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Author(s): V. M. Belyaev and M. B. Johnson

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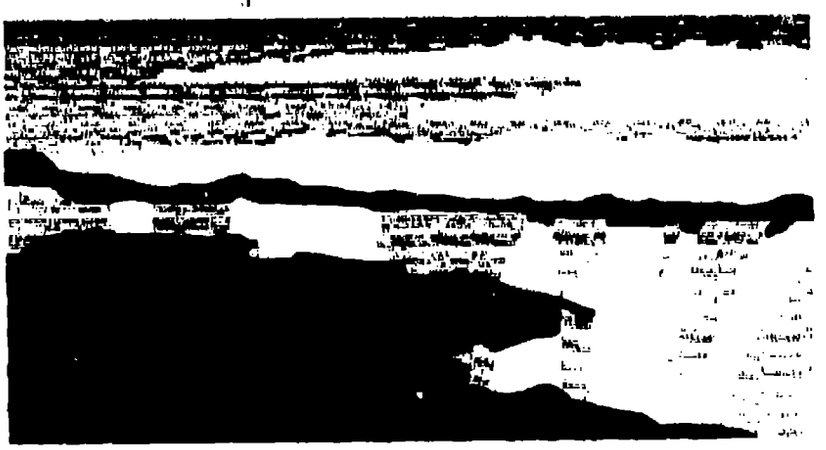

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DEEP INELASTIC SCATTERING AND LIGHT-CONE WAVE FUNCTIONS

V.M. BELYAEV

ITEP, 117259 Moscow, Russia.

Mikkel B. JOHNSON

LANL, Los Alamos, NM 87545, USA

In the framework of light-cone QCD sum rules, we study the valence quark distribution function $q(x_B)$ of a pion for moderate x_B. The sum rule with the leading twist-2 wave function gives $q(x_B) \approx \phi_\pi(x_B)$. Twist-4 wave functions give about 30% for $x_B \sim 0.5$. It is shown that QCD sum rule predictions, with the asymptotic pion wave function, are in good agreement with experimental data. We found that a two-hump profile for the twist-2 wave function leads to a valence quark distribution function that contradicts experimental data.

1 Introduction

Two types of objects in hadron physics are used to represent quark and gluon distributions in a hadron. One is the quark and gluon distribution functions that appear in deep-inelastic lepton-hadron scattering, and the other is the light-cone wave functions of hadrons introduced in pQCD by Chernyak and Zhitnitsky [1] to describe hadron form factors at large Q^2. In this paper, we investigate a relationship between these objects. To make a prediction for the valence-quark distribution in terms of light-cone wave functions, we combine the ideas of calculating the deep-inelastic scattering amplitudes in the QCD sum rule approach [2] as suggested by Ioffe [3] with the so-called light-cone QCD sum rules [5].

In this work we propose a method to calculate the pion structure function directly in terms of light-cone wave functions. The starting point of this method was formulated in Ref[4], where it was pointed that if $x_B = Q^2/(2pq)$ is not close to the boundary values $x \approx 0$ and $x \approx 1$, then the imaginary part of the deep-inelastic scattering amplitude is determined by small distances in the t-channel. A general proof of this statement follows from the fact that at large $|p^2|$ and $|q^2|$, the nearest singularity in the t-channel is at $t = -\frac{1-x_B}{x_B}p^2$ in the kinematics that is used in this paper. Thus, in the case of intermediate x_B we can use the OPE to construct QCD sum rules.

2 Correlator

To calculate the quark distribution function for a pion, we consider the following correlator:

$$T_{\mu\rho\lambda}(p,q,k) = -i\int d^4x d^4z e^{ipx+iqz} < 0|T\{j^5_\mu(x), j^d_\rho(z), j^d_\lambda(0)\}|\pi^-(k)> \quad (1)$$

where k is a pion momentum, and where

$$j^5_\mu = u\gamma_\mu\gamma_5 d, \quad j^d_\rho = \bar{d}\gamma_\rho d, \quad (2)$$

$$k^2 = 0, \quad q^2 = (p+q-k)^2, \quad t = (p-k)^2 = 0, \quad s = (p+q)^2, \quad Q^2 = -q^2. \quad (3)$$

¿From relations (3) it is easy to determine that

$$(2k, p+q) = s + Q^2, \quad (2pk) = p^2 \quad (4)$$

We calculate the discontinuity in s at fixed p^2 and Q^2 of the correlator (1),

$$ImT_{\mu\rho\lambda} = \frac{1}{2i}\left[T_{\mu\rho\lambda}(p^2, q^2, s+i\varepsilon) - T_{\mu\rho\lambda}(p^2, q^2, s-i\varepsilon)\right], \quad (5)$$

where p^2 and q^2 are space-like vectors, $p^2 < 0$, $q^2 < 0$, such that $|p^2|, |q^2| \gg \Lambda_{QCD}$. In the scaling limit, we assume that $|p^2| \ll |q^2|$ and keep only the first nonvanishing terms in an expansion in powers of p^2/q^2. Perturbative logarithmic corrections are taken into account by the Altarelli-Parisi equation.

We calculate $ImT_{\mu\rho\lambda}$ in the physical region of the s channel, and the pion contribution in this amplitude has the following form:

$$ImT_{\mu\rho\lambda} = p_\mu \frac{f_\pi}{p^2} Im\left\{\int d^4z e^{iqz} < \pi(p)|T\{j^d_\rho(z), j^d_\lambda(0)\}|\pi(k)>\right\} \quad (6)$$

The general form for $Im\left\{\int d^4z e^{iqz} < \pi(p)|T\{j^d_\rho(z), j^d_\lambda(0)\}|\pi(k)>\right\}$ is

$$\begin{aligned}
&A_1(s,Q^2)(q^2 g_{\rho\lambda} - q^{(2)}_\rho q^{(1)}_\lambda) \\
&+A_2(s,Q^2)(q^2 g_{\rho\lambda} - q^{(1)}_\rho q^{(1)}_\lambda - q^{(2)}_\rho q^{(2)}_\lambda + q^{(1)}_\rho q^{(2)}_\lambda) \\
&+B_1(s,Q^2)\left(p - \frac{pq^{(1)}}{q^2}q^{(1)}\right)_\rho \left(p - \frac{pq^{(2)}}{q^2}q^{(1)}\right)_\lambda \\
&+B_2(s,Q^2)\left(p - \frac{pq^{(1)}}{q^2}q^{(1)}\right)_\rho \left(p - \frac{pq^{(2)}}{q^2}q^{(2)}\right)_\lambda
\end{aligned}$$

$$+B_3(s,Q^2)\left(p-\frac{pq^{(1)}}{q^2}q^{(2)}\right)_\rho\left(p-\frac{pq^{(2)}}{q^2}q^{(1)}\right)_\lambda$$
$$+B_4(s,Q^2)\left(p-\frac{pq^{(1)}}{q^2}q^{(2)}\right)_\rho\left(p-\frac{pq^{(2)}}{q^2}q^{(2)}\right)_\lambda \qquad (7)$$

where $q^{(1)}=q$ and $q^{(2)}=p+q-k$ are the momenta of the virtual photons; $(q^{(1)})^2=(q^{(2)})^2=q^2$. It is clear that

$$4x_B^2q^d(x_B)=\frac{Q^2}{\pi}\left(B_1(s,Q^2)+B_2(s,Q^2)+B_3(s,Q^2)+B_4(s,Q^2)\right)\quad Q^2\to\infty \qquad (8)$$

where

$$Im\left\{\int d^4z e^{iqz}<\pi(p)|T(J_\mu^d(z),J_\lambda^d(0))|\pi(p)>\right\}$$
$$=4\pi\frac{x_B^2q^d(x_B)}{Q^2}\left(p-\frac{pq}{q^2}q\right)_\mu\left(p-\frac{pq}{q^2}q\right)_\lambda+\ldots \qquad (9)$$

Here $q^d(x_B)$ is d-quark distribution function of a pion.

To find the quark distribution function in this paper, wwe consider the tensor structure $p_\mu p_\rho p_\lambda$ in correlator (1). We define the imaginary part of the correlation function for these tensor structures as $\frac{4\pi x_B^2}{Q^2}t(p^2,x_B)$. We can write the following dispersion relation for this function

$$t(p^2,x_B)=f_\pi\left(\frac{q^d(x_B)}{p^2}+\int ds\frac{\rho(s,x_B)}{s-p^2}ds\right), \qquad (10)$$

where the second term in the right-hand side is the higher-states contribution. To supress the exited states contribution, as usually done in QCD sum rules, we will consider instead of $t(p^2,x)$ its Borel transform in p^2:

$$t(M^2,x_B)=\frac{(-p^2)^{n+1}}{n!}\left(\frac{d}{dp^2}\right)^n t(p^2,x_B)$$
$$=-f_\pi\left(x_B^2q^d(x_B)+\int\rho(s,x_B)e^{-s/M^2}ds\right) \qquad (11)$$

3 QCD sum rule

To find $q^d(x_B)$, we apply the OPE near the light-cone $x^2=0$. In contrast to conventional QCD sum rules based on the OPE of a T product of currents at

small distances, we apply an expansion near the light-cone expressed in terms of nonlocal operators, i.e. matrix elements that define hadron light-cone wave functions of increasing twist. The amplitude of two-quark nonlocal operator has the following form:

$$<0|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|\pi(k)> = ik_\mu f_\pi \int_0^1 du e^{-ikxu}(\varphi_\pi(u) + x^2 g_1(u) + O(x^4))$$
$$+ f_\pi \left(x_\mu - \frac{x^2}{kx}k_\mu\right) \int_0^1 du e^{-ikxu}(g_2(u) + O(x^2)), \quad (12)$$

where $\varphi_\pi(u)$, and $g_1(u)$ and $g_2(u)$, are the leading twist-2 and the twist-4 pion light-cone wave functions, respectively.

The leading twist-2 operator gives the following contribution to $t(p^2, x)$

$$f_\pi \frac{\varphi_\pi(xn)}{p^2}. \quad (13)$$

Using eq.(10) for $t(p^2, xn)$, the leading twist-2 wave function contribution is

$$q^d(x_n) = \varphi_\pi(x_n). \quad (14)$$

It is clear that this relation corresponds to a pure parton picture in which the pion consists of two valence quarks only:

$$\int_0^1 q^d(x_n)dx_n = 1, \quad \int_0^1 x_n q^d(x_n)dx_n = 1/2. \quad (15)$$

These relations follow from the normalization of the twist-2 pion wave function ($\int_0^1 \varphi_\pi(u)du = 1$), and from its symmetry: $\varphi_\pi(u) = \varphi_\pi(1-u)$.

In this paper we use the asymptotic pion wave function:

$$\varphi_\pi(u) = \varphi^{(asym)}(u) = 6u(1-u). \quad (16)$$

Note that there are alternative models for φ_π, the Chernyak-Zhitnitsky wave functions φ_π^{CZ} and the function that was introduced in Ref.[9]. Because of their two-humped shape, these alternative models for the pion wave function lead to poor agreement between the QCD sum rule prediction and experiment.

To make a reasonable prediction, we have to estimate the twist-4 wave function contribution. In this paper, we take into account the contribution of the two-particle wave functions of twist-4. This contribution to $t(p^2, x_n)$ is

$$\frac{f_\pi}{p^4} f_A(x_n) = \frac{4f_\pi}{p^4}\left(\frac{g_1(x_n) + g_2(x_n)}{x_n} + \frac{1}{2}g_2(x_n) - \frac{dg_1(x_n)}{dx_n}\right) \quad (17)$$

where $G_2(u) = -\int_0^u dv g_2(v)$.

Using eqs.(10,13,16), after making a Borel transformation, we have

$$\varphi_\pi(x_B) = \frac{4}{M^2}\left(\frac{g_1(x_B) + G_2(x_B)}{x} + \frac{1}{2}g_2(x_B) - \frac{dg_1(x_B)}{x_B}\right)$$
$$= q^d(x_B) + C(x_B)e^{-m^2_{A_1}/M^2} \qquad (18)$$

Here, we take into account the contribution of the A_1-meson to evaluate the higher states contribution.

To estimate twist-4 contribution, we use the following set of twist-4 pion wave functions (see Ref.[8] and Ref.[7]):

$$g_1(u) = \delta^2\frac{5}{2} + \frac{1}{2}\epsilon\delta^2(\bar{u}u(2 + 13uu$$
$$+2u^3(10 - 15u + 6u^3)\ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})),$$
$$g_2(u) = \frac{10}{3}\delta^2\bar{u}u(u - \bar{u}), \qquad (19)$$

where $\bar{u} \equiv 1 - u$. One of the parameters is defined by the matrix element

$$< 0|g_s u\tilde{G}_{\alpha\beta}\gamma_\beta u|\pi(p) > = i\delta^2 f_\pi p_\alpha \qquad (20)$$

The QCD sum rule estimate of Ref.[8] yelds $\delta^2 = 0.2GeV^2$ at $\mu \simeq 1GeV$. At $\mu \simeq 1GeV$, $\epsilon \simeq 0.5$ (see Ref.[8]).

To fix the function $C(x_B)$, we take the QCD sum rule (17) in the limit $M^2 \to \infty$ to be valid up to terms $1/M^2$. This condition leads to the relation: $C(x_B) = \frac{f_A(x_B)}{m^2_{A_1}}$.

Finally we obtain the following QCD sum rule

$$q^d(x_B) = \varphi_\pi(x_B) - f_A(x_B)\left(\frac{1}{M^2} + \frac{1}{m^2_{A_1}}e^{-m^2_{A_1}/M^2}\right) \qquad (21)$$

For $M^2 \sim 1GeV^2$ and $0.2 \leq x_B \leq 0.8$, the contribution of A_1 meson, which imitates the higher state contribution, and the contribution of twist-4 wave functions, is less than 30%. The results are depicted in Fig.1

Note that in our calculations we do not take into consideration the contribution of leading logarithms. So, in order to make a comparison with experiment, we have to use the quark distribution function at low Q^2 where these logarithms are small. Evolution of experimental data to low Q^2 has been done in Ref.[9], and we use the results of this paper as experimental data

It is interesting to estimate the second momentum of the quark distribution function. Assuming that the region near the end points $x_B = 0, 1$ (where our

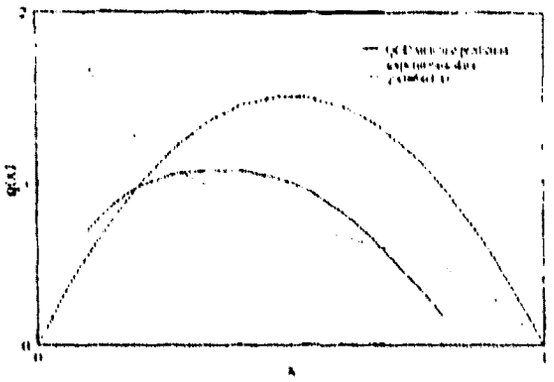

Figure 1:

consideration is not valid) gives a small contribution to the second moment, the sum rule (21) gives the following sum rule for the second moment:

$$\frac{1}{2} - \frac{\delta^2}{M^2} = M_2^d + (higher\ \ resonance\ \ contribution). \qquad (22)$$

The analysis of this sum rule gives $M_2^d = 0.27 \pm 0.05$ at low normalization point, which is in a good agreement with experimental data: $M_2^d \simeq 0.3$ (see Ref.[9]).

4 Conclusions

The light-cone QCD sum rules considered in this paper lead to satisfactory agreement with experimental data. To improve the prediction of the QCD sum rule, it is nessesary to take into consideration twist 4 quark gluon wave functions. One should note that the present experimental data for the quark distribution functions was obtained from an analysis of the Drell-Yan process, which is uncertain through the so-called K-factor. The choice $\mu_\pi = \mu_\pi^{asymp.}$ gives the best agreement between the QCD sum rule prediction and the experimental data. Alternative models for the pion wave function having a two humped profile lead to a poor agreement with the experimental data.

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References

1. V.L. Chernyak and A.R. Zhitnitsky, *Phys. Rep.* **112**, 173 (1984).
2. M.A. Shifman A.I. Vainshtein and V.I. Zakharov, NPB **147**, 285 (1979); B **147**,448 (1979).
3. B.L. Ioffe, *Pisma ZhETF* **42**, 266 (1985); **43**, 316 (1986).
4. V.M. Belyaev and B.L. Ioffe, *Nucl. Phys.* B **310**, 548 (1988).
5. I.I Balitsky, V.M. Braun, and A.V. Kolesnichenko, *Sov. J. Nucl. Phys* **44**, 1028 (1986); *Nucl. Phys.* B **312**, 509 (1989).
6. V.M. Braun, and I.B. Filyanov, ZPC **48**, 239 ((1990)).
7. A.S. Gorsky,*Sov. J. Nucl. Phys* **41**, 1008 (1985); **45**, 212 (1987); **50**, 198 (1989).
8. V.A. Novikov, M.A Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* B **237**, 525 (1984).
9. M. Glück, E. Reya, A. Vogt, *Z. Phys* C **53**, 651 (1992).

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