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RELATION BETWEEN THE BDMPS TRANSPORT COEFFICIENT AND THE DIPOLE CROSS SECTION

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Abstract
In the BDMPS formalism, the transverse momentum accumulated by a quark propagating through nuclear matter is proportional to the so-called transport coefficient $\hat{q}$. On the other hand, transverse momentum broadening can also be calculated within the color dipole approach where it is expressed in terms of the dipole cross section $\sigma_{qg}$. Since both approaches are equivalent, it is possible to find a relation between $\hat{q}$ and $\sigma_{qg}$.

Transverse momentum broadening of a fast parton (i.e. quark or gluon) propagating through nuclear matter has received much attention during the past years, because it is intimately related to the medium induced energy loss of that parton [1, 2]. Indeed, it was found by the BDMPS collaboration [1] that in QCD the induced radiative energy loss per unit length is proportional to the transverse momentum acquired by the parton. This leads to the seemingly counterintuitive result $\hat{q} \propto L$, where $L$ is the length of the medium traversed by the parton. The same result was obtained by Zakharov [3] in the color dipole approach. Even though these two approaches use very different languages to describe induced energy loss, it was found in [4] that they are equivalent. In this note, we present a quantitative relation between the two main non-perturbative inputs of both approaches, the BDMPS transport coefficient $\hat{q}$ and the dipole cross section $\sigma_{qg}$, Eq. (5). To our knowledge, this relation was first published in [5].

Broadening of transverse momentum of a fast quark propagating in nuclear matter (but not energy loss) was first investigated within the dipole approach in [6]. The authors of [6] study broadening of Drell-Yan pairs in hadron-nucleus collisions. The result of this phenomenological analysis is that the projectile quark performs a random walk as it propagates through the nucleus and thereby acquires the transverse momentum

$$\delta(p_T^2) \propto 2 C(\bar{x}, \bar{Q}^2) \rho \bar{A} x L, \quad (1)$$

where $\rho \bar{A}$ is the nuclear density. The factor $C(\bar{x}, \bar{Q}^2)$ originates from the dipole cross section, which can be written for small separations $\rho$ as

$$\sigma_{qg}(\bar{q} \bar{t}) = C(\bar{q} \bar{t}) \rho^2. \quad (2)$$

A theoretically more profound derivation of Eq. (1) can be found in [5, 7]. Note that $C$ depends on energy $\bar{x}$ and on a hard scale $\bar{Q}^2$. At leading order, these scales cannot be calculated exactly and one has to rely on plausible arguments to find their values, see [7]. The coefficient $C$ can be expressed in terms of the gluon density $G_N$ of a nucleon [8],

$$C(\bar{q} \bar{t}) \equiv \frac{\pi^2}{3} \alpha_s(\lambda/\rho^2) \bar{x} G_N(\bar{\rho} \rho^2)^2, \quad (3)$$

where $\lambda$ is a dimensionless number. Note that in order to obtain broadening for gluons, one would have to multiply $C$ by the ratio of Casimir factors for adjoint and fundamental representation of $\mathfrak{g}$ (3), i.e. $C_A/C_F \equiv \theta/4$.

On the other hand, for cold nuclear matter the BDMPS transport coefficient $\hat{q}$ can also be expressed in terms of the gluon density of a nucleon [1]. For the quark case we are interested in, one finds

$$\hat{q} = \frac{2 \pi^2 \alpha_s(Q^2)}{3} \rho \bar{A} x' G_N(x', Q^2). \quad (4)$$
Again, the scales $x'$ and $Q^2$ cannot be determined exactly. The transport coefficient describes the “scattering power” of the medium. Our final result is obtained by comparing Eqs. (3) and (4), which yields

$$\hat{q}(x', Q^2) = 2\rho_A C(\hat{x}, \hat{Q}^2).$$

(5)

This equality holds to logarithmic accuracy because of the scale dependence of $\hat{q}$ and $C$. In addition, one finds from Eqs. (1), (3) and (4)

$$\delta(p_{F}^2) = \hat{q}L = p_{1W}^2,$$

(6)

where (following the notation of [1]) $p_{1W}^2$ is the characteristic transverse momentum squared of a parton produced in the medium after traversing nuclear matter of length $L$. The second equality in Eq. (6) was found within the BDMPS formalism in [1].

We stress that Eq. (5) was not obtained by equating Eq. (1) with the corresponding result from the BDMPS approach, i.e. $p_{1W}^2 = \hat{q}L$. Instead, we have shown that $\delta(p_{F}^2) = p_{1W}^2$, confirming the equivalence of the two approaches. Note that it is not clear a priori that these two quantities should be equal, since $\delta(p_{F}^2)$ is the broadening of an incident quark, while $p_{1W}^2$ is the broadening for a quark produced in the medium. However, since both quantities are proportional to the length of the medium, the result $\delta(p_{F}^2) = p_{1W}^2$ is plausible.

As a concluding remark, we mention that one could attempt to set $\delta(p_{F}^2)$ equal to the nuclear broadening obtained in the higher twist factorization approach of [9]. However, even though the relation between BDMPS/color dipole formalism and LQS approach is not fully understood yet, the LQS approach probably contains different physics and is not equivalent to the other two approaches. A more detailed discussion can be found in [1, 5]. Therefore, the relation between the LQS parameter $\lambda_{QS}$ [9] and the gluon density such a procedure would yield [1, 5] should be interpreted with great care. The connection between LQS and BDMPS/color dipole approach certainly needs further study.

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References


[2] F. Arleo, these proceedings (and references therein).


