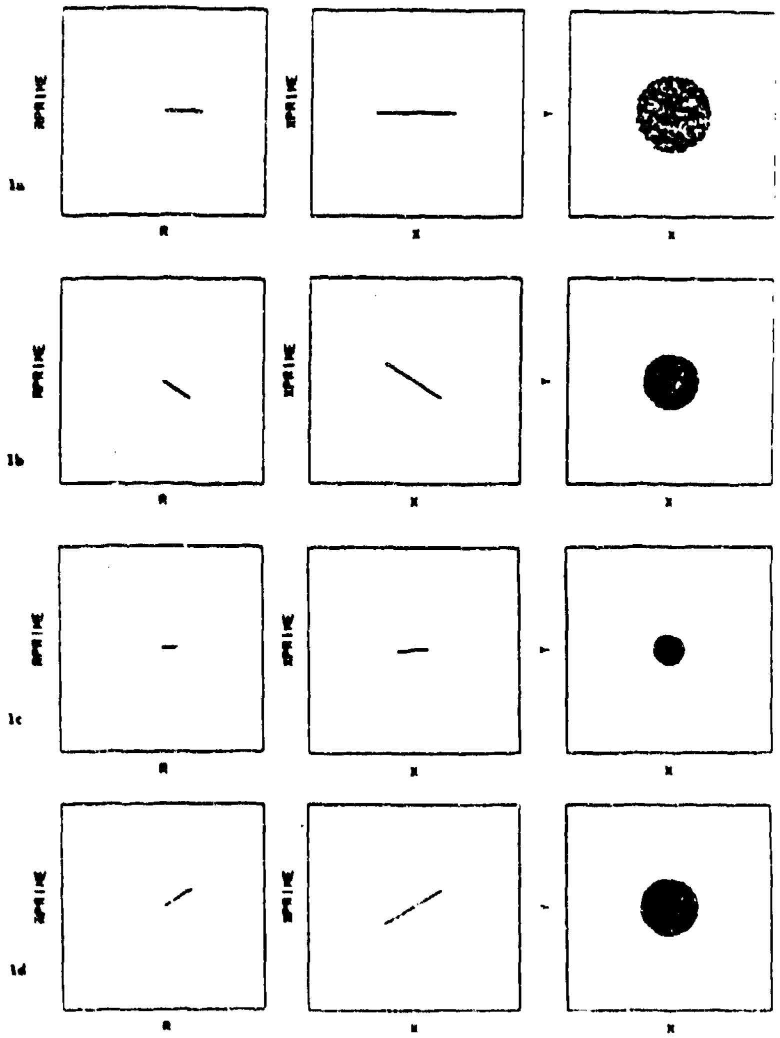

Fig. 1. Radial r - r' phase space, transverse x - x' and y - y' phase space, and cross section x - y from simulation of an initial uniform-density laminar beam in a uniform linear focusing channel for a) t = 0, b) t = 0.25, c) t = 0.50, and d) t = 0.75 in units of beam-plasma periods. The initial rms beam sizes in x and y are 50% larger than the matched size

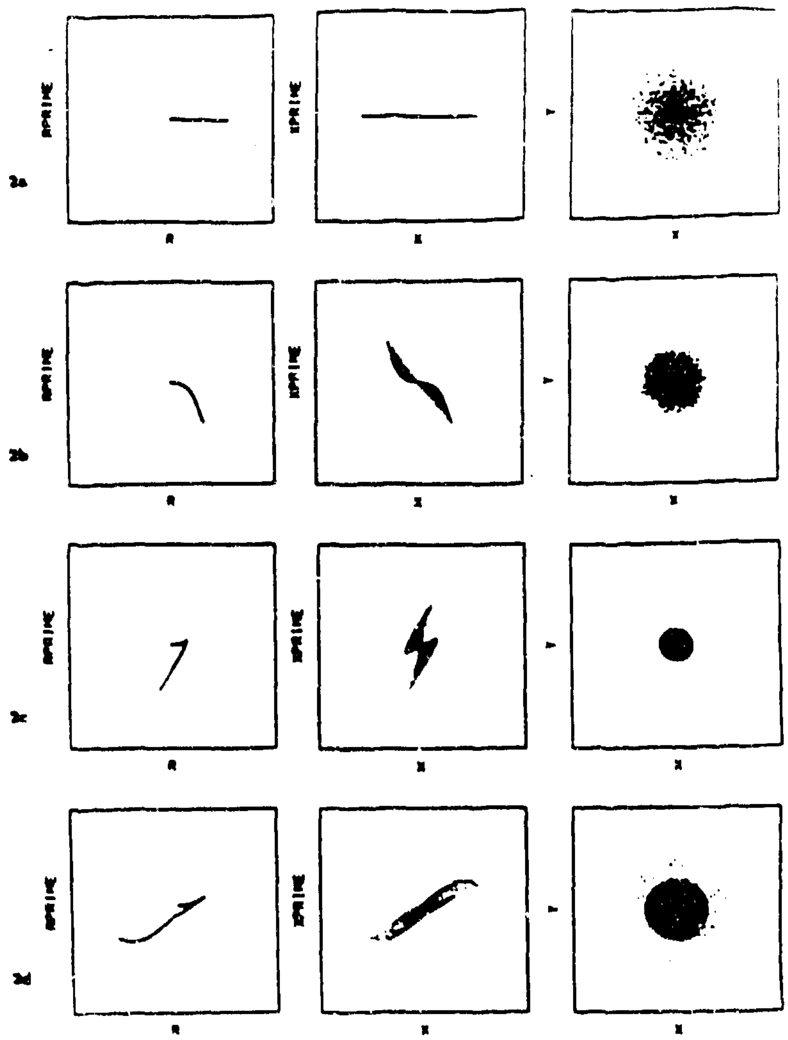

Fig. 2. Radial (r - r') phase space, transverse x - x' and y - y' phase space, and cross section (x - y) from simulation of an initial Gaussian-density laminar beam in a uniform linear focusing channe . for a) t = 0, b) t = 0.25, c) t = 0.50, d) t = 0.75, e) t = 1.00, f) t = 1.50, g) t = 2.00, h) t = 3.00, i) t = 4.00, j) t = 5.00, k) t = 10.00, and l) t = 20.00, in units of beam-plasma periods. The initial rms beam sizes in x and y are 50% larger than the matched size.

We show the r - r' phase space because we expect the dynamics to appear simpler in r - r' space when only radial forces act on a laminar beam. We assign an initial positive radius to all particles, but if during the simulation a particle crosses the axis, we change the sign of the radius before plotting a point in r-r' space.

Mismatched Uniform Density Laminar Beam

We begin by studying the dynamics of an initial space-charge dominated uniform-density laminar beam with zero velocity spread, which is rms mismatched so that the initial rms beam size is larger than the matched value by a factor of 1.5. Figures 1a through 1d show the beam characteristics for four different times, 0, 0.25, 0.50, and 0.75, measured in beam-plasma periods. The beam-plasma period for a uniform beam

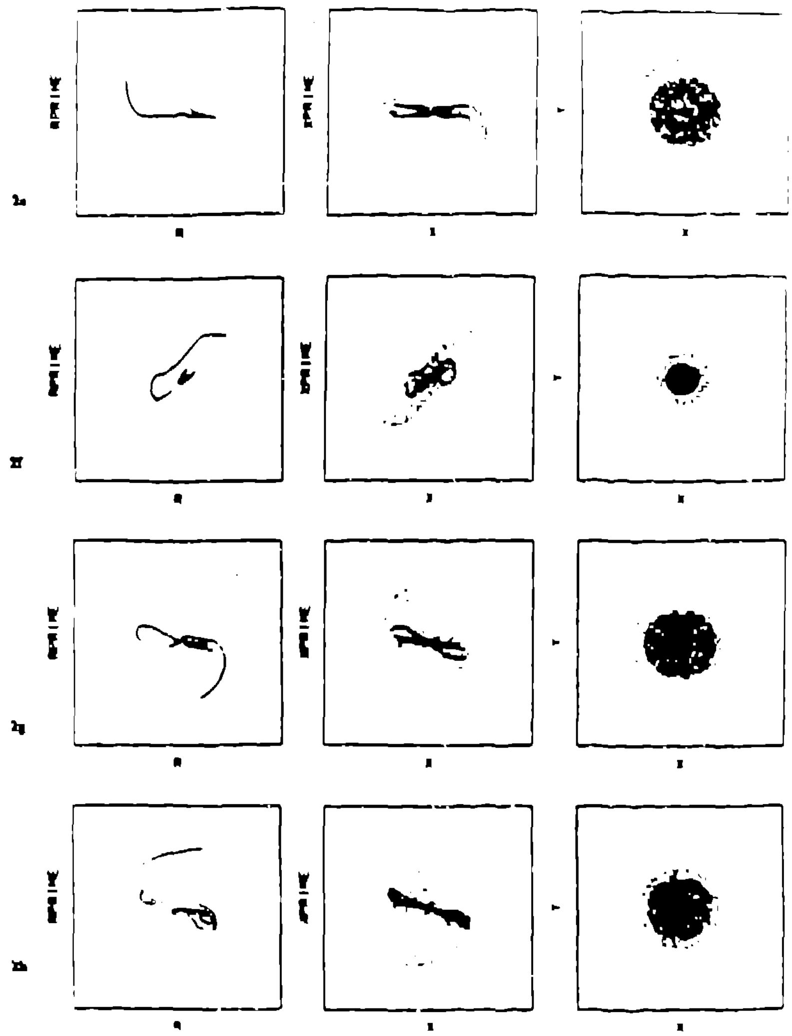

Fig. 2. (cont.)

of density n_0 is defined in the usual way; thus $T_p = 2\pi / \omega_p$, and $\omega_p^2 = q^2 n_o / \varepsilon_o m$ is the beam-plasma frequency. The phase-space plots show density (plasma) oscillations that are excited by the unbalanced external focusing and internal space-charge forces. The total force alternates at the beam-plasma frequency between focusing and defocusing. The charge distribution always remains uniform so that only linear forces act on the beam, and the emittance remains zero. In the absence of space-charge forces particles would experience only the external fields and would cross the axis as they execute betatron oscillations. For a space-charge dominated beam, the particles do not cross the axis, but each particle oscillates about an equilibrium radius.

Gaussian Density Laminar Beam

Next we examine the dynamics of an initial Gaussian-density laminar beam with zero initial velocity spread, which is rms mismatched by the same factor 1.5. Figures 2a through 2l show a sequence of plots for different times in units of the plasma period (defined for the equivalent uniform beam with the same rms size). For this case, the external force is linear, but the space-charge force is nonlinear. Several new features are present. Most of the small amplitude trajectories undergo plasma oscillations (they

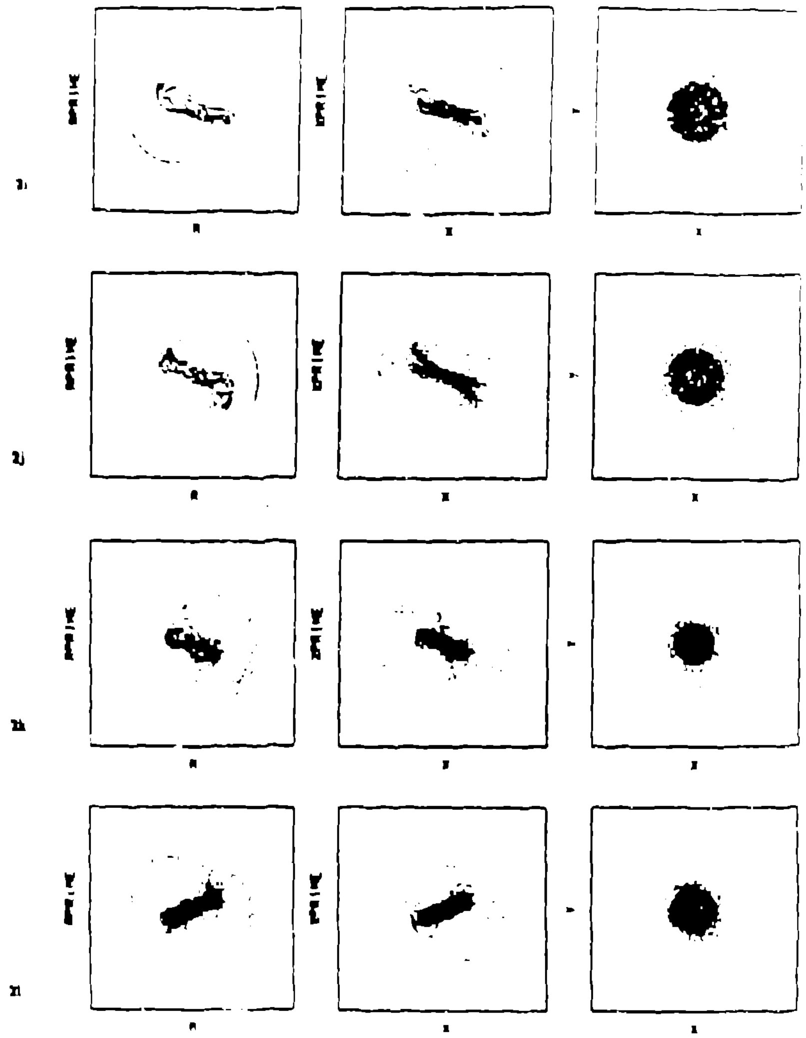

Fig. 2. (cont.)

do not cross the axis) and form an inner core. The large amplitude trajectories correspond to betatron oscillations (they cross the axis) and develop into an outer halo. In r - r' space the halo evolves like a ring-shaped filament. In x - x' space the ring appears as a low-density disk. These differences are the result of the fact that any arbitrary point in r - r' space projects to a straight line in x - x' space that passes through the origin and ranges between (-r,-r') and (r,r'). Although effective emittance growth has often been identified with a process of filamentation, we see that the filament in this problem is observed in the r - r' phase space. In the usual x - x' projected phase space the outer part of the filament becomes a diffuse disk-like halo. This will be discussed in more detail in the following section.

Even within a few plasma periods the nonlinear space-charge force produces a random-looking distribution of points within the core. This randomization or thermalization is the result of a process in which the inner part of the filament in r - r' space is stretched and folded many times. The stretching and folding is associated with variations of the magnitude and sign of the space-charge force. The halo produced after several plasma periods is a common feature of the nonlinear space-charge force. We find that the outer filaments seen in r - r' space contain mostly the particles with large initial amplitudes but also contain a few particles with small initial amplitudes that were launched during the initial stages of randomization of the core. For our example, the halo is a distinctive structure in r - r' space even after 20 plasma periods; unlike the core, the halo is not yet thermalized.

At present there is no established criterion for defining the halo. For the present example of an rms-mismatched Gaussian laminar beam, we find that an ellipse with the same Courant-Snyder parameters as the rms ellipse and with an emittance five times larger than the rms ellipse in r-r' space, appears to enclose most of the core and exclude most of the halo. If we define the core particles to be all those contained within this ellipse, and define halo particles as those outside, we find that after about 10 plasma periods, 6% of the particles are contained within the halo. For this example the core and the halo contribute about equally to the final rms emittance. Furthermore, the rms emittance of the core grows to its final value in about one-quarter of a plasma period, like that of an rms-matched beam[3]. Most of the growth of the halo occurs over about 10 plasma periods[4]. More study is needed to determine how these results vary with the amount of mismatch and to determine what happens when using more realistic beams with nonzero initial emittance. One concern for this problem with pure radial dynamics is that because all particles that cross the axis must pass through a common point at the origin (x=y=0), there may be singular density fluctuations at the origin that may produce unrealistic forces. Previous work on emittance growth for this 2-D mismatched beam is given in Refs. 4 and 5, and for 1-D sheet beams in Ref. 6.

2.4 Filamentation and Beam Halo

The phase-space plots in Fig. 2 show a complex filamentation pattern in r-r' space, where the halo forms an outer ring. The presence of filamentation is a well known effect of nonlinear forces in 1-D, and in this 2-D problem it is observed clearly in r-r' phase-space. However, in the projected phase-spaces x-x', and y-y', the halo does not appear as a ring, but forms the diffuse structures, observed in Fig. 2. This is explained from the fact that each point in r-r' phase space projects into a straight line in x-x' or y-y' phase space, as is shown in Fig. 3.

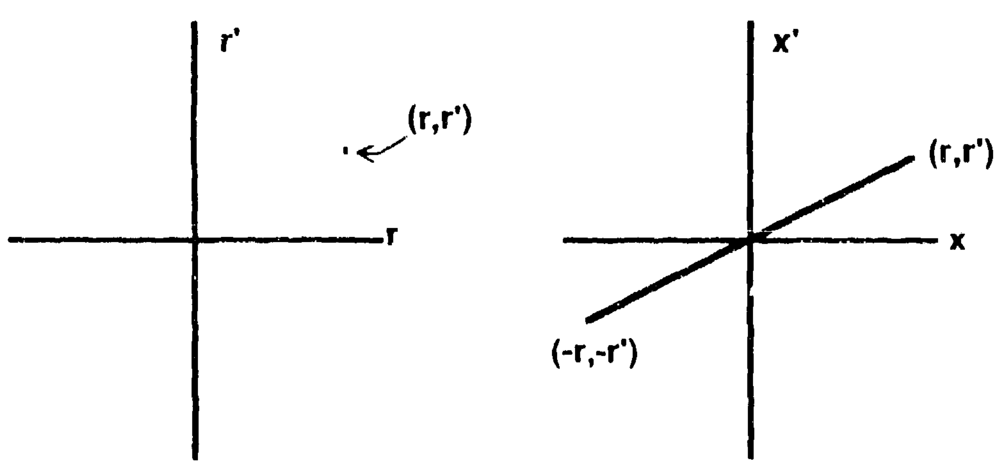

Fig. 3. Projection of a point in r-r' phase space into x-x' phase space as a straight line.

To understand this we consider a particle with polar coordinates (r,q), which moves only radially with divergence r'= dr/dz. In Cartesian coordinates we have x= rcosq, y=rsinq, x'=r'cosq, and y'=r'sinq, and we obtain x'/x = y'/y = r'/r. Therefore, x' = x r'/r and y' = y r'/r, and for fixed r and r' the relationship between x' and x is expressed by a straight line that passes through the origin. The values of x and x' for fixed r and r' depend on the angle q. The extreme values are $x = \pm r$, and $x' = \pm r'$. This results in points or straight lines in the r-r' plane transforming into straight lines in the x-x' plane, while curved lines in the r-r' plane transform into a fanlike butterfly or bow-tie pattern in the x-x' plane. In general, an arbitrary filament in the r-r' plane appears as a more diffuse distribution of points in the x-x' plane. A more complete treatment, including beams with nonzero angular momentum is given in Ref. 7.

BEAM ENTROPY

We are interested in identifying some general principles, which allow us to understand why a charged-particle beam distribution evolves as it does, and towards which steady-state distribution the beam is evolving. We are naturally led to reexamine the concept of beam entropy, which was discussed by Lawson, Lapostolle, and Gluckstern[8] in 1973. In this paper the authors concluded that the beam entropy is a measure of disorder, which is related to emittance, the more conventional measure of disorder in beams. The authors concluded that beam entropy depends on 1) rms emittance, 2) the distribution function, and 3) the phase-space cell size chosen to define the entropy. The dependence of entropy on the distribution function implies the entropy depends not only on the second moments of the distribution, but also on the higher moments. The phase-space cell size of interest may be determined by the experimental resolution. We will see later that the entropy concept may indeed be a useful one if one distinguishes between the microscopic characterization of the distribution and a macroscopic or coarse-grained one. Of course, this distinction has also been important for emittance.

To proceed further we review the entropy concept. First, given an arbitrary phase-space distribution, we divide phase-space into cells of equal volume. One distinguishes between microscopic and macroscopic states. The microscopic state is defined by specifying the cell where each particle is located. The macroscopic state is defined by specifying only the total population of each cell, regardless of which particles are present. The measured properties of the beam, such as the particle distribution and the rms emittance clearly depend only on the macroscopic state. Following Boltzmann's approach, we define the disorder of the macroscopic state as the number of ways of permuting the individual particles between the cells, while maintaining the same macroscopic state. Thus the disorder is the number of distinct microscopic states that result in the same macroscopic state. That this represents a measure of disorder follows from the assumption that more ordered macroscopic states will have fewer microscopic ways in which they can be constructed. Given a system of N particles to be distributed among m cells, and with n_i particles in the ith cell, the disorder W is given by

$$W = \frac{N!}{\prod_{i=1}^{m} n_i!}.$$

The entropy is defined using Boltzmann's constant k, as

$$S = k \log W = k\left[\log N! - \sum_i \log n_i!\right].$$

For example, the most ordered macroscopic state has all particles in one cell, and the above definitions result in S=0. As the particles become distributed in more cells, the disorder and entropy both increase.

It is interesting to consider the limiting case of very large numbers of particles and cells with infinitesimal volume. For very large numbers of particles per cell, it follows from Stirling's formula that $\log n! \cong n \log n - n$. Then we obtain $\log W \cong N \log N - \sum_i n_i \log n_i$. If the phase- space density is f, and the cell size dV becomes infinitesimal, the number in the ith cell may be expressed as $n_i = f dV$. Changing the summation to an integral, we write $\log W \cong N \log N - N \log dV + \int dV f \log f$. For fixed N and dV, the first two terms are constant, and if only changes in S with respect to time are of interest, we need only be concerned with the third term. The rate of change of S with respect to time is

$$\frac{dS}{dt} = k\frac{d}{dt}\log W = k\left[\int dV \frac{df}{dt}(1 + \log f)\right].$$

For a typical particle-accelerator beam, which satisfies the Vlasov-Poisson equation, df/dt = 0, which implies that dS/dt = 0. Therefore, on a microscopic or fine-grained scale, the entropy is constant. This is not surprising, because from Liouville's theorem the microscopic phase-space density along the trajectory of any particle is constant. However, for entropy defined using phase-space cells with a finite volume, the entropy is not necessarily constant. This situation is not unlike that of emittance, which may change on a coarse-grained scale, even though Liouville's theorem guarantees a constant phase-space volume on a microscopic scale.

Intuitively, a laminar beam is a highly ordered state with zero emittance. Although, particle collisions would produce disorder on a microscopic scale, the space-charge forces, which satisfy the Vlasov-Poisson equation cannot increase the microscopic entropy, as we have seen. However, one may ask whether the nonlinear space-charge forces increase disorder and entropy on a coarse-grained scale. To answer this question, we have carried out a numerical simulation study for an initial Gaussian, rms matched, space-charge-dominated or laminar 2-D continuous beam, in a uniform linear focusing channel with radial symmetry. We divided the phase-space distribution into 100 by 200 equal-volume cells. The initial beam had zero divergence, a Gaussian distribution in real space, truncated at 3s, and populated 74 phase-space cells. Using 2000 particles, at each time step the entropy S = k logW was computed. Fig. 4 shows the computed entropy plotted versus the distance along the channel, expressed in plasma wavelengths. We see oscillations at the plasma period that damp out by about 15 periods. The time-averaged entropy increases most rapidly during the first 15 plasma periods, and continues to increase afterwards very slowly. This example shows that the coarse-grained entropy, averaged over time, does increase. For comparison, the rms emittance (not shown) increases during the first quarter plasma period[3], after which it remains essentially constant. When the same simulation is done for a mismatched, laminar beam, we observe oscillations in the coarse-grained entropy that are associated with the oscillating beam radius. For a uniform-density, laminar beam the time averaged, coarse-grained entropy remains constant.

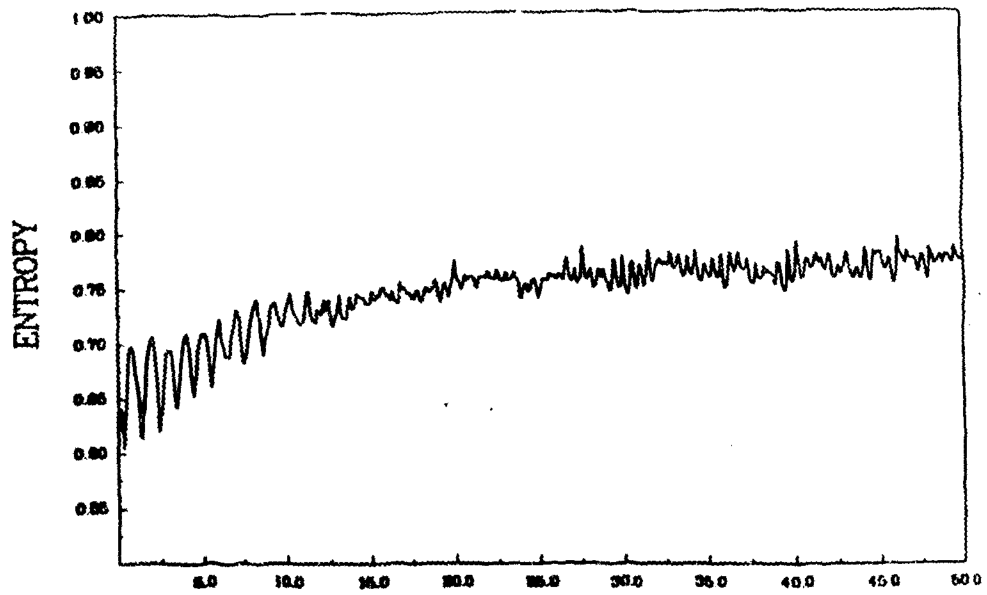

Fig. 4. Beam entropy versus distance measured in plasma wavelengths for an initially rms-matched Gaussian density laminar beam.

A law of coarse grained entropy increase might be used to describe two previously unexplained characteristics of space-charge dominated beams. First, it is observed in numerical simulations that such beams do not appear to evolve toward highly ordered equilibrium states such as the well known K-V distribution, but rather towards more disordered states with a core velocity distribution similar to Maxwellian. Recently for example, the maximum-entropy hypothesis was used to describe the characteristics of the final distribution for a high-intensity expanding beam in free space[9]. Secondly, a tendency for beams with more than one degree of freedom to equipartition has been discussed, especially in high-intensity linacs[10]. Thus, it may be of interest to explore the idea of coarse-grained entropy increase as a means for a conceptual understanding of these important observations.[11]

CONCLUSIONS

Even after more than 20 years we find that there are many questions about space-charge effects that have not been resolved. An important general question concerns the nature of the state of the beam after a few tens to a few hundred plasma periods, a time scale of practical interest for many linear accelerators and transport systems. Is the beam or at least the core of the beam in some approximate equilibrium state? Why are certain equilibrium distributions more representative of real beams than others? Is a coarse-grained Maxwell-Boltzmann distribution a good description? If equipartitioning is a characteristic of the beam, why is it so? One interesting and plausible hypothesis is that enough nonlinearity is provided by the space-charge forces for the beam to approach a state of maximum (coarse-grained) entropy. This may explain why beams in numerical simulations do not evolve toward highly ordered equilibrium states like the K V distribution, and why we observe a tendency for beams to equipartition their kinetic energies. It is clear that a lot of work still remains before we have a complete understanding of this interesting and important area of charged-particle-beam physics. A more general discussion of space-charge dominated beams, including a more complete set of references on the subject can be found in Ref. 10.

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