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EVALUATION OF CONDUCTOR STRESSES IN A PULSED HIGH-CURRENT TOROIDAL TRANSFORMER

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Abstract

The Precision, High-Energy Density, Liner Implosion eXperiment (PHELIX) pulsed power driver is currently under development at Los Alamos National Laboratory. When operational PHELIX will provide 5-10 MAmps of peak current with pulse rise-time of ~5-10 ms. Crucial to the performance of PHELIX is a multi-turn primary, single-turn secondary, current step-up toroidal transformer, $R_{\text{major}} \sim 30$ cm, $R_{\text{minor}} \sim 10$ cm. The transformer lifetime should exceed 100 shots. Therefore it is essential that the design be robust enough to survive the magnetic stresses produced by high currents. In order to evaluate our design, two methods have been utilized. First, an analytical evaluation has been performed. By identifying the magnetic forces as $J_1^2/2 \nabla L_1 + J_1 J_2 \nabla M_{12}$, where $J_1$ and $J_2$ are currents in two circuits, coupled by mutual inductance $M_{12}$ and $L_1$ is the self-inductance of the circuit carrying current $J_1$, analytical estimates of stress can be obtained. These results are then compared to a computational MHD model of the same system and to a full finite-element, electromagnetic simulation.

I. INTRODUCTION

The notion of performing electromagnetically-driven, high energy-density hydrodynamic implosion experiments at greatly reduced total energy was introduced in a concept called PHELIX (Precision, High Energy-Density, Liner Implosion experiment) [1-3]. The impetus for this approach was the recognition that precision diagnostic techniques, such as proton radiography, could usefully examine implosions at cm and mm scales that would not require application of multi-megajoule capacitor banks (e.g., Atlas). The costs and difficulties of building and operating such banks can preclude pursuit of pulsed power techniques for important hydrodynamic studies. The more modest inductance change associated with smaller implosions, however, demands that we employ transformer circuitry in driving the liner. Such circuitry can involve complex electromagnetic forces, the analysis of which we discuss here.

II. ANALYTICAL MODELING

We can use the inductance-gradient method to estimate forces and stresses in the PHELIX transformer. The general formula for the force on circuit '1' due to its current $J_1$ and to current $J_2$ in circuit '2' is [4]:

$$F_1 = (J_1^2/2) \nabla L_1 + J_1 J_2 \nabla M_{12} \tag{1}$$

Where $L_1$ is the self-inductance of circuit '1' and $M_{12}$ is the mutual inductance between the two circuits. The gradient operators are taken in the direction of the change in magnitude of each geometric parameter that occurs in the expressions for self and mutual inductance. For example, if the self-inductance of a solenoid with $N_i$ turns is:

$$L_i = \mu \pi N_i^2 r_i^2/h \tag{2}$$

then the gradient would be:
\[ \nabla = r \partial / \partial r + k \partial / \partial h \]  

where \( r \) and \( k \) are unit vectors in the radial (\( r \)) and axial (\( h \)) directions, respectively. The forces in the radial and axial directions are then:

\[ F_r = \mu \pi N_p^2 J_p^2 (r/h) \]  

and

\[ F_z = -\mu \pi N_p^2 J_p^2 (r/h)^2/2 \]  

Under the action of its own current, the coil tries to blow apart radially and squeeze axially.

For the PHELIX transformer, we essentially have a multi-turn solenoid inside a single-turn solenoid, with negligible fringing field because the field-lines are all contained within the solenoids, (thereby corresponding to the idealized situation to which Eqn. 2 applies). In this case, the coil “length” \( h \) is the circumference \( 2\pi R \) of the magnetic field-line that threads the common centers of the cross-sections of the primary and secondary coils. The self-inductances for the primary and secondary, respectively, are:

\[ L_p = \mu N_p^2 r_p^2/2R \]  

\[ L_s = \mu N_s^2 r_s^2/2R \]  

The mutual inductance is:

\[ M_{ps} = (\mu N_p N_p r_p^2/J_p) \]  

\[ = (\mu N_p r_p/2\pi R)N_p r_p^2/J_p \]  

\[ = \mu N_p r_p^2 J_p^2/2R \]  

The forces on each coil due their own currents are then:

\[ F_{rp} = \mu N_p^2 J_p^2 (r_p/2R) \]  

\[ F_{rp} = -\mu N_p^2 J_p^2 (r_p/R)^3/4 \]  

The forces due to the coupling between the two coils are:

\[ F_{rpm} = \mu N_p N_s J_p J_s r_p/R \]  

\[ F_{rpm} = -\mu N_p N_s J_p J_s^2 r_p^2/2R^2 \]  

\[ F_{rm} = 0 \]  

\[ F_{rm} = -\mu N_p N_s J_p J_s^2 r_p^2/2R^2 \]  

The total forces on the coils in their respective major and minor radii are then:

\[ F_{r_p} = \mu N_p^2 J_p^2 (r_p/2R) + m N_p N_s J_p J_s r_p/R \]  

\[ F_{r_p} = -\mu N_p^2 J_p^2 (r_p/R)^3/4 - m N_p N_s J_p J_s^2 r_p^2/2R^2 \]  

The force equations then simplify to:

\[ F_{rp} = -\mu N_p^2 J_p^2 (r_p/R) \left( (r_p/R)^2 - 1/2 \right) \]  

\[ F_{rpm} = \mu N_p^2 J_p^2 (r_p/R)^3 \left( (r_p/R)^2 - 1/2 \right)/2 \]  

\[ F_{rs} = \mu N_s^2 J_s^2 (r_s/R)(r_s/r_p)^4 \]  

\[ F_{rs} = \mu N_p^2 J_p^2 (r_p/R)^3 (r_p/r_s)^4/4 \]  

For \((r_p/r_s)^4 = k = 0.9\), the primary turns are under compression in the minor radius and
pulling apart along their major circumference. The secondary circuit experiences tension along both the minor and major circumferential directions. Basically, for a well-coupled transformer (without complications of other inductances, resistances and secondary load), the magnetic field inside the primary is largely cancelled, leaving the magnetic field between the primary and secondary conductors. The magnetic pressure of this outer field presses the primary inward and the secondary outward. The "magnetic tension" is overcome by the magnetic pressure, so the secondary would attempt to expand its major radius. Interestingly, the primary turns would pull apart along their major radius.

The mechanical stresses opposing the movement of the conductors may be estimated from the preceding force equations, if we make some simplifying assumptions about the geometry. For example, by dividing Eqn. 14a by the surface area of the secondary, we obtain an equivalent pressure:

\[ p_s = \frac{F_s}{(4\pi^2Rr_s)} = \frac{\mu N_p^2 J_p^2 (r_p/r_s)^4}{8\pi^2R^2} \] (15)

Note that the magnetic field associated with the secondary current is:

\[ B_s = \mu N_p^2 J_p (r_p/r_s)^2 / 2\pi R \] (16)

so \( p_s \) is merely the magnetic pressure \( B_s^2 / 2m \).

If we ignore the complex geometry of the toroid, in favor of the simpler situation of a thin-walled cylinder, the stress in the minor circumference is:

\[ S_r = p_s r_s/d_s = \frac{\mu N_p^2 J_p^2 (r_p/r_s)^4 (r_r/r_0)^4}{8\pi^2R^2} \] (17)

The stress along the major circumference is similarly:

\[ S_{rs} = \frac{F_s}{2\pi r_s d_s} = \frac{\mu N_p^2 J_p^2 (r_p/R)^2 (r_r/r_0)^4}{8\pi r_r d} \] (18)

As a numerical example, suppose \( N_p = 4 \), \( J_p = 3 \) MA, \( k = 0.9 \), \( R = 25 \) cm, \( r_s = 5 \) cm, and \( d_s = 1 \) cm, the minor hoop stress is \( S_r = 21.8 \) kpsi, while the major hoop stress is \( S_{rs} = 68.5 \) kpsi. The former stress is about a factor of two below the yield strength of aluminum, so we may have an adequate solution, even with a static calculation; the latter stress suggests that (statically) we may have a problem. We still need to examine the situation for the actual waveforms from the time-dependent circuit calculation, using the earlier force formulas (Eqns. 11-12). Furthermore, we should integrate these forces over time to compute the impulses, which then permit calculation of the strains that might occur.

If we equate the kinetic energy associated with the impulse to the elastic strain energy, we may estimate the strains and associated stresses. For an impulse \( I \) shared by a mass \( M \), the kinetic energy is:

\[ W_k = I^2/2M \] (19)

where \( I = Ft \), based on the circuit calculations, and \( M = rAh \) for a cross-section \( A \) and a length \( h \) in the direction of the stress; this assumes that the impulse can indeed be shared with the full cross-section and length. The energy associated with simple elastic deformation is:

\[ W_e = \int SAh \, de = EAh \, e^2/2 \] (20)

where \( e \) is the strain and \( E \) is the elastic (Young's) modulus of the material. In terms of the stress \( S_m \) at the maximum strain:

\[ W_e = S_m^2 Ah/2E \] (21)

This stress is then related to the impulse by:

\[ S_m = (E/r)^{1/2} I/Ah \] (22)

As a numerical example, let's look at the stress in the major circumference of the secondary. For this, \( A = 2pr_s d_s, h = 2\pi R \), and the impulse, based on Eqn. 14b, is:

\[ I = \mu \pi N_p^2 J_p^2 (r_p/R)^2 (r_r/r_0)^4/4 \] (23)
With values as before, and choosing an effective pulse time $\tau = 10 \mu$s, the impulse is $I = 14.7$ nC. For aluminum, with $E = 10$ Mpsi ($= 6.8 \times 10^{6}$ nt/m$^2$) and $r = 2.7 \times 10^3$ kg/m$^3$, the peak stress is $S_m = 2190$ psi. This is considerably lower than the static value previously computed.

A separate concern is the combination of stresses on the primary coils. Around the major circumference, the individual coils will tend to move apart. They are wrapped, however, in a way that will limit their basic azimuthal motion. It may nevertheless reduce frictional coupling between adjacent coils. The force in the minor radial direction is inward, so we may need to support the primary coils against such motion, perhaps with internal dielectric sections. Calculations based on impulse should indicate whether or not this will be necessary.

Finally, we must address the mechanical support of the disc transmission plates and their connections to the secondary. In addition to the usual forces driving the plates apart, we have the resultant forces needed to connect the hoop stress components in the minor circumference of the secondary. This hoop stress is “cut” by the exit gap needed to release magnetic energy to the liner load and must be replaced by some sort of clamping/buttress arrangement, operating across the secondary-side voltage difference. Such arrangements, of course, are needed for the disc plates as well. With the self-inductance of the disc transmission-plates:

$$L_{op} = (\mu/2\pi)d \ln(r_{o2}/r_{o1})$$  \hspace{1cm} (24)

where $r_{o2}$ is the plate radius near the connection to the secondary, $r_{o1}$ is the radius of connection near the liner and $d$ is the plate separation, the total force, attempting to push the plates apart is:

$$F_p = (\mu/4\pi)\ln(r_{o2}/r_{o1}) N_s^2 I^2$$

$$= (\mu/4\pi)\ln(r_{o2}/r_{o1}) N_s^2 I^2 (r_p/r_o)^4 \hspace{1cm} (25)$$

As a numerical example, with $r_{o2}/r_{o1} = 10$ and other values as before, the required clamping force to hold the plates together statically is $F_p = 26.9$ Mnt = 6 Mlbf. An estimate based on impulse requires a specification of the clamping arrangement and is deferred to later design.

**III. MHD MODELING**

Simplified modeling has been undertaken utilizing the RAVEN 1-D MHD code [5]. This modeling takes an expected driving current waveform ($T_{rise} \sim 2 \mu$s, $I_{max} \sim 5$ MA) and applies it to concentric cylindrical electrodes in a theta-pin geometry. The electrode materials have tabular EOS and simple strength models incorporated into the hydrodynamics. The purpose here is to solve for the stresses in the outer electrode, compare to the theoretical prediction, and judge whether the electrode will survive repeated shots without encountering permanent damage due to plastic deformation.

In Fig. 1, the radial profiles of various quantities of interest are plotted at peak current. The material density (blue) is the nominal 2.7 g/cc of aluminum for an inner conductor (2 mm thick) and outer conductor (1 cm thick) with nominal 12 cm radius. The pressure generated (red) is a negligible fraction ($< 100$ bar) of the nominal yield strength (2.6 kbar) of aluminum even with current densities of ($\sim 10^{10}$ A/m$^2$). From this analysis, it is expected multiple shots on the PHelix transformer should not cause permanent damage.
simulations of concentric electrodes in in a $\theta$-pinch configuration.

VI. REFERENCES


APPENDIX A: ESTIMATE OF CRUSHING OF PRIMARY TURNS

In the same spirit as the earlier estimates of the stresses in the secondary, we may consider the effects of compression in the minor radial direction of the primary turns. This consideration is complicated by several factors, including the helical layout of the material (vs the simple cylindrical approximations for the secondary circuit) and the concentration of stress first in the conducting material (e.g., Cu), but partially supported by the larger diameter of insulation (with a much lower elastic modulus). The total force (for the shorted secondary example) is:
We may estimate an average equivalent pressure by dividing this by the surface area of the primary, \( 4\pi r_p R \):

\[
p = \frac{\mu N_p^2 J_p^2 (r_p/R) [ (r_p/r_t)^2 - 1/2 ]}{4\pi^2 r_p R}
\]  

(27)

The stress in the primary (as compression in the minor hoop direction) may then be written as:

\[
S_{hp} = p (r_p/d_p) f_{cp}
\]

(28)

where \( d_p \) is the thickness of the primary turn and \( f_{cp} \) is a concentration factor for the uniformly applied stress compared to the discrete size of the turns:

\[
f_{cp} = 2\pi R d_p / N_p N_c (\pi d_p^2 / 4)
\]

(29)

with \( N_c \) the number of cables that are used to assemble the primary. Substitution provides:

\[
S_{hp} = 2\mu N_p J_p^2 (r_p/R) [ (r_p/r_t)^2 - 1/2 ] / N_c \pi^2 d_p^2
\]

(30)

Based on the same values previously employed, and with \( N_c = 48 \), we have:

\[
S_{hp} = 2.13 \times 10^6 \text{ [psi]} / d_p^2 \text{[mm]}
\]

(31)

For a center conductor diameter of 0.115" (2.92 mm), the stress in the copper (unsupported by the surrounding plastic) is about 250 kpsi, so the copper would yield, if this were a static load. If we ignore the copper and use only the insulator, with a diameter of 0.375" (9.53 mm), the stress is reduced to 23.5 kpsi, which is more than the plastic would allow statically.

To consider the effects of the short pulse duration, we return to the impulse formulation, writing here the kinetic energy provided to a single turn in time \( \tau \) as:

\[
w_K = (S_{hp} A \tau^2 / 2) (2\pi r_p A) = S_{hp} A \tau^2 / 4\pi r_p
\]

(32)

where \( A = \pi d_p^2 / 4 \) is the cross-sectional area of the turn. The elastic energy is:

\[
w_e = S_m^2 (2\pi r_p A) / 2E
\]

(33)

so the maximum stress obtained by equating the kinetic and elastic energies is:

\[
S_m = S_{hp} (E/r)^{1/2} \tau / 2\pi r_p
\]

(34)

For plastic, with \( E = 100 \text{ kpsi} \) and \( r = 1.2 \text{ g/cm}^3 \), the elastic speed is \((E/r)^{1/2} = 753 \text{ m/s}\). With \( r_p = 4.74 \text{ cm} \) and \( \tau = 10 \mu s \) again, the maximum stress would be \( S_m = 594 \text{ psi} \). If we apply the same factor to reduce the stress in unsupported copper, the stress would become 6.3 kpsi. These impulsive stress values are not too far away from the yield strengths of plastic and copper, respectively, so more detailed calculations (e.g., including the actual current waveforms) are warranted.