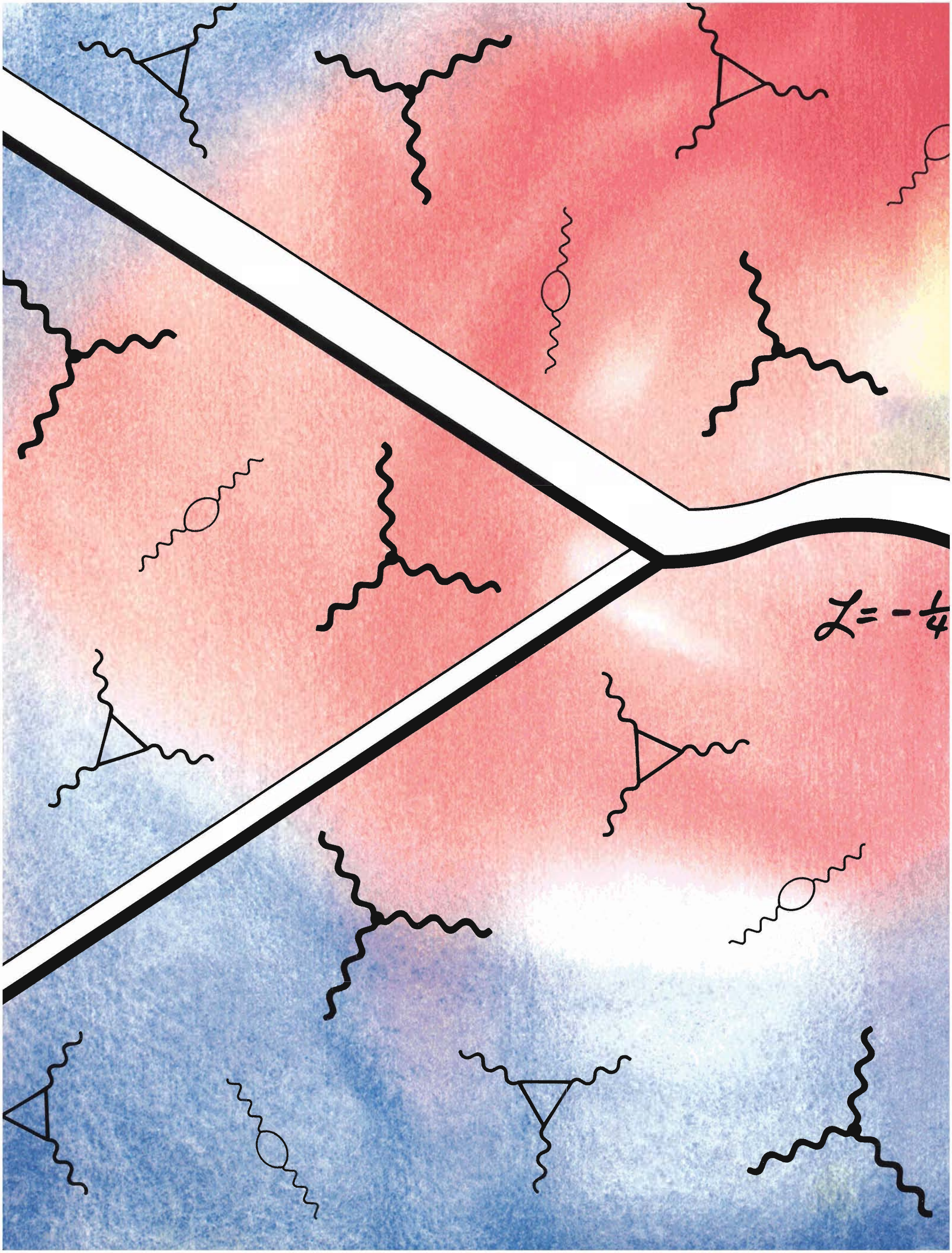

$\mathcal{L}=-\frac{1}{4}$

Particle Physics and the Standard Model

$$F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\gamma^{\mu}(\partial_{\mu} - ieA_{\mu})\Psi$$

by Stuart A. Raby, Richard C. Slansky, and Geoffrey B. West

Until the 1930s all natural phenomena were presumed to have their origin in just two basic forces—gravitation and electromagnetism. Both were described by classical fields that permeated all space. These fields extended out to infinity from well-defined sources, mass in the one case and electric charge in the other. Their benign rule over the physical universe seemed securely established.

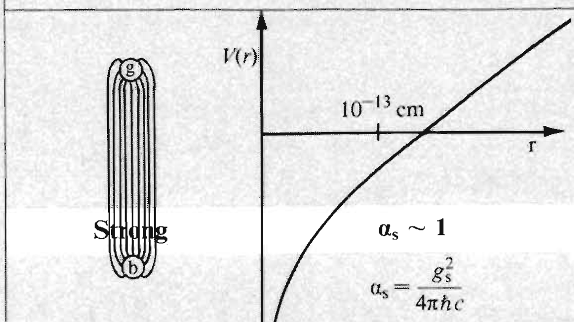
As atomic and subatomic phenomena were explored, it became apparent that two completely novel forces had to be added to the list; they were dubbed the weak and the strong. The strong force was necessary in order to understand how the nucleus is held together: protons bound together in a tight nuclear ball (10^{-12} centimeter across) must be subject to a force much stronger than electromagentism to prevent their flying apart. The weak force was invoked to understand the transmutation of a neutron in the nucleus into a proton during the particularly slow form of radioactive decay known as beta decay.

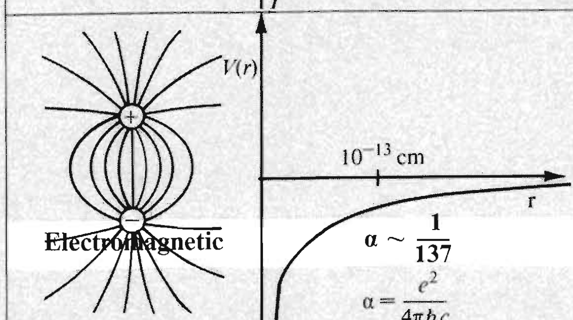
Since neither the weak force nor the strong force is directly observed in the macroscopic world, both must be very short-range relative to the more familiar gravitational and electromagnetic forces. Furthermore, the relative strengths of the forces associated with all four interactions are quite different, as can be seen in Table 1. It is therefore not too surprising that for a very long period these interactions were thought to be quite separate. In spite of this, there has always been a lingering suspicion (and hope) that in some miraculous fashion all four were simply manifestations of one source or principle and could therefore be described by a single unified field theory.

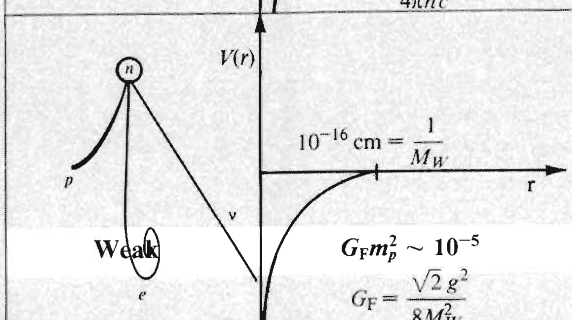
The color force among quarks and gluons is described by a generalization of the Lagrangian $\mathscr{L}$ of quantum electrodynamics shown above. The large interaction vertex dominating these pages is a common feature of the strong, the weak, and the electromagnetic forces. A feature unique to the strong force, the self-interaction of colored gluons, is suggested by the spiral in the background.

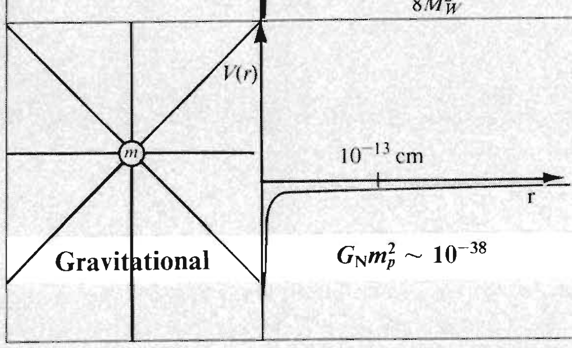
Table I

The four basic forces. Differences in strengths among the basic interactions are observed by comparing characteristic cross sections and particle lifetimes. (Cross sections are often expressed in barns because the cross-sectional areas of nuclei are of this order of magnitude; one barn equals 10^{-24} square centimeter.) The stronger the force, the larger is the effective scattering area, or cross section, and the shorter the lifetime of the particle state. At 1 GeV strong processes take place 10^2 times faster than electromagnetic processes and 10^5 times faster than weak processes.









Name	Relative Strength between Two Protons at 10^{-13}cm	Typical Scattering Cross Sections at 1 GeV in millibarns	Typical Lifetimes of Particle States	Representative Effects
Strong	$\boldsymbol{\alpha_s \sim 1}$ $\alpha_s = \dfrac{g_s^2}{4\pi\hbar c}$	Proton-Neutron Elastic Scattering $n + p \rightarrow n + p$ $\boldsymbol{\sigma \sim 10^1}$ **mb**	Strong Decay of the Delta $\Delta^+ \rightarrow p + \pi^0$ $\boldsymbol{\tau \sim 10^{-23}}$ **s**	Stable Proton (bound states of quarks) Stable Nuclei Fission Fusion
Electromagnetic	$\boldsymbol{\alpha \sim \dfrac{1}{137}}$ $\alpha = \dfrac{e^2}{4\pi\hbar c}$	Scattering of Light $\gamma + p \rightarrow \gamma + p$ $\boldsymbol{\sigma \sim 10^{-1}}$ **mb**	Electromagnetic Decay of the Rho $\rho \rightarrow \pi + \gamma$ $\boldsymbol{\tau \sim 10^{-21}}$ **s**	Light Radiowaves Stable Atoms and Molecules Chemical Reactions
Weak	$\boldsymbol{G_F m_p^2 \sim 10^{-5}}$ $G_F = \dfrac{\sqrt{2}\,g^2}{8M_W^2}$	Neutrino-Nucleon Scattering $\nu + p \rightarrow \nu + p$ $\boldsymbol{\sigma \sim 10^{-11}}$ **mb**	Weak Decay of the Muon $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ $\boldsymbol{\tau \sim 10^{-6}}$ **s**	Beta Decay Unstable Neutron Pion Decay Muon Decay
Gravitational	$\boldsymbol{G_N m_p^2 \sim 10^{-38}}$	Graviton-Proton Scattering $g + p \rightarrow g + p$ $\boldsymbol{\sigma \sim 10^{-77}}$ **mb**		Galaxies Solar Systems Curved Space-Time and Cosmology

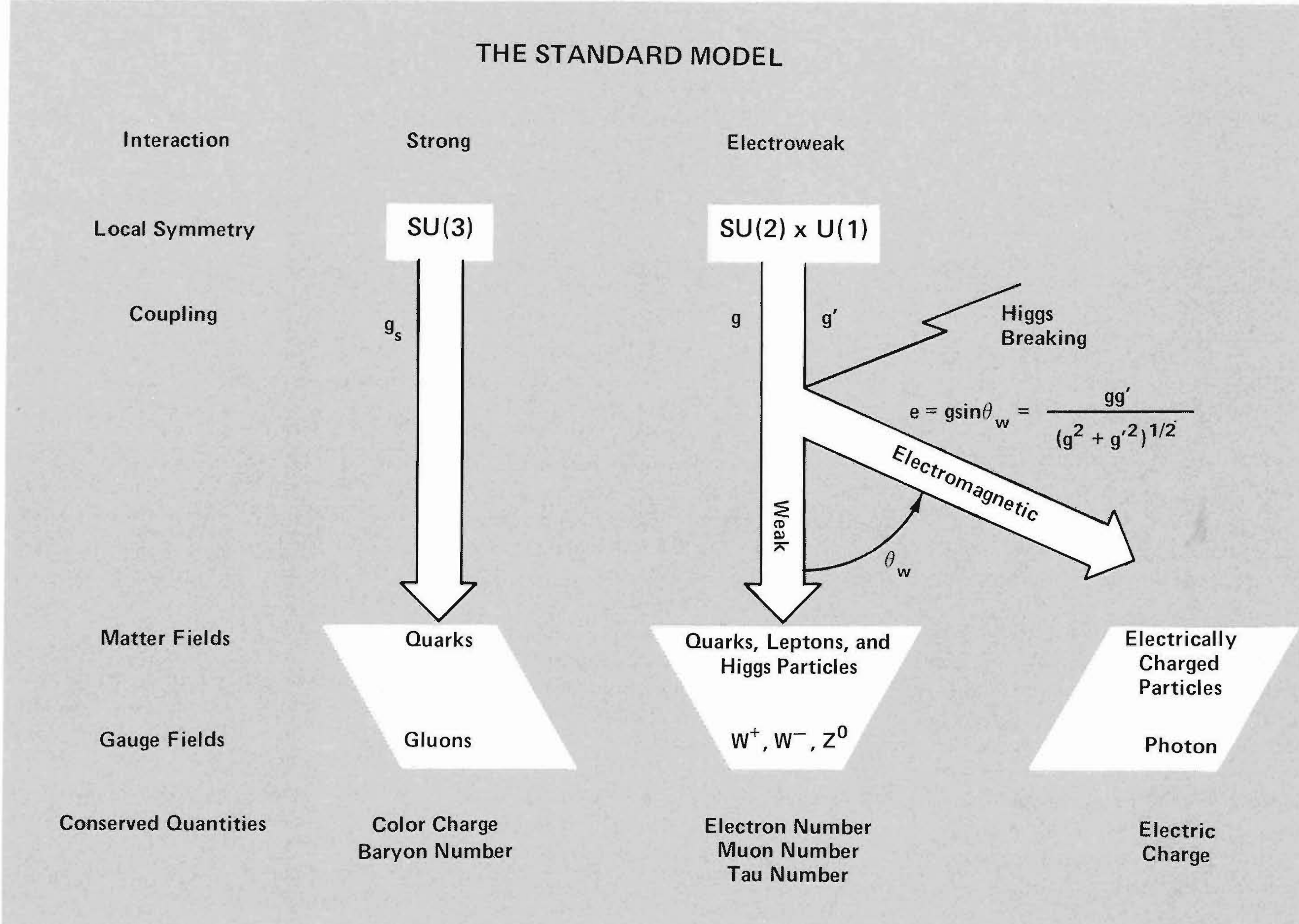


Fig. 1. The main features of the standard model. The strong force and the electroweak force are each induced by a local symmetry group, SU(3) and SU(2) × U(1), respectively. These two symmetries are entirely independent of each other. SU(3) symmetry (called the color symmetry) is exact and therefore predicts conservation of color charge. The SU(2) × U(1) symmetry of the electroweak theory is an exact symmetry of the Lagrangian of the theory but not of the solutions to the theory. The standard model ascribes this symmetry breaking to the Higgs particles, particles that create a nonzero weak charge in the vacuum (the lowest energy state of the system). The only conserved quantity that remains after the symmetry breaking is electric charge.

The spectacular progress in particle physics over the past ten years or so has renewed this dream; many physicists today believe that we are on the verge of uncovering the structure of this unified theory. The theoretical description of the strong, weak, and electromagnetic interactions is now considered well established, and, amazingly enough, the theory shows these forces to be quite similar despite their experimental differences. The weak and strong forces have sources analogous to, but more complicated than, electric charge, and, like the electromagnetic force, both can be described by a special type of field theory called a local gauge theory. This formulation has been so successful at explaining all known phenomenology up to energies of 100 GeV (1 GeV = 10^9 electron volts) that it has been coined "the standard model" and serves as the point of departure for discussing a grand unification of all forces, including that of gravitation.

The elements of the standard model are summarized in Fig. 1. In this description the basic constituents of matter are quarks and leptons, and these constituents interact with each other through the exchange of gauge particles (vector bosons), the modern analogue of force fields. These so-called local gauge interactions are inscribed in the language of Lagrangian quantum field theory, whose rich formalism contains mysteries that escape even its most faithful practitioners. Here we will introduce the central themes and concepts that have led to the standard model, emphasizing how its formalism enables us to describe all phenomenology of the strong, weak, and electromagnetic interactions as different manifestations of a single symmetry principle, the principle of local symmetry. As we shall see, the standard model has many arbitrary parameters and leaves unanswered a number of important questions. It can hardly be regarded as a thing of great beauty—unless one keeps in mind that it embodies a single unifying principle and therefore seems to point the way toward a grander unification.

For those readers who are more mathematically inclined, the arguments here are complemented by a series of lecture notes immediately following the main text and entitled "From Simple Field Theories to the Standard Model." The lecture notes introduce Lagrangian formalism and stress the symmetry principles underlying construction of the standard model. The main emphasis is on the classical limit of the model, but indications of its quantum generalizations are also included.

Unification and Extension

Two central themes of physics that have led to the present synthesis are "unification" and "extension." By "unification" we mean the coherent description of phenomena that are at first sight totally unrelated. This takes the form of a mathematical description with specific rules of application. A theory must not only describe the known phenomena but also make predictions of new ones. Almost all theories are incomplete in that they provide a description of phenomena only within a specific range of parameters. Typically, a theory changes as it is extended to explain phenomena over a larger range of parameters, and sometimes it even simplifies. Hence, the second theme is called extension—and refers in particular to the extension of theories to new length or energy scales. It is usually extension and the resulting simplification that enable unification.

Perhaps the best-known example of extension and unification is Newton's theory of gravity (1666), which unifies the description of ordinary-sized objects falling to earth with that of the planets revolving around the sun. It describes phenomena over distance scales ranging from a few centimeters up to 10^{25} centimeters (galactic scales). Newton's theory is superceded by Einstein's theory of relativity only when one tries to describe phenomena at extremely high densities and/or velocities or relate events over cosmological distance and time scales.

The other outstanding example of unification in classical physics is Maxwell's theory of electrodynamics, which unifies electricity with magnetism. Coulomb (1785) had established the famous inverse square law for the force between electrically charged bodies, and Biot and Savart (1820) and Ampère (1820-1825) had established the law relating the magnetic field **B** to the electric current as well as the law for the force between two electric currents. Thus it was known that static charges give rise to an electric field **E** and that moving charges give rise to a magnetic field **B**. Then in 1831 Faraday discovered that the field itself has a life of its own, independent of the sources. A time-dependent magnetic field induces an electric field. This was the first clear hint that electric and magnetic phenomena were manifestations of the same force field.

Until the time of Maxwell, the basic laws of electricity and magnetism were expressed in a variety of different mathematical forms, all of which left the central role of the fields obscure. One of Maxwell's great achievements was to rewrite these laws in a single formalism using the fields **E** and **B** as the fundamental physical entities, whose sources are the charge density ρ and the current density **J**, respectively. In this formalism the laws of electricity and magnetism are expressed as differential equations that manifest a clear interrelationship between the two fields. Nowadays they are usually written in standard vector notation as follows.

Coulomb's law:	$\nabla \cdot \mathbf{E} = 4\pi\rho/\varepsilon_0;$
Ampère's law:	$\nabla \times \mathbf{B} = 4\pi\mu_0\mathbf{J};$
Faraday's law:	$\nabla \times \mathbf{E} + \partial\mathbf{B}/\partial t = 0;$
and the absence of magnetic monopoles:	$\nabla \cdot \mathbf{B} = 0\,.$

The parameters ε_0 and μ_0 are determined by measuring Coulomb's force between two static charges and Ampère's force between two current-carrying wires, respectively.

Although these equations clearly "unite" **E** with **B**, they are incomplete. In 1865 Maxwell realized that the above equations were not consistent with the conservation of electric charge, which requires that

$$\nabla \cdot \mathbf{J} + \partial\rho/\partial t = 0\,.$$

This inconsistency can be seen from Ampère's law, which in its primitive form requires that

$$\nabla \cdot \mathbf{J} = (4\pi\mu_0)^{-1}\nabla \cdot (\nabla \times \mathbf{B}) \equiv 0\,.$$

Maxwell obtained a consistent solution by amending Ampère's law to read

$$\nabla \times \mathbf{B} = 4\pi\mu_0\mathbf{J} + \varepsilon_0\mu_0\frac{\partial\mathbf{E}}{\partial t}\,.$$

With this new equation, Maxwell showed that both **E** and **B** satisfy the wave equation. For example,

$$\left(\nabla^2 - \varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = 0\,.$$

This fact led him to propose the electromagnetic theory of light. Thus, from Maxwell's unification of electric and magnetic phenomena emerged the concept of electromagnetic waves. Moreover, the speed c of the electromagnetic waves, or light, is given by $(\varepsilon_0\mu_0)^{-1/2}$ and is thus determined uniquely in terms of purely static electric and magnetic measurements alone!

It is worth emphasizing that apart from the crucial change in Ampère's law, Maxwell's equations were well known to natural philosophers before the advent of Maxwell! The unification, however, became manifest only through his masterstroke of expressing them in terms of the "right" set of variables, namely, the fields **E** and **B**.

Extension to Small Distance Scales

Maxwell's unification provides an accurate description of large-scale electromagnetic phenomena such as radio waves, current flow, and electromagnets. This theory can also account for the effects of a medium, provided macroscopic concepts such as conductivity and permeability are introduced. However, if we try to extend it to very short distance scales, we run into trouble; the granularity, or quantum nature, of matter and of the field itself becomes important, and Maxwell's theory must be altered.

Determining the physics appropriate to each length scale is a crucial issue and has been known to cause confusion (see "Fundamental Constants and the Rayleigh-

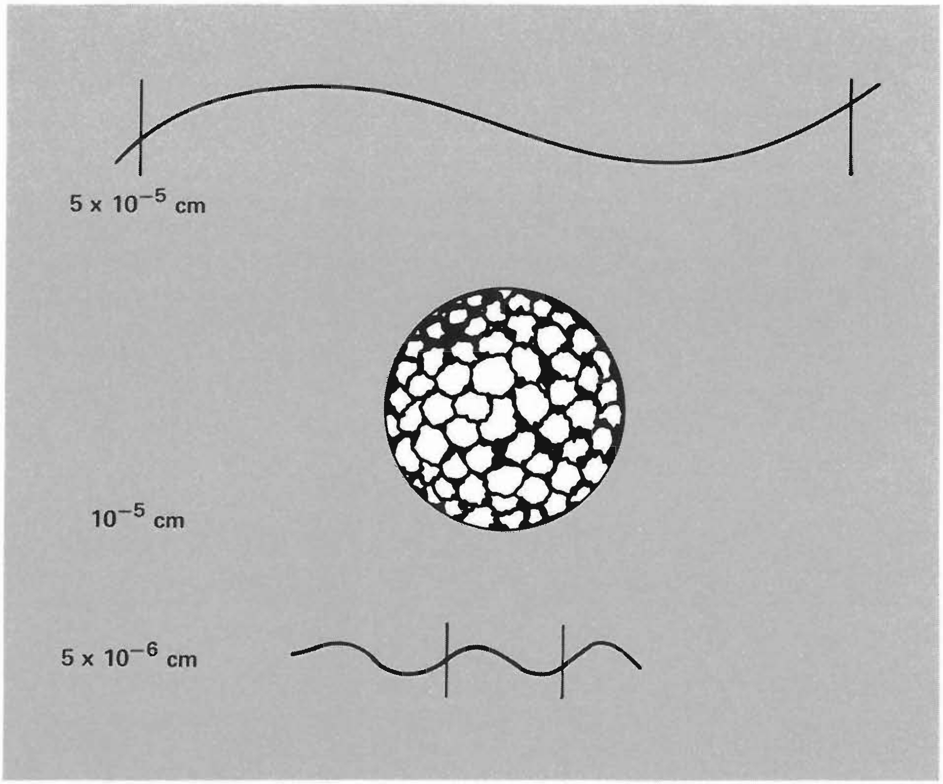


Fig. 2. The wavelength of the probe must be smaller than the scale of the structure one wants to resolve. Viruses, which are approximately 10^{-5} centimeter in extent, cannot be resolved with visible light, the average wavelength of which is 5×10^{-5} centimeter. However, electrons with momentum* p *of about 20 eV/*c *have de Broglie wavelengths short enough to resolve them.

Riabouchinsky Paradox"). For example, the structure of the nucleus is completely irrelevant when dealing with macroscopic distances of, say, 1 centimeter, so it would be absurd to try to describe the conductivity of iron over this distance in terms of its quark and lepton structure. On the other hand, it would be equally absurd to extrapolate Ohm's law to distance intervals of 10^{-13} centimeter to determine the flow of electric current. Relevant physics changes with scale!

The thrust of particle physics has been to study the behavior of matter at shorter and shorter distance scales in hopes of understanding nature at its most fundamental level. As we probe shorter distance scales, we encounter two types of changes in the physics. First there is the fundamental change resulting from having to use quantum mechanics and special relativity to describe phenomena at very short distances. According to quantum mechanics, particles have both wave and particle properties. Electrons can produce interference patterns as waves and can deposit all their energy at a point as a particle. The wavelength λ associated with the particle of momentum p is given by the de Broglie relation

$$\lambda = \frac{h}{p},$$

where h is Planck's constant ($h/2\pi \equiv \hbar = 1.0546 \times 10^{-27}$ erg $\cdot$ second). This relation is the basis of the often-stated fact that resolving smaller distances requires particles of greater momentum or energy. Notice, incidentally, that for sufficiently short wavelengths, one is *forced* to incorporate special relativity since the corresponding particle momentum becomes so large that Newtonian mechanics fails.

The marriage of quantum mechanics and special relativity gave birth to quantum field theory, the mathematical and physical language used to construct theories of the elementary particles. Below we will give a brief review of its salient features. Here we simply want to remind the reader that quantum field theory automatically incorporates quantum ideas such as Heisenberg's uncertainty principle and the dual wave-particle properties of all of matter, as well as the equivalence of mass and energy.

Since the wavelength of our probe determines the size of the object that can be studied (Fig. 2), we need extremely short wavelength (high energy) probes to investigate particle phenomena. To gain some perspective, consider the fact that with visible light we can see without aid objects as small as an amoeba (about 10^{-1} centimeter) and with an optical microscope we can open up the world of bacteria at about 10^{-4} centimeter. This is the limiting scale of light probes because wavelengths in the visible spectrum are on the order of 5×10^{-5} centimeter.

To resolve even smaller objects we can exploit the wave-like aspects of energetic particles as is done in an electron microscope. For example, with "high-energy" electrons ($E \approx 20$ eV) we can view the world of viruses at a length scale of about 10^{-5} centimeter. With even higher energy electrons we can see individual molecules (about 10^{-7} centimeter) and atoms (10^{-8} centimeter). To probe down to nuclear (10^{-12} centimeter) and subnuclear scales, we need the particles available from high-energy accelerators. Today's highest energy accelerators produce 100-GeV particles, which probe distance scales as small as 10^{-16} centimeter.

This brings us to the second type of change in appropriate physics with change in scale, namely, changes in the forces themselves. Down to distances of approximately 10^{-12} centimeter, electromagnetism is the dominant force among the elementary particles. However, at this distance the strong force, heretofore absent, suddenly comes into play and completely dominates the interparticle dynamics. The weak force, on the other hand, is present at all scales but only as a small effect. At the shortest distances being probed by present-day accelerators, the weak and electromagnetic forces become comparable in strength but remain several orders of magnitude weaker than the strong force. It is at this scale however, that the fundamental similarity of all three forces begins to emerge. Thus, as the scale changes, not only does each force itself change, but its relationship to the other forces undergoes a remarkable evolution. In our modern way of thinking, which has come from an understanding of the renormalization, or scaling, properties of quantum field theory, these changes in physics are in some ways analogous to the paradigm of phase transitions. To a young and naive child, ice, water, and steam appear to be quite different entities, yet rudimentary observations quickly teach that they are different manifestations of the same stuff, each associated with a different temperature scale. The modern lesson from renormalization group analysis, as discussed in "Scale and Dimension—From Animals to Quarks," is that the physics of the weak, electromagnetic, and strong forces may well represent different aspects of the same unified interaction. This is the philosophy behind grand unified theories of all the interactions.

Antiproton

By 1931 the negative energy states yielded for the electron by Dirac's relativistic quantum theory of 1928 had been interpreted, not as states of the proton (Dirac's initial thought), but as states of a particle with the same mass as the electron but opposite electric charge. Such a particle was found by chance in 1932 among the products of cosmic-ray collisions with nuclei. Searches for the antiproton (or negative proton) in the same environment proved unsuccessful, and physicists began considering its production by bombarding nuclei with energetic protons from an accelerator. Since electric charge and baryon number must be conserved in strong interactions, the production process involves creation of a proton-antiproton pair:

$$p + p\,(\text{or } n) \rightarrow (p + \bar{p}) + p + p\,(\text{or } n)\,.$$

This reaction has a threshold of approximately 5.7 GeV for the kinetic energy of the incident proton.

The Berkeley Bevatron was designed with antiproton production in mind, and this 6-GeV synchrotron enabled O. Chamberlain, E. Segrè, C. Wiegand, and T. Ypsilantis to make the first laboratory observation of the antiproton in 1955. Their identification method involved sorting out, from among the many products of the proton-nucleon collisions, negatively charged particles of a certain momentum with a bending magnet and further sorting out particles of the appropriate velocity, and hence mass, with two scintillation detectors spaced a known distance apart. Discovery of the antiproton strongly supported the idea that for every particle there exists an antiparticle with the same mass but opposite values of electric charge or other quantum properties. ■

Quantum Electrodynamics and Field Theory

Let us now return to the subject of electromagnetism at small distances and describe quantum electrodynamics (QED), the relativistic quantum field theory, developed in the 1930s and 1940s, that extends Maxwell's theory to atomic scales. We emphasize that the standard model is a generalization of this first and most successful quantum field theory.

In quantum field theory every particle has associated with it a mathematical operator, called a quantum field, that carries the particle's characteristic quantum numbers. Probably the most familiar quantum number is spin, which corresponds to an intrinsic angular momentum. In classical mechanics angular momentum is a continuous variable, whereas in quantum mechanics it is restricted to multiples of ½ when measured in units of $\hbar$. Particles with ½-integral spin (1/2, 3/2, 5/2, ...) are called fermions; particles with integral spin (0, 1, 2, 3, ...) are called bosons. Since no two identical fermions can occupy the same position at the same time (the famous Pauli exclusion principle), a collection of identical fermions must necessarily take up some space. This special property of fermions makes it natural to associate them with matter. Bosons, on the other hand, can crowd together at a point in space-time to form a classical field and are naturally regarded as the mediators of forces.

In the quantized version of Maxwell's theory, the electromagnetic field (usually in the guise of the vector potential A_μ) is a boson field that carries the quantum numbers of the photon, namely, mass $m = 0$, spin $s = 1$, and electric charge $Q = 0$. This quantized field, by the very nature of the mathematics, automatically manifests dual wave-particle properties. Electrically charged particles,

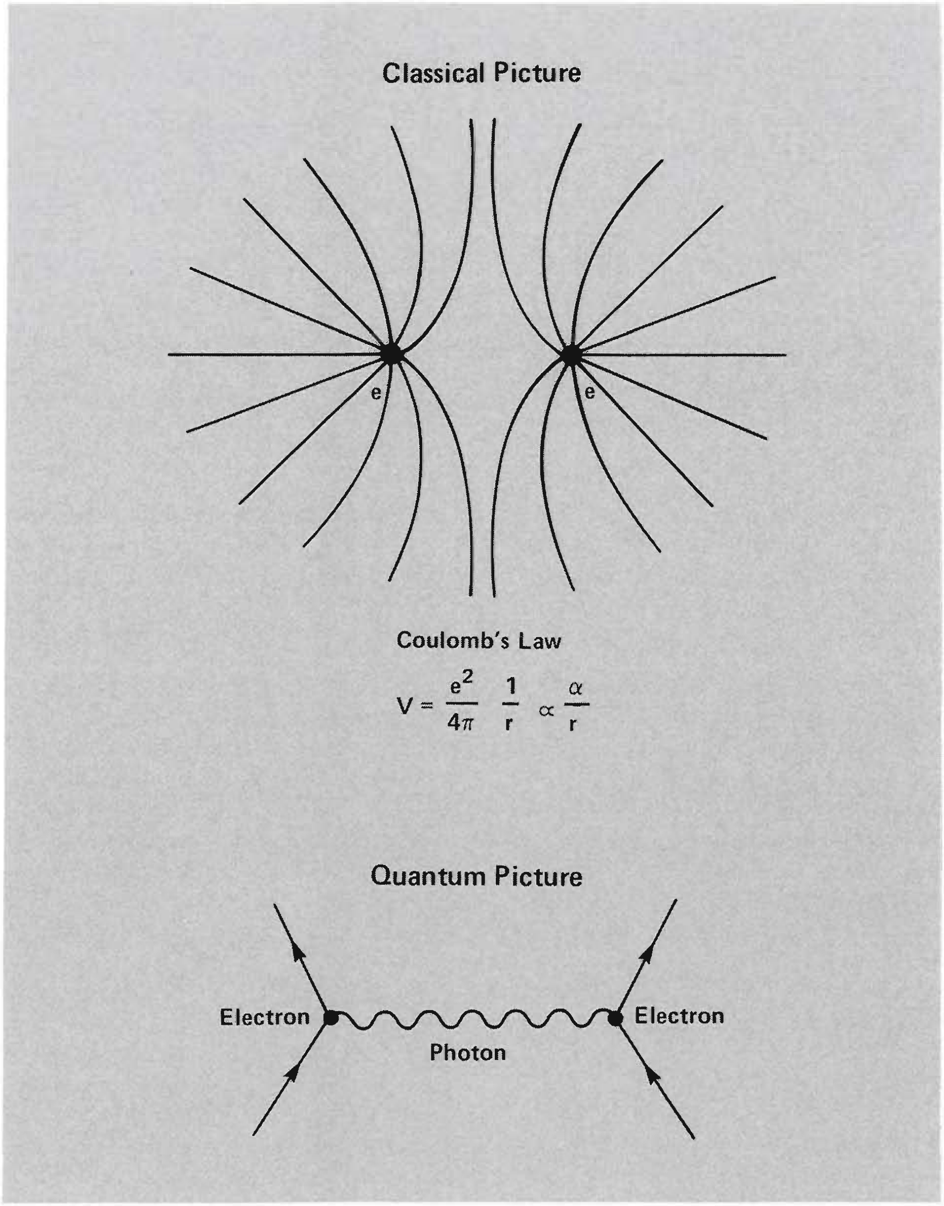


Fig. 3. (a) The force between two electrons is described classically by Coulomb's law. Each electron creates a force field (shown as lines emanating from the charge (e) that is felt by the other electron. The potential energy V is the energy needed to bring the two electrons to within a distance r of each other. (b) In quantum field theory two electrons feel each other's presence by exchanging virtual photons, or virtual particles of light. Photons are the quanta of the electromagnetic field. The Feynman diagram above represents the (lowest order, see Fig. 5) interaction between two electrons (straight lines) through the exchange of a virtual photon (wavy line).

such as electrons and positrons, are also represented by fields, and, as in the classical theory, they interact with each other through the electromagnetic field. In QED, however, the interaction takes place via an exchange of photons. Two electrons "feel" each other's presence by passing photons back and forth between them. Figure 3 pictures the interaction with a "Feynman diagram": the straight lines represent charged particles and the wavy line represents a photon. (In QED such diagrams correspond to terms in a perturbative expansion for the scattering between charged particles (see Fig. 5). Similarly, most Feynman diagrams in this issue represent lowest order contributions to the particle reactions shown.)

These exchanged photons are rather special. A real photon, say in the light by which you see, must be massless since only a massless particle can move at the speed of light. On the other hand, consider the left-hand vertex of Fig. 3, where a photon is emitted by an electron; it is not difficult to convince oneself that if the photon is massless, energy and momentum are not conserved! This is no sin in quantum mechanics, however, as Heisenberg's uncertainty principle permits such violations provided they occur over sufficiently small space-time intervals. Such is the case here: the violating photon is absorbed at the right-hand vertex by another electron in such a way that, overall, energy and momentum are conserved. The exchanged photon is "alive" only for a period concomitant with the constraints of the uncertainty principle. Such photons are referred to as virtual photons to distinguish them from real ones, which can, of course, live forever.

The uncertainty principle permits all sorts of virtual processes that momentarily violate energy-momentum conservation. As illustrated in Fig. 4, a virtual photon being exchanged between two electrons can, for a very short time, turn into a virtual electron-positron pair. This conversion of energy into mass is allowed by the famous equation of special relativity, $E = mc^2$. In a similar fashion almost anything that *can* happen *will* happen, given a sufficiently small space-time interval. It is the countless multitude of such virtual processes that makes quantum field theory so rich and so difficult.

Given the immense complexity of the theory, one wonders how any reliable calculation can ever be made. The saving grace of quantum electrodynamics, which has made its predictions the most accurate in all of physics, is the smallness of the coupling between the electrons and the photons. The coupling strength at each vertex where an electron spews out a virtual photon is just the electronic charge e, and, since the virtual photon must be absorbed by some other electron, which also has charge e, the probability for this virtual process is of magnitude e^2. The corresponding *dimensionless* parameter that occurs naturally in this theory is denoted by α and defined as $e^2/4\pi\hbar c$. It is approximately equal to 1/137. The probabilities of more complicated virtual processes involving many virtual particles are proportional to higher powers of α and are therefore very much smaller relative to the probabilities for simpler ones. Put slightly differently, the smallness of α implies that perturbation theory is applicable, and we can control the level of accuracy of our calculations by including higher and higher order virtual processes (Fig. 5). In fact, quantum electrodynamic calculations of certain atomic and electronic properties agree with experiment to within one part in a billion.

As we will elaborate on below, the quantum field theories of the electroweak and the strong interactions that compose the standard model bear many resemblances to quantum electrodynamics. Not too surprisingly, the coupling strength of the weak interaction is also small (and in fact remains small at all energy or distance scales), so perturbation theory is always valid. However, the analogue of α for the strong interaction is not always small, and in many calculations perturbation theory is inadequate. Only at the high energies above 1 GeV, where the theory is said to be asymptotically free, is the analogue of α so small that perturbation theory is valid. At low and moderate energies

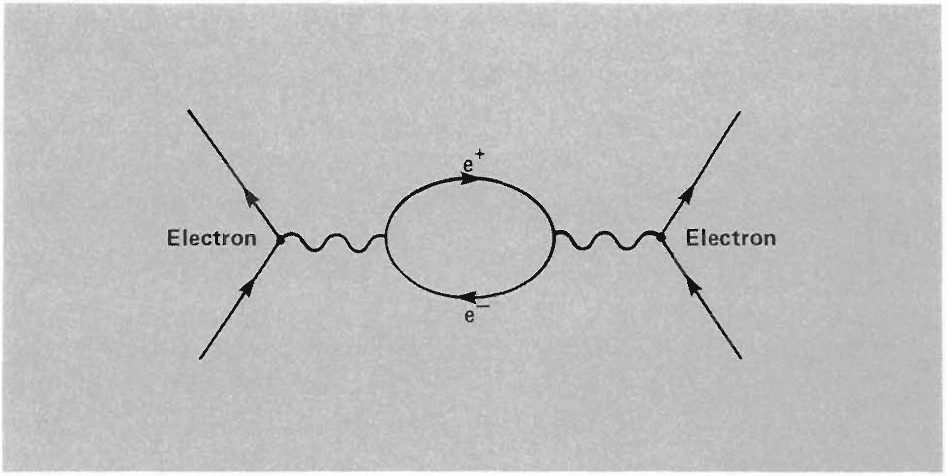


Fig. 4. A virtual photon being exchanged between two electrons can, for a very short time, turn into a virtual electron-positron (e^+-e^-) pair. This virtual process is one of many that contribute to the electromagnetic interaction between electrically charged particles (see Fig. 5).

(for example, those that determine the properties of protons and neutrons) the strong-interaction coupling strength is large, and analytic techniques beyond perturbation theory are necessary. So far such techniques have not been very successful, and one has had to resort to the nasty business of numerical simulations!

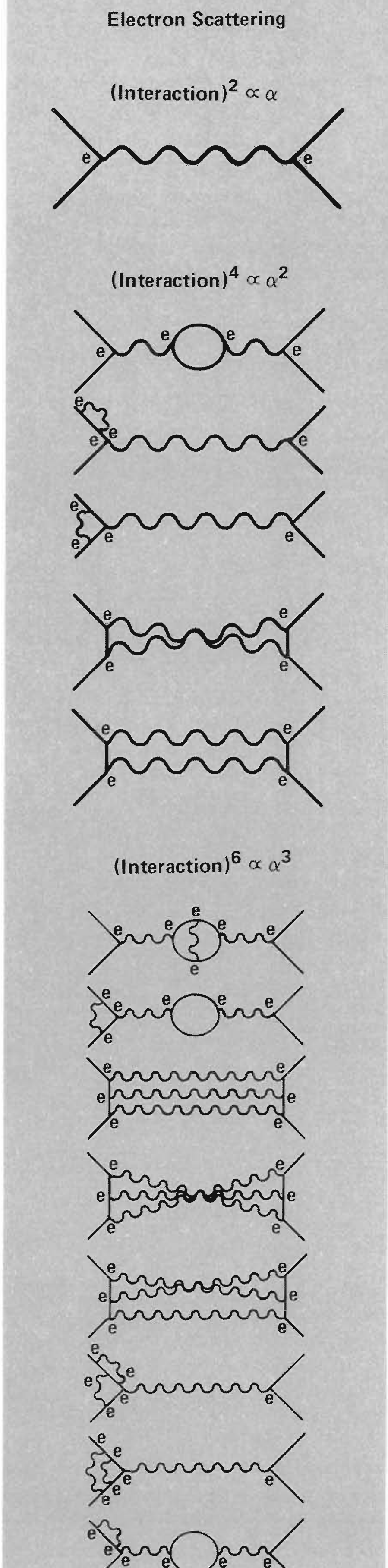
As discussed at the end of the previous section, these changes in coupling strengths with changes in scale are the origin of the changes in the forces that might lead to a unified theory. For an example see Fig. 3 in "Toward a Unified Theory."

Symmetries

One cannot discuss the standard model without introducing the concept of symmetry. It has played a central role in classifying the known particle states (the ground states of 200 or so particles plus excited states) and in predicting new ones. Just as the chemical elements fall into groups in the periodic table, the particles fall into multiplets characterized by similar quantum numbers. However, the use of symmetry in particle physics goes well beyond mere classification. In the construction of the standard model, the special kind of symmetry known as local symmetry has become *the* guiding dynamical principle; its aesthetic influence in the search for unification is reminiscent of the quest for beauty among the ancient Greeks. Before we can discuss this dynamical principle, we must first review the general concept of symmetry in particle physics.

In addition to electric charge and mass, particles are characterized by other quantum numbers such as spin, isospin, strangeness, color, and so forth. These quantum numbers reflect the symmetries of physical laws and are used as a basis for classification and, ultimately, unification.

Although quantum numbers such as spin and isospin are typically the distinguishing features of a particle, it is probably less well known that the mass of a particle is sometimes its only distinguishing feature. For example, a muon (μ) is distinguished from an electron (e) only because its mass is 200 times greater that that of the electron. Indeed, when the muon was discovered in 1938, Rabi was reputed to have made the remark, "Who ordered *that*?" And the tau



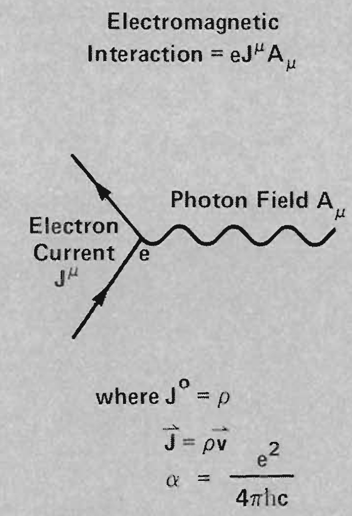


Fig. 5. As shown above, the basic interaction vertex of quantum electrodynamics is an electron current J^μ ***interacting with the electromagnetic field*** A_μ***. Because the coupling strength*** α ***is small, the amplitude for processes involving such interactions can be approximated by a perturbation expansion on a free field theory. The terms in such an expansion, shown at left for electron scattering, are proportional to various powers of*** α***. The largest contribution to the electron-scattering amplitude is proportional to*** α ***and is represented by a Feynmann diagram in which the interaction vertex appears twice. Successively smaller contributions arise from terms proportional to*** α^2 ***with four interaction vertices, from terms proportional to*** α^3 ***with six interaction vertices, and so on.***

(τ), discovered in 1973, is 3500 times heavier than an electron yet again identical to the electron in other respects. One of the great unsolved mysteries of particle physics is the origin of this apparent hierarchy of mass among these leptons. (A lepton is a fundamental fermion that has no strong interactions.) Are there even more such particles? Is there a reason why the mass hierarchy among the leptons is paralleled (as we will describe below) by a similar hierarchy among the quarks? It is believed that when we understand the origin of fermion masses, we will also understand the origin of CP violation in nature (see box). These questions are frequently called the family problem and are discussed in the article by Goldman and Nieto.

Groups and Group Multiplets. Whether or not the similarity among e, μ, and τ reflects a fundamental symmetry of nature is not known. However, we will present several possibilities for this family symmetry to introduce the language of groups and the significance of internal symmetries.

Consider a world in which the three leptons have the same mass. In this world atoms with muons or taus replacing electrons would be indistinguishable: they would have identical electromagnetic absorption or emission bands and would form identical elements. We would say that this world is *invariant* under the interchange of electrons, muons, and taus, and we would call this invariance a *symmetry of nature*. In the real world these particles don't have the same mass; therefore our hypothetical symmetry, if it exists, is broken and we can distinguish a muonic atom from, say, its electronic counterpart.

We can describe our hypothetical invariance or family symmetry among the three leptons by a set of symmetry operations that form a mathematical construct called a *group*. One property of a group is that any two symmetry operations performed in succession also corresponds to a symmetry operation in that group. For example, replacing an electron with a muon, and then replacing a muon with a tau can be defined as two *discrete* symmetry operations that when performed in succession are equivalent to the discrete symmetry operation of replacing an electron with a tau. Another group property is that every operation must have an inverse. The inverse of replacing an electron with a muon is replacing a muon with an electron. This set of discrete operations on e, μ, and τ forms the discrete six-element group π_3 (with π standing for permutation). In this language e, μ, and τ are called a *multiplet* or *representation* of π_3 and are said to transform as a triplet under π_3.

Another possibility is that the particles e, μ, and τ transform as a triplet under a group of *continuous* symmetry operations. Consider Fig. 6, where e, μ, and τ are represented as three orthogonal vectors in an abstract three-dimensional space. The set of continuous rotations of the three vectors about three independent axes composes the group known as the three-dimensional rotation group and denoted by SO(3). As shown in Fig. 6, SO(3) has three independent transformations, which are represented by orthogonal 3×3 matrices. (Note that π_3 is a subset of SO(3).)

Suppose that SO(3) were an unbroken family symmetry of nature and e, μ, and τ transformed as a triplet under this symmetry. How would it be revealed experimentally? The SO(3) symmetry would add an extra degree of freedom to the states that could be formed by e, μ, and τ. For example, the spatially symmetric ground state of helium, which ordinarily must be antisymmetric under the interchange of the two electron spins, could now be antisymmetric under the interchange of either the spin or the family quantum number of the two leptons. In particular, the ground state would have three different antisymmetric configurations and the threefold degeneracy might be split by spin-spin interactions among the leptons and by any SO(3) symmetric interaction. Thus the ground state of known helium would probably be replaced by sets of degenerate levels with small hyperfine energy splittings.

In particle physics we are always interested in the largest group of operations that leaves all properties of a system unchanged. Since e, μ, and τ are described by complex fields, the largest group of operations that could act on this triplet is U(3) (the group of all unitary 3×3 matrices U satisfying $U^\dagger U = 1$). Another possibility is SU(3), a subgroup of U(3) satisfying the additional constraint that $\det U = 1$.

This list of symmetries that may be reflected in the similarity of e, μ, and τ is not exhaustive. We could invoke a group of symmetry operations that acts on any subset of the three particles, such as SU(2) (the group of 2×2 unitary matrices with $\det U = 1$) acting, say, on e and μ as a doublet and on τ as a singlet. Any one of these possibilities may be realized in nature, and each possibility has different experimentally observable

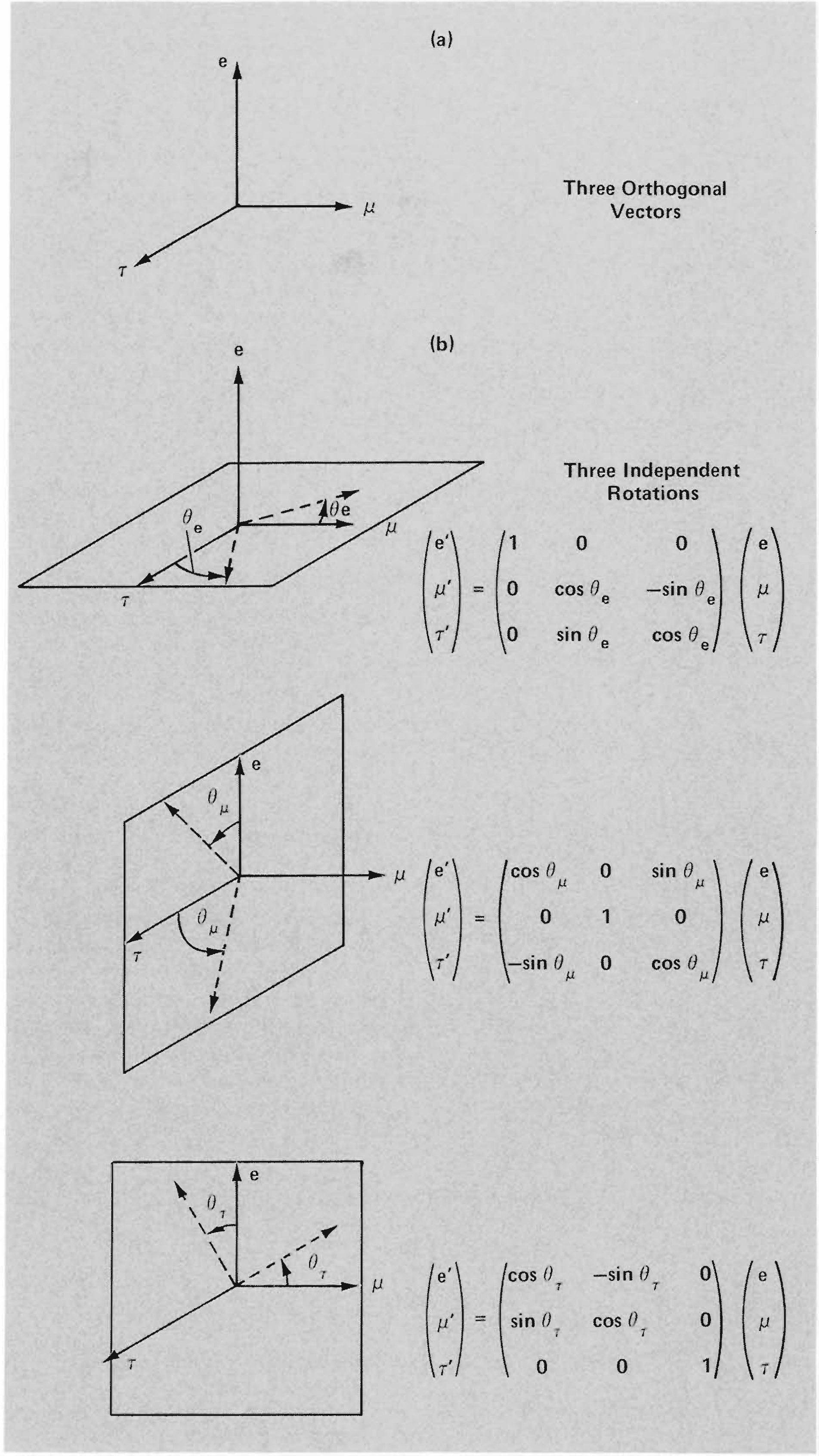


Fig. 6. (a) The three leptons e, μ, and τ are represented as three orthogonal vectors in an abstract three-dimensional space. (b) The set of rotations about the three orthogonal axes defines SO(3), the three-dimensional rotation group. SO(3) has three charges (or generators) associated with the infinitesimal transformations about the three independent axes. These generators have the same Lie algebra as the generators of the group SU(2), as discussed in Lecture Note 4 following this article.

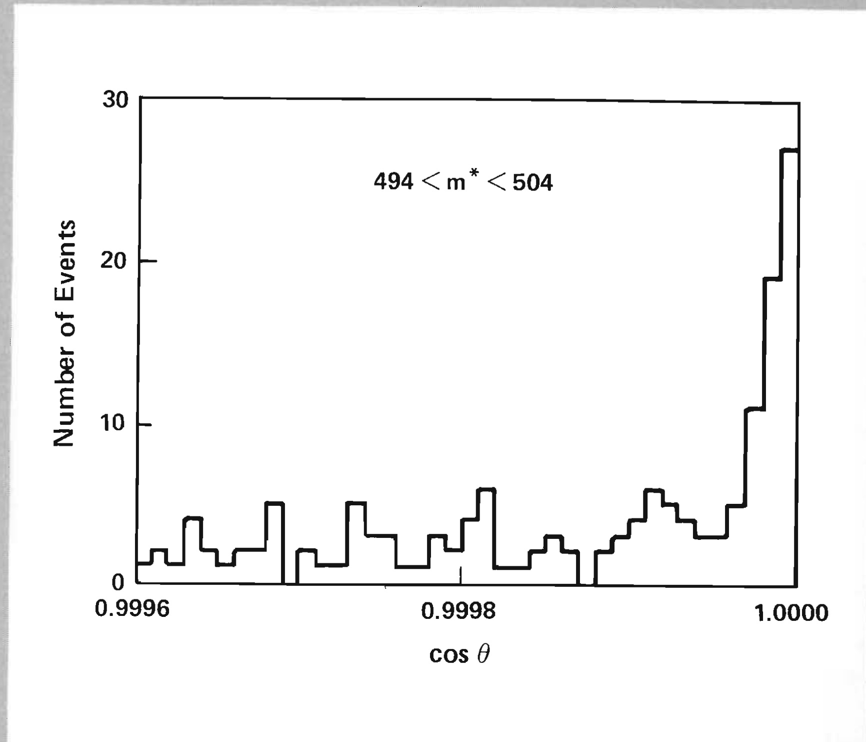
consequences. However, the known differences in the masses of e, μ, and τ imply that *any* symmetry used to describe the similarity among them is a broken symmetry. Still, a broken symmetry will retain traces of its consequences (if the symmetry is broken by a small amount) and thus also provides useful predictions.

Our hypothetical broken symmetry among e, μ, and τ is but one example of an approximate *internal* global symmetry. Another is the symmetry between, say, the neutron and the proton in strong interactions, which is described by the group known as strong-isospin SU(2). The neutron and proton transform as a doublet under this symmetry and the three pions transform as a triplet. We will discuss below the classifica-

CP Violation

The faith of physicists in symmetries of nature, so shaken by the observation of parity violation in 1956, was soon restored by invocation of a new symmetry principle—CP conservation—to interpret parity-violating processes. This principle states that a process is indistinguishable from its mirror image provided all particles in the mirror image are replaced by their antiparticles. Alas, in 1964 this principle also was shattered with the results of an experiment on the decay of neutral kaons.

According to the classic analysis of M. Gell-Mann and A. Pais, neutral kaons exist in two forms: K^0_S, with an even CP eigenvalue and decaying with a relatively short lifetime of 10^{-10} second into two pions, and K^0_L, with an odd CP eigenvalue and decaying with a lifetime of about 5×10^{-8} second into three pions. CP conservation prohibited the decay of the longer lived K^0_L into two pions. But in an experiment at Brookhaven, J. Christenson, J. Cronin, V. Fitch, and R. Turlay found that about 1 in 500 K^0_L mesons decays into two pions. This first observation of CP violation has been confirmed in many other experiments on the neutral kaon system, but to date no other CP-violating effects have been found. The underlying mechanism of CP violation remains to be understood, and an implication of the phenomenon, the breakdown of time-reversal invariance (which is necessary to maintain CPT conservation), remains to be observed. ■



Evidence for the CP-violating decay of K^0_L into two pions. Here the number of events in which the invariant mass (m*) of the decay products was in close proximity to the mass of the neutral kaon is plotted versus the cosine of the angle θ between the K^0_L beam and the vector sum of the momenta of the decay products. The peak in the number of events at cos θ ≅ 1 (indicative of two-body decays) could only be explained as the decay of K^0_L into two pions with a branching ratio of about 2×10^{-3}. (Adapted from "Evidence for the 2π Decay of the K^0_L Meson" by J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Physical Review Letters ***13(1964):138.)***

tion of strongly interacting particles into multiplets of SU(3), a scheme that combines strong isospin with the quantum number called strangeness, or strong hypercharge. (For a more complete discussion of continuous symmetries and internal global symmetries such as SU(2), see Lecture Notes 2 and 4.)

Exact, or unbroken, symmetries also play a fundamental role in the construction of theories: exact rotational invariance leads to the exact conservation of angular momentum, and exact translational invariance in space-time leads to the exact conservation of energy and momentum. We will now discuss how the exact phase invariance of electrodynamics leads to the exact conservation of electric charge.

Global U(1) Invariance and Conservation Laws. In quantum field theory the dynamics of a system are encoded in a function of the fields called a Lagrangian, which is related to the energy of the system. The Lagrangian is the most convenient means for studying the symmetries of the theory because it is usually a simple task to check if the Lagrangian remains unchanged under particular symmetry operations.

An electron is described in quantum field theory by a complex field,

$$\psi_{\text{electron}} = (\psi_1 + i\psi_2)/\sqrt{2}\,,$$

and a positron is described by the complex conjugate of that field,

$$\psi_{\text{positron}} = (\psi_1 - i\psi_2)/\sqrt{2}\,.$$

Although the real fields ψ_1 and ψ_2 are separately each able to describe a spin-½ particle, the two together are necessary to describe a particle with electric charge.*

The Lagrangian of quantum electrodynamics is unchanged by the continuous operation of multiplying the electron field by an arbitrary phase, that is, by the transformation

$$\psi \rightarrow e^{i\Lambda Q}\psi\,,$$

where Λ is an arbitary real number and Q is the electric charge operator associated with the field. The eigenvalue of Q is -1 for an electron and $+1$ for a positron. This set of phase transformations forms the global symmetry group U(1) (the set of unitary 1×1 matrices). In QED this symmetry is unbroken, and electric charge is a conserved quantum number of the system.

There are other global U(1) symmetries relevant in particle physics, and each one implies a conserved quantum number. For example, baryon number conservation is associated with a U(1) phase rotation of all baryon fields by an amount $e^{i\Lambda B}$, where $B = 1$ for protons and neutrons, $B = 1/3$ for quarks, and $B = 0$ for leptons. Analogously, electron number is conserved if the field of the electron neutrino is assigned the same electron number as the field of the electron and all other fields are assigned an electron number of zero. The same holds true for muon number and tau number. Thus a global U(1) phase symmetry seems to operate on each type of lepton. (Possible violation of muon-number conservation is discussed in "Experiments To Test Unification Schemes.")

The Principle of Local Symmetry

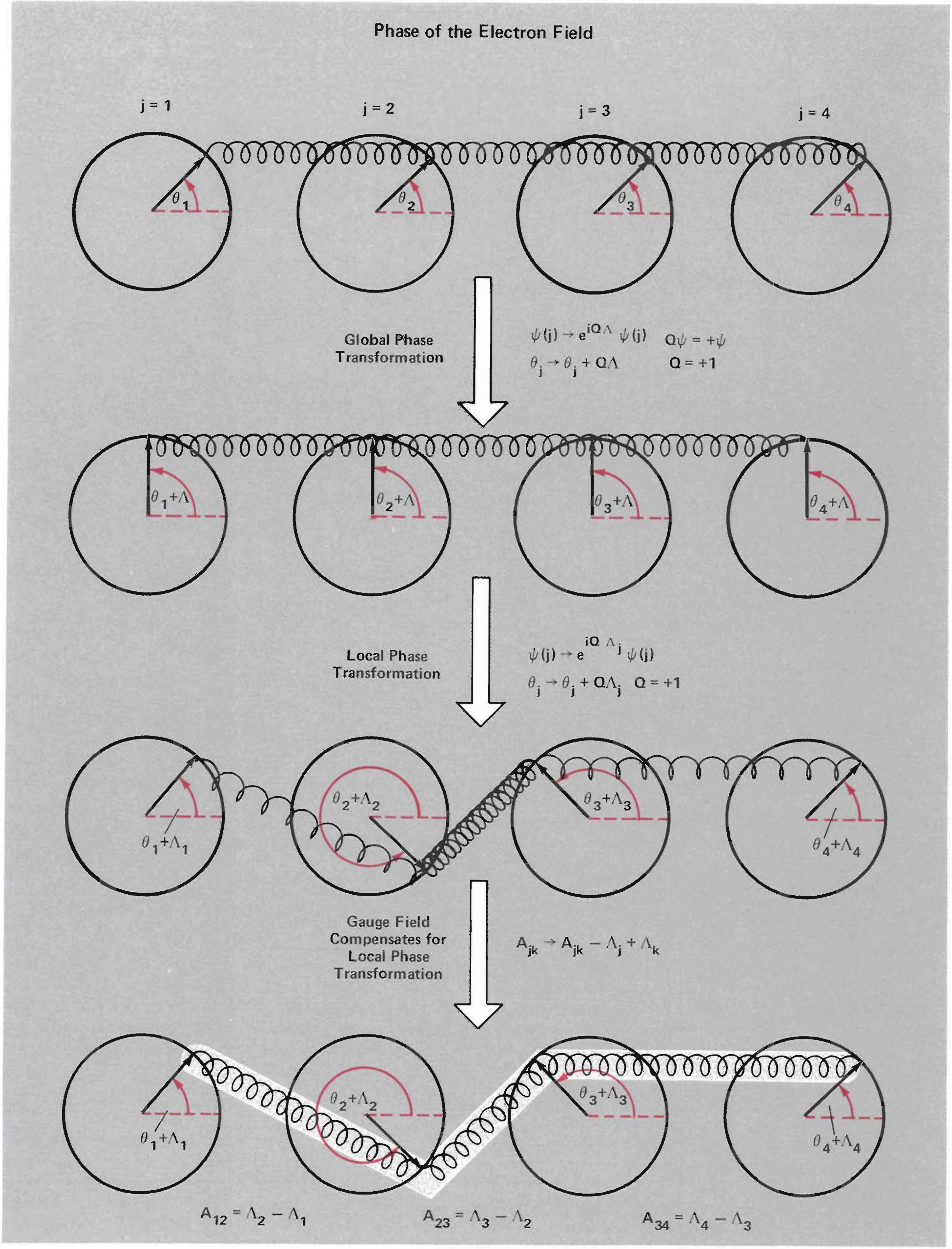
We are now ready to distinguish a global phase symmetry from a local one and examine the dynamical consequences that emerge from the latter. Figure 7 illustrates what happens to the electron field under the global phase transformation $\psi \rightarrow e^{i\Lambda Q}\psi$. For convenience, space-time is represented by a set of discrete points labeled by the index j. The phase of the electron field at each point is represented by an arrow that rotates about the point, and the kinetic energy of the field is represented by springs connecting the arrows at different space-time points. A *global* U(1) transformation rotates every two-dimensional vector by the same arbitrary angle Λ: $\theta_j \rightarrow \theta_j + Q\Lambda$, where Q is the electric charge. In order for the Lagrangian to be invariant under this global phase rotation, it is clearly sufficient for it to be a function only of the phase *differences* $(\theta_i - \theta_j)$. Both the free electron terms and the interaction terms in the QED Lagrangian are invariant under this continuous global symmetry.

A local U(1) transformation, in contrast, rotates every two-dimensional vector by a *different* angle Λ_j. This local transformation, unlike its global counterpart, does *not* leave the Lagrangian of the free electron invariant. As represented in Fig. 7 by the stretching and compressing of the springs, the kinetic energy of the electron changes under local phase transformations. Nevertheless, the full Lagrangian of quantum electrodynamics *is* invariant under these local U(1) transformations. The electromagnetic field (A_μ) precisely compensates for the local phase rotation and the Lagrangian is left invariant. This is represented in Fig. 7 by restoring the stretched and compressed springs to their initial tension. Thus, the kinetic energy of the electron (the energy stored in the springs) is the same before and after the local phase transformation.

In our discrete notation, the full La-

Fig. 7. Global versus local phase transformations. The arrows represent the phases of an electron field at four discrete points labeled by j = 1, 2, 3, and 4. The springs represent the kinetic energy of the electrons. A global phase transformation does not change the tension in the springs and therefore costs no energy. A local phase transformation* without *gauge interactions stretches and compresses the springs and thus does cost energy. However introduction of the gauge field (represented by the white haze) exactly compensates for the local phase transformation of the electron field and the springs return to their original tension so that local phase transformations* with *gauge interactions do not cost energy.

**The real fields ψ_1 and ψ_2 are four-component Majorana fields that together make up the standard four-component complex Dirac spinor field.*


Phase of the Electron Field
j = 1
j = 2
j = 3
j = 4
θ_1
θ_2
θ_3
θ_4
Global Phase Transformation
$\psi(j) \rightarrow e^{iQ\Lambda}\,\psi(j)$
$Q\psi = +\psi$
$\theta_j \rightarrow \theta_j + Q\Lambda$
$Q = +1$
$\theta_1+\Lambda$
$\theta_2+\Lambda$
$\theta_3+\Lambda$
$\theta_4+\Lambda$
Local Phase Transformation
$\psi(j) \rightarrow e^{iQ\Lambda_j}\,\psi(j)$
$\theta_j \rightarrow \theta_j + Q\Lambda_j$
$Q = +1$
$\theta_1+\Lambda_1$
$\theta_2+\Lambda_2$
$\theta_3+\Lambda_3$
$\theta_4+\Lambda_4$
Gauge Field Compensates for Local Phase Transformation
$A_{jk} \rightarrow A_{jk} - \Lambda_j + \Lambda_k$
$\theta_1+\Lambda_1$
$\theta_2+\Lambda_2$
$\theta_3+\Lambda_3$
$\theta_4+\Lambda_4$
$A_{12} = \Lambda_2 - \Lambda_1$
$A_{23} = \Lambda_3 - \Lambda_2$
$A_{34} = \Lambda_4 - \Lambda_3$

grangian is a function of $\theta_j - \theta_k + A_{jk}Q$ and is invariant under the simultaneous transformations

$$\theta_j \rightarrow \theta_j + Q\Lambda_j \quad \text{and} \quad A_{jk} \rightarrow A_{jk} - \Lambda_j + \Lambda_k \,.$$

The matrix with elements A_{jk} is the discrete space-time analogue of the electromagnetic potential defined on the links between the points k and j. Thus, if one starts with a theory of free electrons with no interactions and demands that the physics remain invariant under local phase transformation of the electron fields, then one induces the standard electromagnetic interactions between the electron current J^μ and photon field A_μ, as shown in Figs. 5 and 8. From this point of view, Maxwell's equations can be viewed as a *consequence* of the local U(1) phase invariance. Although this local invariance was originally viewed as a curiosity of QED, it is now viewed as the guiding principle for constructing field theories. The invariance is usually termed *gauge invariance*, and the photon is referred to as a gauge particle since it mediates the U(1) gauge interaction. It is worth emphasizing that local U(1) invariance implies that the photon is massless because the term that would describe a massive photon is *not* itself invariant under local U(1) transformations.

The local gauge invariance of QED is the prototype for theories of both the weak and the strong interactions. Obviously, since neither of these is a long-range interaction, some additional features must be at work to account for their different properties. Before turning to a discussion of these features, we stress that in theories based on local gauge invariance, currents always play an important role. In classical electromagnetism the fundamental interaction takes place between the vector potential and the electron current; this is reflected in quantum electrodynamics by Feynman diagrams: the virtual photon (the gauge field) ties into the current produced by the moving electron (see Fig. 8). As will become clear below, a similar situation exists in the strong interaction and, more important, in the weak interaction.

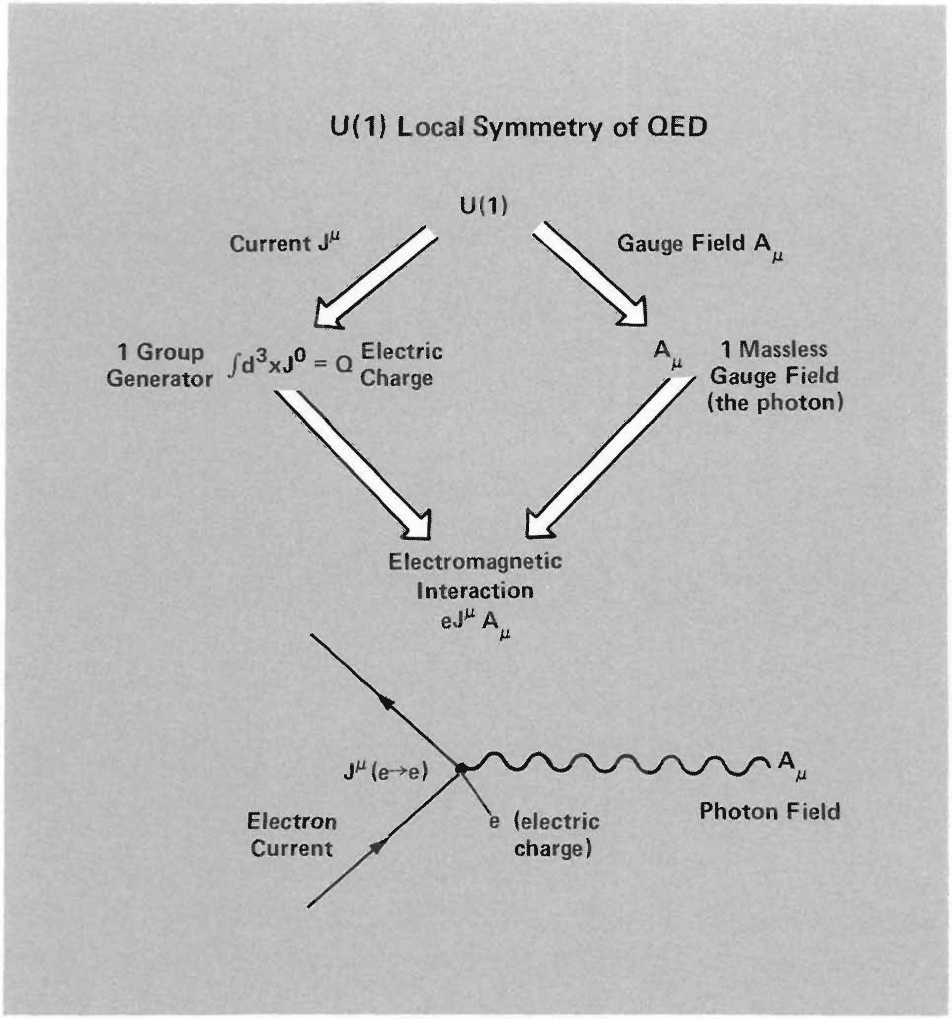


Fig. 8. The U(1) local symmetry of QED implies the existence of a gauge field to compensate for the local phase transformation of the electrically charged matter fields. The generator of the U(1) local phase transformation is Q, the electric charge operator defined in the figure in terms of the current density J^0. The gauge field A_μ interacts with the electrically charged matter fields through the current J^μ. The coupling strength is e, the charge of the electron.

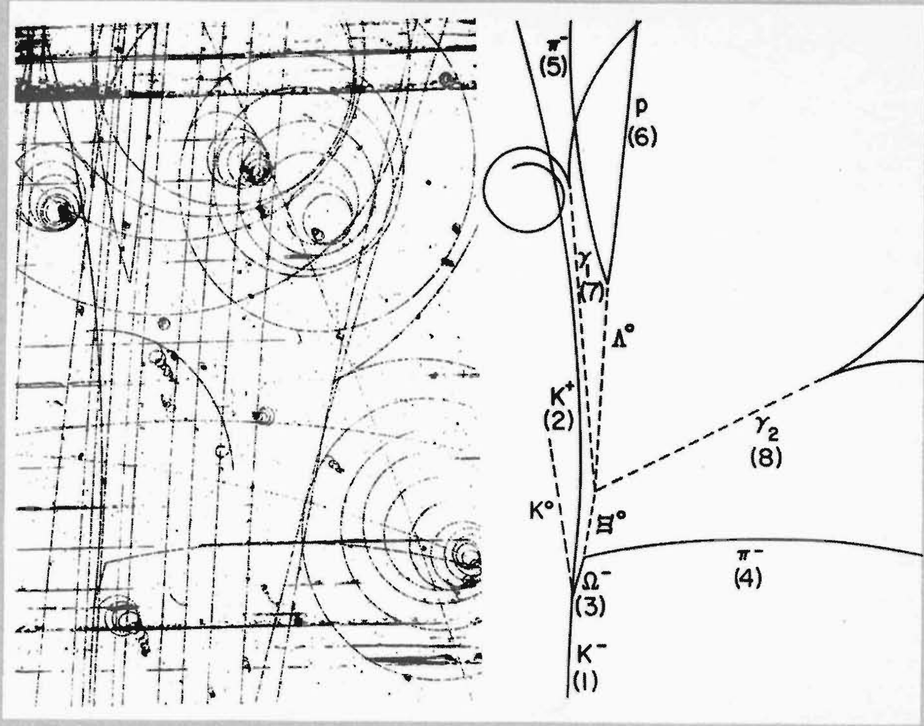
The Strong Interaction

In an atom electrons are bound to the nucleus by the Coulomb force and occupy a region about 10^{-8} centimeter in extent. The nucleus itself is a tightly bound collection of protons and neutrons confined to a region about 10^{-12} centimeter across. As already emphasized, the force that binds the protons and neutrons together to form the nucleus is much stronger and considerably shorter in range than the electromagnetic force. Leptons do not feel this strong force; particles that *do* participate in the strong interactions are called hadrons.

Ω^-

In 1961 M. Gell-Mann and independently Y. Ne'eman proposed a system for classifying the roughly one hundred baryons and mesons known at the time. This "Eightfold Way" was based on the SU(3) group, which has eight independent symmetry operations. According to this system, hadrons with the same baryon number, spin angular momentum, and parity and with electric charge, strangeness (or hypercharge), isotopic spin, and mass related by certain rules were grouped into large multiplets encompassing the already established isospin multiplets, such as the neutron and proton doublet or the negative, neutral, and positive pion triplet. Most of the known hadrons fit quite neatly into octets. However, the decuplet partly filled by the quartet of Δ baryons and the triplet of $\Sigma(1385)$ baryons lacked three members. Discovery of the $\Xi(1520)$ doublet was announced in 1962, and these baryons satisfied the criteria for membership in the decuplet. This partial confirmation of the Eightfold Way motivated a search at Brookhaven for the remaining member, already named Ω^- and predicted to be stable against strong and electromagnetic interactions, decaying (relatively slowly) by the weak interaction. Other properties predicted for this particle were a baryon number of 1, a spin angular momentum of 3/2, positive parity, negative electric charge, a strangeness of -3, an isotopic spin of 0, and a mass of about 1676 MeV.

A beam of 5-GeV negative kaons produced at the AGS was directed into a liquid-hydrogen bubble chamber, where the Ω^- was to be produced by reaction of the kaons with protons. The tracks of the decay products of the new particle were then sought in the bubble-chamber photographs. In early 1964 a candidate event was found for decay of an Ω^- into a π^- and a Ξ^0, one of three possible decay modes. Within several weeks, by coincidence and good fortune, another Ω^- was found, this time decaying into a Λ^0 and a K^-, the mode now known to be dominant. Analysis of the tracks for these two events confirmed the predicted mass and strangeness, and further studies confirmed the the predicted spin and parity. Discovery of the Ω^- established the Eightfold Way as a viable description of hadronic states. ■



The Ω^- was first detected in the bubble-chamber photograph reproduced above. A K^- entered the bubble chamber from the bottom (track 1) and collided with a proton. The collision produced an Ω^- (track 3), a K^+ (track 2), and a K^0, which, being neutral, left no track and must have decayed outside the bubble chamber. The Ω^- decayed into a π^- (track 4) and a Ξ^0. The Ξ^0 in turn decayed into a Λ^0 and a π^0. The Λ^0 decayed into a π^- (track 5) and a proton (track 6), and the π^0 very quickly decayed into two gamma rays, one of which (track 7) created an e^--e^+ pair within the bubble chamber. (Photo courtesy of the Niels Bohr Library of the American Institute of Physics and Brookhaven National Laboratory.)

The mystery of the strong force and the structure of nuclei seemed very intractable as little as fifteen years ago. Studying the relevant distance scales requires machines that can accelerate protons or electrons to energies of 1 GeV and beyond. Experiments with less energetic probes during the 1950s revealed two very interesting facts. First, the strong force does not distinguish between protons and neutrons. (In more technical language, the proton and the neutron transform into each other under isospin rotations, and the Lagrangian of the strong interaction is invariant under these rotations.) Second, the structure of protons and neutrons is as rich as that of nuclei. Furthermore, many new hadrons were discovered that were apparently just as "elementary" as protons and neutrons.

The table of "elementary particles" in the mid-1960s displayed much of the same complexity and symmetry as the periodic table of the elements. In 1961 both Gell-Mann and Ne'eman proposed that all hadrons could be classified in multiplets of the symmetry group called SU(3). The great triumph of this proposal was the prediction and subsequent discovery of a new hadron, the omega minus. This hadron was needed to fill a vacant space in one of the SU(3) multiplets (Fig. 9).

In spite of the SU(3) classification scheme, the belief that all of these so-called elementary particles were truly elementary became more and more untenable. The most contradictory evidence was the finite size of hadrons (about 10^{-13} centimeter), which drastically contrasted with the point-like nature of the leptons. Just as the periodic table was eventually explained in terms of a few basic building blocks, so the hadronic zoo was eventually tamed by postulating the existence of a small number of "truly elementary point-like particles" called quarks. In 1963 Gell-Mann and, independently, Zweig realized that all hadrons could be constructed from three spin-½ fermions, designated u, d, and s (up, down, and strange). The SU(3) symmetry that manifested itself in the table of "elementary particles" arose from an invariance of the La-

Table of "Elementary Particles"

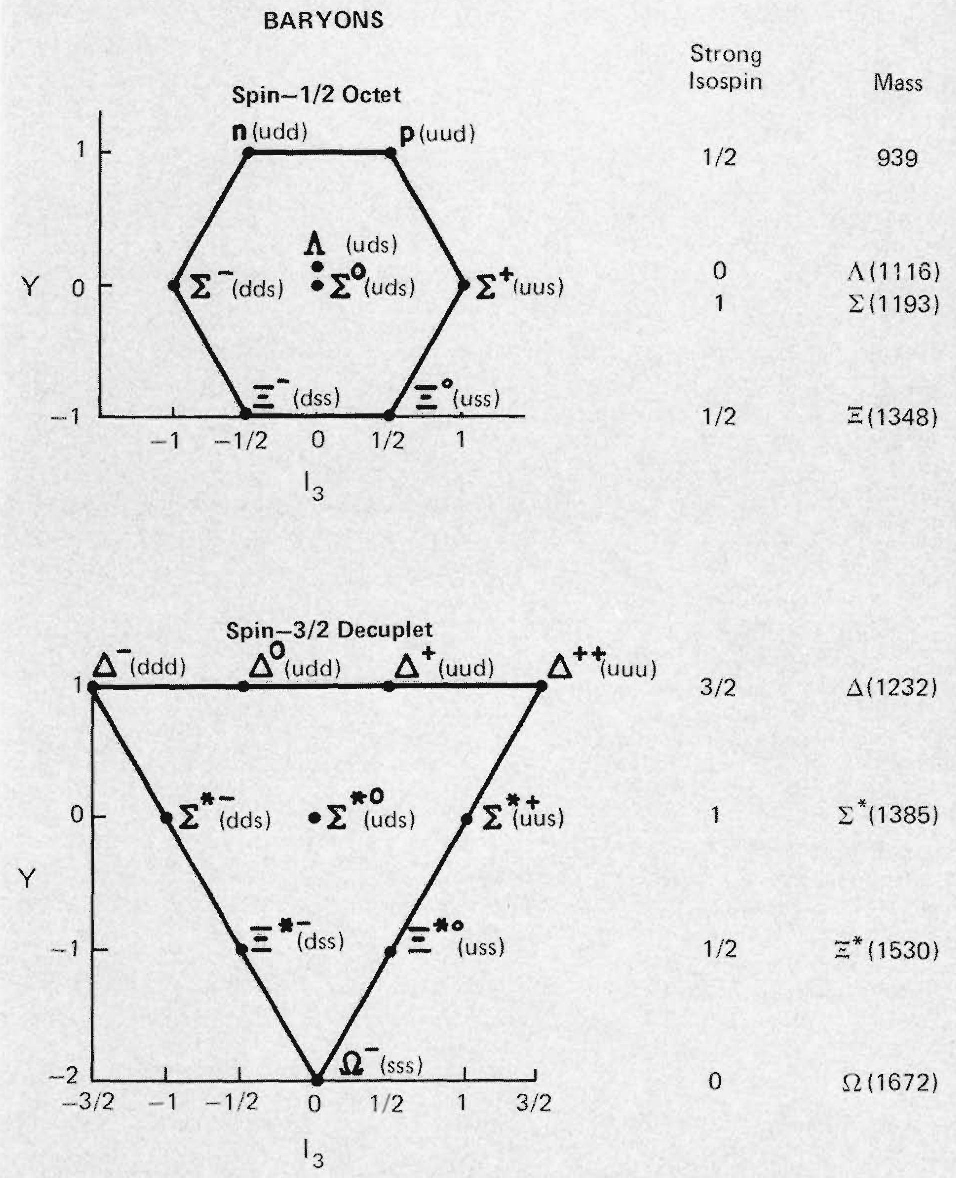
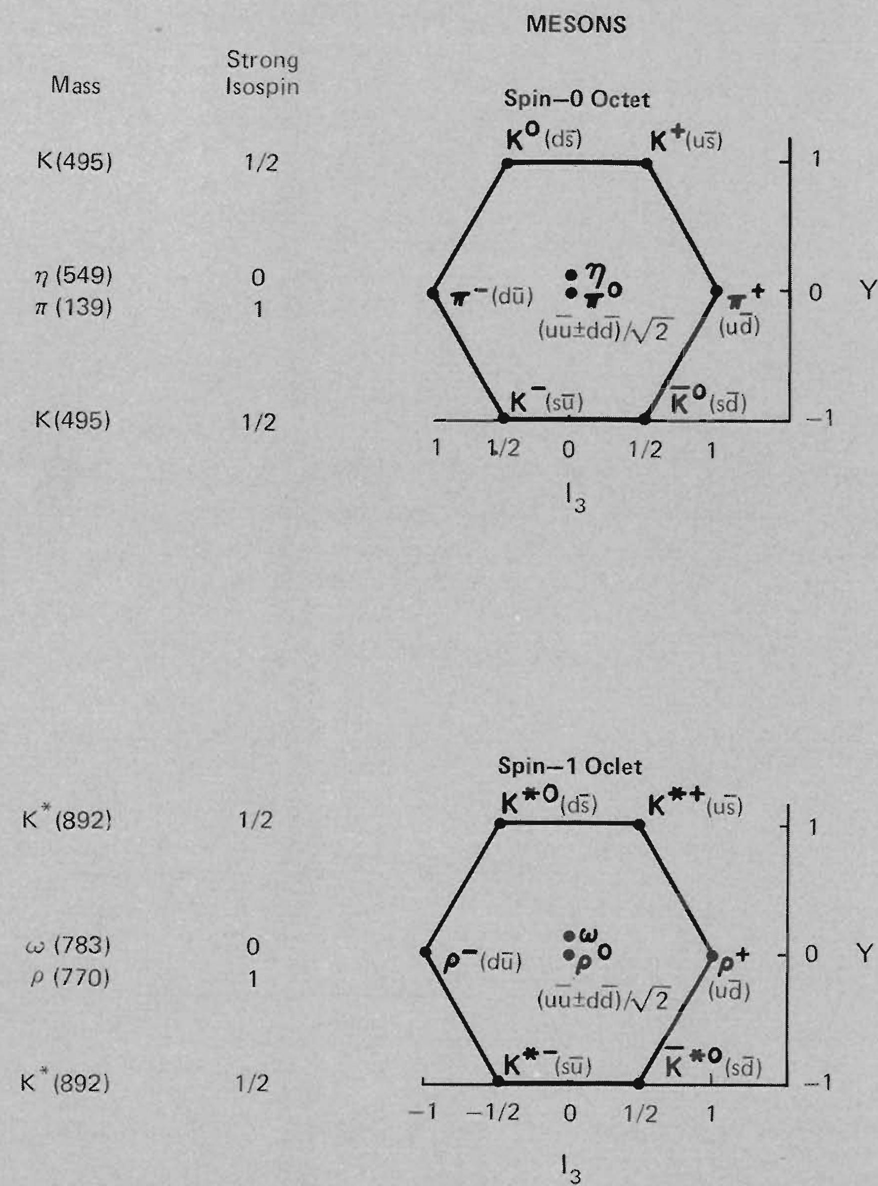


Fig. 9. The Eightfold Way classified the hadrons into multiplets of the symmetry group SU(3). Particles of each SU(3) multiplet that lie on a horizontal line form strong-isospin (SU(2)) multiplets. Each particle is plotted according to the quantum numbers I_3 (the third component of strong isospin) and strong hypercharge Y ($Y = S + B$, where S is strangeness and B is baryon number). These quantum numbers correspond to the two diagonal generators of SU(3). The quantum numbers of each particle are easily understood in terms of its fundamental quark constituents. Baryons contain three quarks and mesons contain quark-antiquark pairs. Baryons in the spin-3/2 decuplet are obtained from baryons in the spin-½ octet by changing the spin and SU(3) flavor quantum numbers of the three quark wave functions. For example, the three quarks that compose the neutron in the spin-½ octet can reorient their spins to form the Δ^0 in the spin-3/2 decuplet. Similar changes in the meson quark-antiquark wave functions change the spin-0 meson octet into the spin-1 meson octet.



Quarks

Name	Symbol	Electric Charge	Y
Up	u	2/3	1/3
Down	d	−1/3	1/3
Strange	s	−1/3	−2/3

grangian of the strong interaction to rotations among these three objects. This global symmetry is exact only if the *u*, *d*, and *s* quarks have identical masses, which implies that the particle states populating a given SU(3) multiplet also have the same mass. Since this is certainly not the case, SU(3) is a broken global symmetry. The dominant breaking is presumed to arise, as in the example of *e*, μ, and τ, from the differences in the masses of the *u*, *d*, and *s* quarks. The origin of these quark masses is one of the great unanswered questions. It is established, however, that SU(3) symmetry among the *u*, *d*, and *s* quarks is preserved by the strong interaction. Nowadays, one refers to this SU(3) as a *flavor* symmetry, with *u*, *d*, and *s* representing different quark flavors. This nomenclature is to distinguish it from another and quite different SU(3) symmetry possessed by quarks, a local symmetry that is associated directly with the strong force and has become known as the SU(3) of *color*. The theory resulting from this symmetry is called quantum chromodynamics (QCD), and we now turn our attention to a discussion of its properties and structure.

The fundamental structure of quantum chromodynamics mimics that of quantum electrodynamics in that it, too, is a gauge theory (Fig. 10). The role of electric charge is played by three "colors" with which each quark is endowed—red, green, and blue. The three color varieties of each quark form a triplet under the SU(3) local gauge symmetry. A local phase transformation of the quark field is now considerably extended since it can rotate the color and thereby change a red quark into a blue one. The local gauge transformations of quantum electrodynamics simply change the phase of an electron, whereas the color transformations of QCD actually change the particle. (Note that these two types of phase transformation are totally independent of each other.)

We explained earlier that the freedom to change the local phase of the electron field forces the introduction of the photon field (sometimes called the gauge field) to keep the Lagrangian (and therefore the resulting physics) invariant under these local phase changes. This is the principle of local symmetry. A similar procedure applied to the quark field induces the so-called chromodynamic force. There are eight independent symmetry transformations that change the color of a quark and these must be compensated for by the introduction of *eight* gauge fields, or spin-1 bosons (analogous to the single photon of quantum electrodynamics). Extension of the local U(1) gauge invariance of QED to more complicated symmetries such as SU(2) and SU(3) was first done by Yang and Mills in 1954. These larger symmetry groups involve so-called non-Abelian, or non-commuting algebras (in which $AB \neq BA$), so it has become customary to refer to this class of theories as "non-Abelian gauge theories." An alternative term is simply "Yang-Mills theories."

The eight gauge bosons of QCD are referred to by the bastardized term "gluon," since they represent the glue that holds the physical hadrons, such as the proton, together. The interactions of gluons with quarks are depicted in Fig. 10. Although gluons are the counterpart to photons in that they have unit spin and are massless, they possess one crucial property not shared by photons: they themselves carry color. Thus they not only mediate the color force but also carry it; it is as if photons were charged. This difference (it is the difference between an Abelian and a non-Abelian gauge theory) has many profound physical consequences. For example, because gluons carry color they can (unlike photons) interact with themselves (see Fig. 10) and, in effect, weaken the force of the color charge at *short* distances. The opposite effect occurs in quantum electrodynamics: screening effects weaken the effective electric charge at *long* distances. (As mentioned above, a virtual photon emanating from an electron can create a virtual electron-positron pair. This polarization screens, or effectively decreases, the electron's charge.)

The weakening of color charge at short distances goes by the name of *asymptotic freedom*. Asymptotic freedom was first ob-

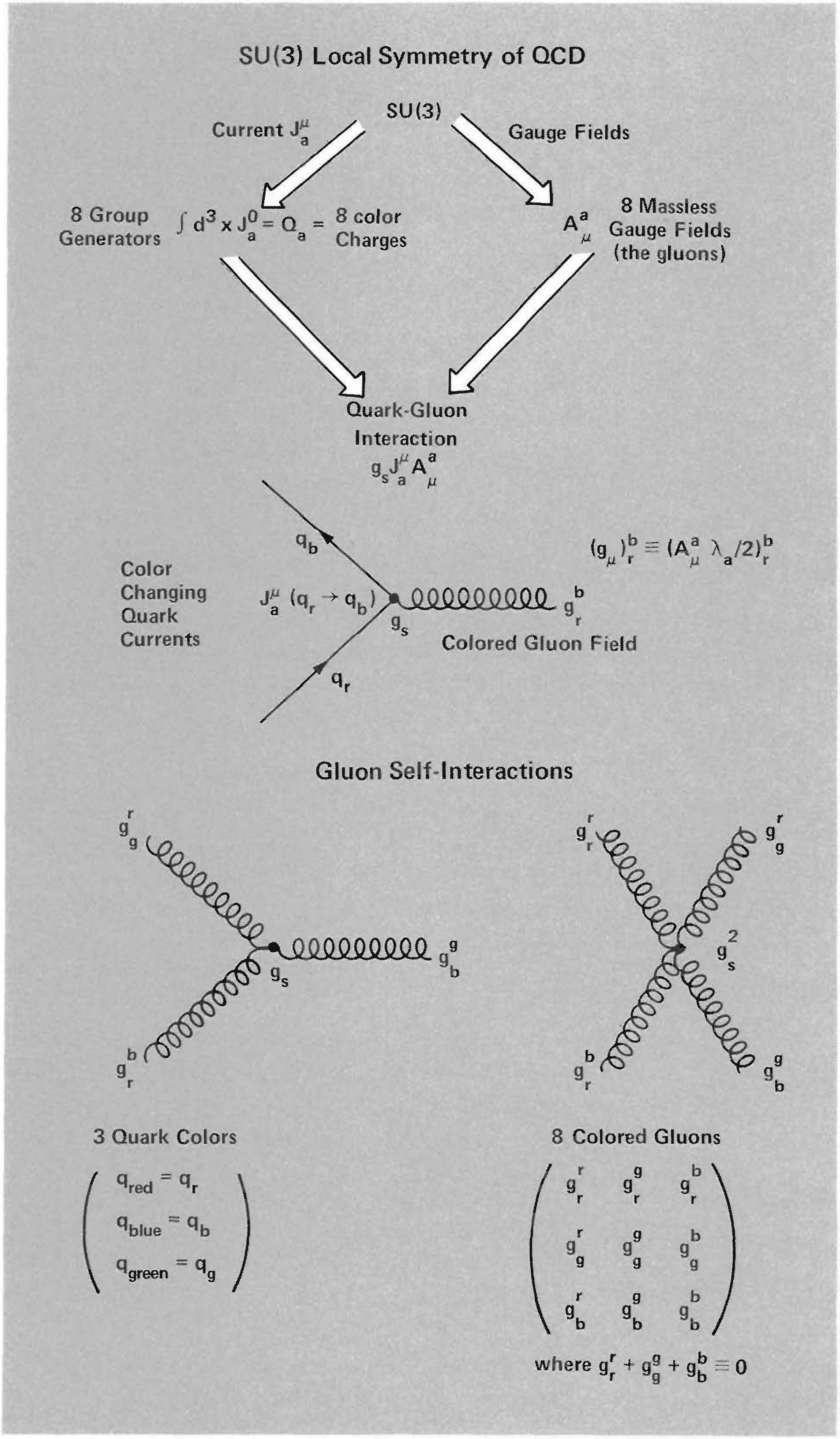


Fig. 10. The SU(3) local color symmetry implies the existence of eight massless gauge fields (the gluons) to compensate for the eight independent local transformations of the colored quark fields. The subscripts* r, g, *and* b *on the gluon and quark fields correspond respectively to red, green, and blue color charges. The eight gluons carry color and obey the non-Abelian algebra of the SU(3) generators (see Lecture Note 4). The interactions induced by the local SU(3) color symmetry include a quark-gluon coupling as well as two types of gluon self-interactions (one proportional to the couping* g_s *and the other proportional to* g_s^2*).

QCD on a Cray: the masses of elementary particles

by Gerald Guralnik, Tony Warnock, and Charles Zemach

How can we extract answers from QCD at energies below 1 GeV? As noted in the text, the confinement of quarks suggests that weak-coupling perturbative methods are *not* going to be successful at these energies. Nevertheless, if QCD is a valid theory it must explain the multiplicities, masses, and couplings of the experimentally observed strongly interacting particles. These would emerge from the theory as bound states and resonances of quarks and gluons. A valid theory must also account for the apparent absence of isolated quark states and might predict the existence and properties of particles (such as glueballs) that have not yet been seen.

The most promising nonperturbative formulation of QCD exploits the Feynman path integral. Physical quantities are expressed as integrals of the quark and gluon fields over the space-time continuum with the QCD Lagrangian appearing in an exponential as a kind of Gibbs weight factor. This is directly analogous to the partition function formulation of statistical machanics. The path integral prescription for strong interaction dynamics becomes well defined mathematically when the space-time continuum is approximated by a discrete four-dimensional lattice of finite size and the integrals are evaluated by Monte Carlo sampling.

The original Monte Carlo ideas of Metropolis and Ulam have now been applied to QCD by many researchers. These efforts have given credibility, but not confirmation, to the hope that computer simulations might indeed provide critical tests of QCD and significant numerical results. With considerable patience (on the order of many months of computer time) a VAX 11/780 can be used to study universes of about 3000 space-time points. Such a universe is barely large enough to contain a proton and not really adequate for a quantitative calculation. Consequently, with these methods, any result from a computer of VAX power is, at best, only an indication of what a well-done numerical simulation might produce.

We believe that a successful computer simulation must combine the following: (1) physical and mathematical ingenuity to search out the best formulations of problems still unsolved in principle; (2) sophisticated numerical analysis and computer programming; and (3) a computer with the speed, memory, and input/output rate of the Cray XMP with a solid-state disk (or better). We have done calculations of particle masses on a lattice of 55,296 space-time points using the Cray XMP. Using new methods developed with coworkers R. Gupta, J. Mandula, and A. Patel, we are examining glueball masses, renormalization group behavior, and the behavior of the theory on much larger lattices. The results to date support the belief that QCD describes interactions of the elementary particles and that these numerical methods are currently the most powerful means for extracting the predictive content of QCD.

The calculations, which have two input parameters (the pion mass and the long-range quark-quark force constant in units of the lattice spacing), provide estimates of many measurable quantities. The accompanying table shows some of our results on elementary particle masses and certain meson coupling strengths. These results represent several hundred hours of Cray time. The quoted relative errors derive from the statistical analysis of the Monte Carlo calculation itself rather than from a comparison with experimental data. Significantly more computer time would significantly reduce the errors in the calculated masses and couplings.

Our work would not have been possible without the support of C Division and many of its staff. We have received generous support from Cray Research and are particularly indebted to Bill Dissly and George Spix for contribution of their skills and their time. ■

Calculated and experimental values for the masses and coupling strengths of some mesons and baryons.

	Calculated Value (MeV/c^2)	Relative Error (%)	Experimental Value (MeV/c^2)
Masses			
ρ meson	767	18	769
Excited ρ	1426	27	1300?
δ meson	1154	15	983
A_1 meson	1413	17	1275
Proton	989	23	940
Δ baryon	1199	17	1210
Couplings			
f_π	121	21	93
f_ρ	211	15	144

Scaling in Deep Inelastic Scattering

In the early part of this century, Rutherford studied the structure of the atom by scattering alpha particles from thin gold foils. The fact that some alpha particles were scattered through unexpectedly large angles implied that an atom must have at its center a very massive, point-like, positively charged nucleus. "Scaling" is an analogous phenomenon in the scattering of leptons from nucleons (neutrons and protons). At very high momentum transfers (the relativistic analogue of large angles) the scattering amplitude depends on kinematic variables only; it has no dependence on the size of the nucleon, its mass, or any other dynamical variable. This scaling behavior implies that the scattering takes place from massive, point-like constituents inside the nucleon. Originally these constituents were named partons by R. Feynman, but, by deducing such properties as their electric charge, J. Bjorken and E. Paschos identified them as objects similar to quarks, the fractionally charged particles proposed independently in 1964 by M. Gell-Mann and G. Zweig as the components of mesons and baryons. Scaling thus joined hadron spectroscopy as a major piece of evidence for the quark model.

The first "deep" elastic and inelastic scattering experiments were carried out in the late 1960s at the two-mile-long electron linac of the Stanford Linear Accelerator Center and involved momentum transfers up to a few GeV/c. ■

served in deep inelastic scattering experiments (see "Scaling in Deep Inelastic Scattering"). This phenomenon explains why hadrons at high energies behave as if they were made of almost free quarks even though one knows that quarks must be tightly bound together since they have never been experimentally observed in their free state. The weakening of the force at high energies means that we can use perturbation theory to calculate hadronic processes at these energies.

The self-interaction of the gluons also explains the apparently permanent confinement of quarks. At long distances it leads to such a proliferation of virtual gluons that the color charge effectively grows without limit, forbidding the propagation of *all* colored particles. Only bleached, or color-neutral, states (such as baryons, which have equal proportions of red, blue, and green, or mesons which have equal proportions of red-antired, green-antigreen, and blue-antiblue) are immune from this confinement. Thus all *observable* hadrons are necessarily colorless, whereas quarks and gluons are permanently confined. This is just as well since gluons are massless, and by analogy with the photon, unconfined massless gluons should give rise to a long-range, Coulomb-like, color force in the strong interactions. Such a force is clearly at variance with experiment! Even though color is confined, residual strong color forces can still "leak out" in the form of color-neutral pions or other hadrons and be responsible for the binding of protons and neutrons in nuclei (much as residual electromagnetic forces bind atoms together to form molecules).

The success of QCD in explaining short-distance behavior and its aesthetic appeal as a generalization of QED have given it its place in the standard model. However, confidence in this theory still awaits convincing calculations of phenomena at distance scales of 10^{-13} centimeter, where the "strong" nature of the force becomes dominant and perturbation theory is no longer valid. (Lattice gauge theory calculations of the hadronic spectrum are becoming more and more reliable. See "QCD on a Cray: The Masses of Elementary Particles.")

The Weak Interaction

Many nuclei are known to be unstable and to emit several kinds of particles when they decay; historically these particles were called alpha particles, beta rays, and gamma rays. These three are now associated with three quite different modes of decay. An alpha particle, itself a helium nucleus, is emitted during the strong-interaction decay mode known as fission. Large nuclei that are only loosely bound by the strong force (such as uranium-238) can split into two stable pieces, one of which is an alpha particle. A gamma ray is simply a photon with "high" energy (above a few MeV) and is emitted during the decay of an excited nucleus. A beta ray is an electron emitted when a neutron in a nucleus decays into a proton, an electron, and an electron antineutrino ($n \rightarrow p + e^- + \bar{\nu}_e$, see Fig. 11). The proton remains in

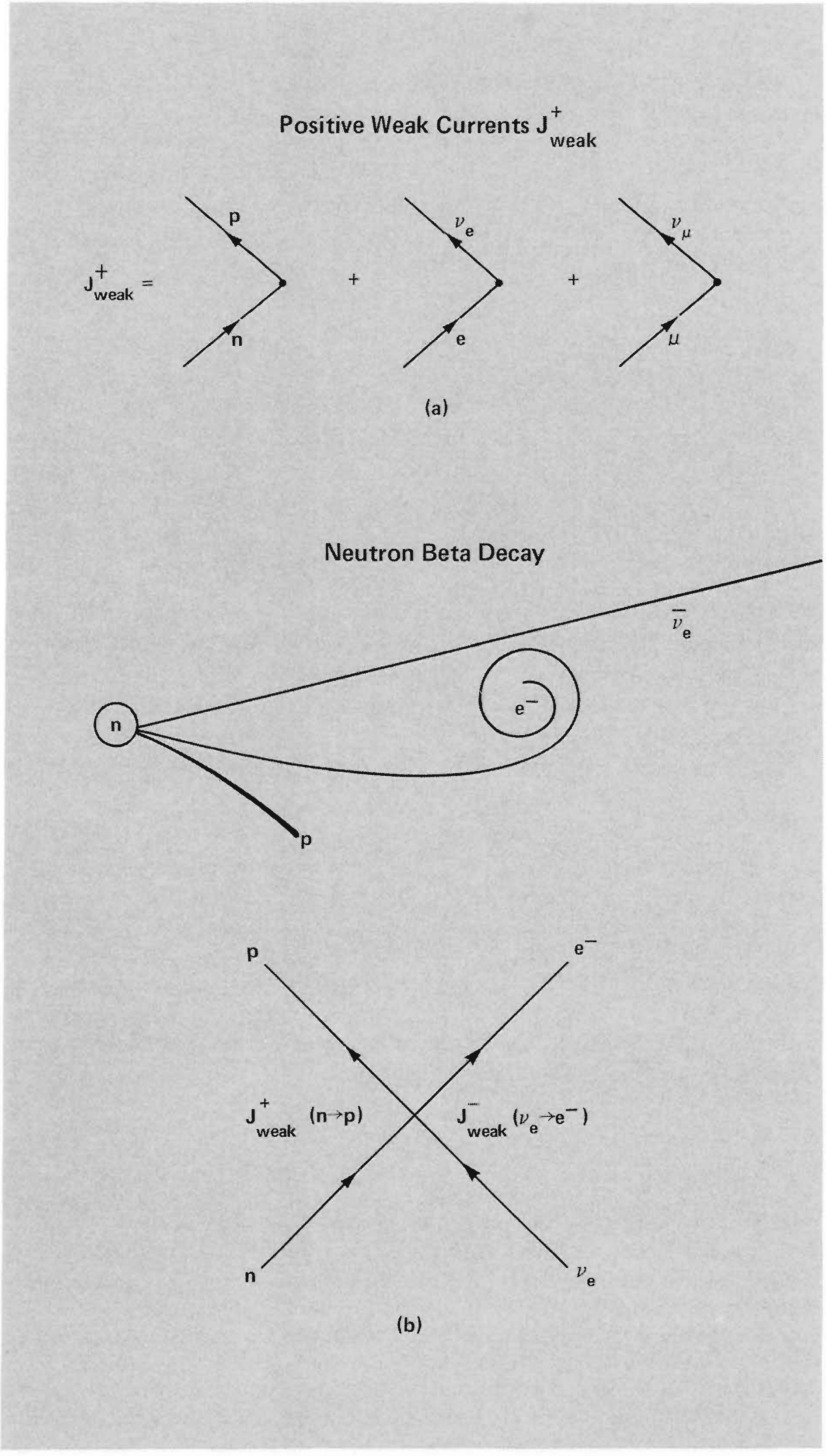


Fig. 11. (a) Components of the charge-raising weak current J^+_{weak} are represented in the figure by Feynman diagrams in which a neutron changes into a proton, an electron into an electron neutrino, and a muon into a muon neutrino. The charge-lowering current J^-_{weak} is represented by reversing the arrows. (b) Beta decay (shown in the figure) and other low-energy weak processes are well described by the Fermi interaction $J^+_{weak} \times J^-_{weak}$. The figure shows the Feynman diagram of the Fermi interaction for beta decay.

the nucleus, and the electron and its antineutrino escape. This decay mode is characterized as weak because it proceeds much more slowly than most electromagnetic decays (see Table 1). Other baryons may also undergo beta decay.

Beta decay remained very mysterious for a long time because it seemed to violate energy-momentum conservation. The free neutron was observed to decay into two particles, a proton and an electron, each with a spectrum of energies, whereas energy-momentum conservation dictates that each should have a unique energy. To solve this dilemma, Pauli invoked the neutrino, a massless, neutral fermion that participates only in weak interactions.

The Fermi Theory. Beta decay is just one of many manifestations of the weak interaction. By the 1950s it was known that all weak processes could be concisely described in terms of the current-current interaction first proposed in 1934 by Fermi. The charged weak currents J^+_{weak} and J^-_{weak} change the electric charge of a fermion by one unit and can be represented by the sum of the Feynman diagrams of Fig. 11a. In order to describe the maximal parity violation, (that is, the maximal right-left asymmetry) observed in weak interactions, the charged weak current includes only left-handed fermion fields. (These are defined in Fig. 12 and Lecture Note 8.)

Fermi's current-current interaction is then given by all the processes included in the product $(G_F/\sqrt{2})\,(J^+_{\text{weak}} \times J^-_{\text{weak}})$ where J^-_{weak} means all arrows in Fig. 11a are reversed. This interaction is in marked contrast to quantum electrodynamics in which two currents interact through the exchange of a virtual photon (see Fig. 3). In weak processes two charge-changing currents appear to interact locally (that is, at a single point) without the help of such an intermediary. The coupling constant for this local interaction, denoted by G_F and called the Fermi constant, is not dimensionless like the coupling parameter α in QED, but has the

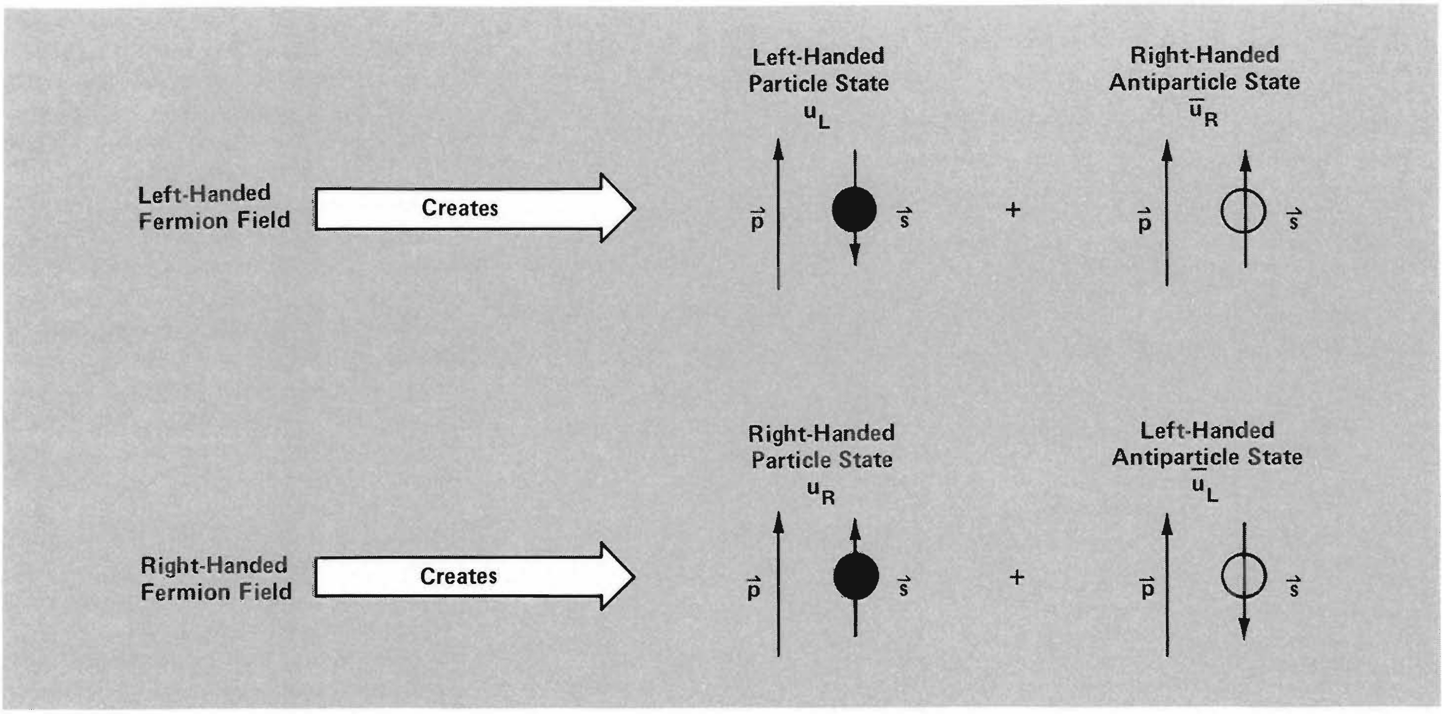


Fig. 12. A Dirac spinor field can be decomposed into left- and right-handed pieces. A left-handed field creates two types of particle states at ultrarelativistic energies—u_L, a particle with spin opposite to the direction of motion, and $\bar{u}_R$, an antiparticle with spin along the direction of motion. Only left-handed fields contribute to the weak charged currents shown in Fig. 11. The left- and right-handedness (or chirality) of a field describes a Lorentz covariant decomposition of Dirac spinor fields.

dimension of mass^{-2} or energy^{-2}. In units of energy, the measured value of $G_F^{-1/2}$ equals 293 GeV. Thus the strength of the weak processes seems to be determined by a specific energy scale. But why?

Predictions of the *W* boson. An explanation emerges if we postulate the existence of an intermediary for the weak interactions. Recall from Fig. 3 that the exchanged, or virtual, photons in QED basically correspond to the Coulomb potential α/r, whose Fourier transform is α/q^2, where q is the momentum of the virtual photon. It is tempting to suggest that the nearly zero range of the weak interaction is only apparent in that the two charged currents interact through a potential of the form $\alpha'[\exp(-M_W r)]/r$ (a form originally proposed by Yukawa for the short-range force between nucleons), where α' is the analogue of α and the mass M_W is so large that this potential has essentially no range. The Fourier transform of this potential, $\alpha'/(q^2 + M_W^2)$, suggests that, if this idea is correct, the interaction between the weak currents is mediated by a "heavy photon" of mass M_W. Nowadays this particle is called the W boson; its existence explains the short range of the weak interactions.

Notice that at low energies, or, equivalently, when $M_W^2 \gg q^2$, the Fourier transform, or so-called propagator of the W boson, reduces to $\alpha'/(M_W^2)$, and since this factor multiplies the two currents, it must be proportional to Fermi's constant. Thus the existence of the W boson gives a natural explanation of why G_F is not dimensionless.

Now, since both the weak and electromagnetic interactions involve electric charge, these two might be manifestations of the same basic force. If they were, then α' might be the same as α and G_F would be proportional to α/M_W^2. Thus the existence of a very massive W boson can explain not only the short range but also the weakness of weak interactions relative to electromagnetic interactions! This argument not only predicts the existence of a W boson but also yields a rough estimate of its mass:

$$M_W \approx \sqrt{\alpha/G_F} = 25\ \mathrm{GeV}/c^2\,.$$

This prediction of a new particle was made in the 1950s, when such energies were well beyond reach of the existing accelerators.

Arguments like the one above convinced physicists that a theoretical unification of

Table 2

Multiplets and quantum numbers in the SU(2) × U(1) electroweak theory.

		Weak Isotopic Charge I_3	Weak Hypercharge Y	Electric Charge Q $(=I_3 + \frac{1}{2}Y)$
Quarks				
SU(2) Doublet	u_L	½	⅓	⅔
	d_L	−½	⅓	−⅓
SU(2) Singlets	u_R	0	4/3	⅔
	d_R	0	−⅔	−⅓
Leptons				
SU(2) Doublet	$(\nu_e)_L$	½	−1	0
	e_L	−½	−1	−1
SU(2)Singlet	e_R	0	−2	−1
Gauge Bosons				
SU(2) Triplet	W^+	1	0	1
	W_3	0	0	0
	W^-	−1	0	−1
SU(2) Singlet	B	0	0	0
Higgs Boson				
SU(2) Doublet	φ^+	½	1	1
	φ^0	−½	1	0

electromagnetic and weak interactions must be possible. Several attempts were made in the 1950s and 1960s, notably by Schwinger and his student Glashow and by Ward and Salam, to construct an "electroweak theory" in terms of a local gauge (Yang-Mills) theory that generalizes QED. Ultimately, Weinberg set forth the modern solution to giving masses to the weak bosons in 1967, although it was not accepted as such until 't Hooft and Veltman showed in 1971 that it constituted a consistent quantum field theory. The success of the electroweak theory culminated in 1982 with the discovery at CERN of the W boson at almost exactly the prediced mass. Notice, incidentally, that at sufficiently high energies, where $q^2 \gg M_W^2$, the weak interaction becomes comparable in strength to the electromagnetic. Thus we see explicitly how the apparent strength of the interaction depends on the wavelength of the probe.

The SU(2) × U(1) Electroweak Theory. Since quantum electrodynamics is a gauge theory based on local U(1) invariance, it is not too surprising that the theory unifying the electromagnetic and weak forces is also a gauge theory. Construction of such a theory required overcoming both technical and phenomenological problems.

The technical problem concerned the fact that an electroweak gauge theory is necessarily a Yang-Mills theory (that is, a theory in which the gauge fields interact with each other); the gauge fields, namely the W bosons, must be charged to mediate the charge-changing weak interactions and therefore by definition must interact with each other electromagnetically through the photon. Moreover, the local gauge symmetry of the theory must be broken because an unbroken symmetry would require all the gauge particles to be massless like the photon and the gluons, whereas the W boson must be massive. A major theoretical difficulty was understanding how to break a Yang-Mills gauge symmetry in a consistent way. (The solution is presented below.)

In addition to the technical issue, there was the phenomenological problem of choosing the correct local symmetry group. The most natural choice was SU(2) because the low-lying states (that is, the observed quarks and leptons) seemed to form doublets under the weak interaction. For example, a W^- changes ν_e into e, ν_μ into μ, or u into d (where all are left-handed fields), and the W^+ effects the reverse operation. Moreover, the three gauge bosons required to compensate for the three independent phase rotations of a local SU(2) symmetry could be identified with the W^+, the W^-, and the photon. Unfortunately, this simplistic scenario does not work: it gives the wrong electric charge assignments for the quarks and leptons in the SU(2) doublets. Specifically, electric charge

Q would be equal to the SU(2) charge I_3, and the values of I_3 for a doublet are $\pm\frac{1}{2}$. This is clearly the wrong charge. In addition, SU(2) would not distinguish the charges of a quark doublet ($\frac{2}{3}$ and $-\frac{1}{3}$) from those of a lepton doublet (0 and -1).

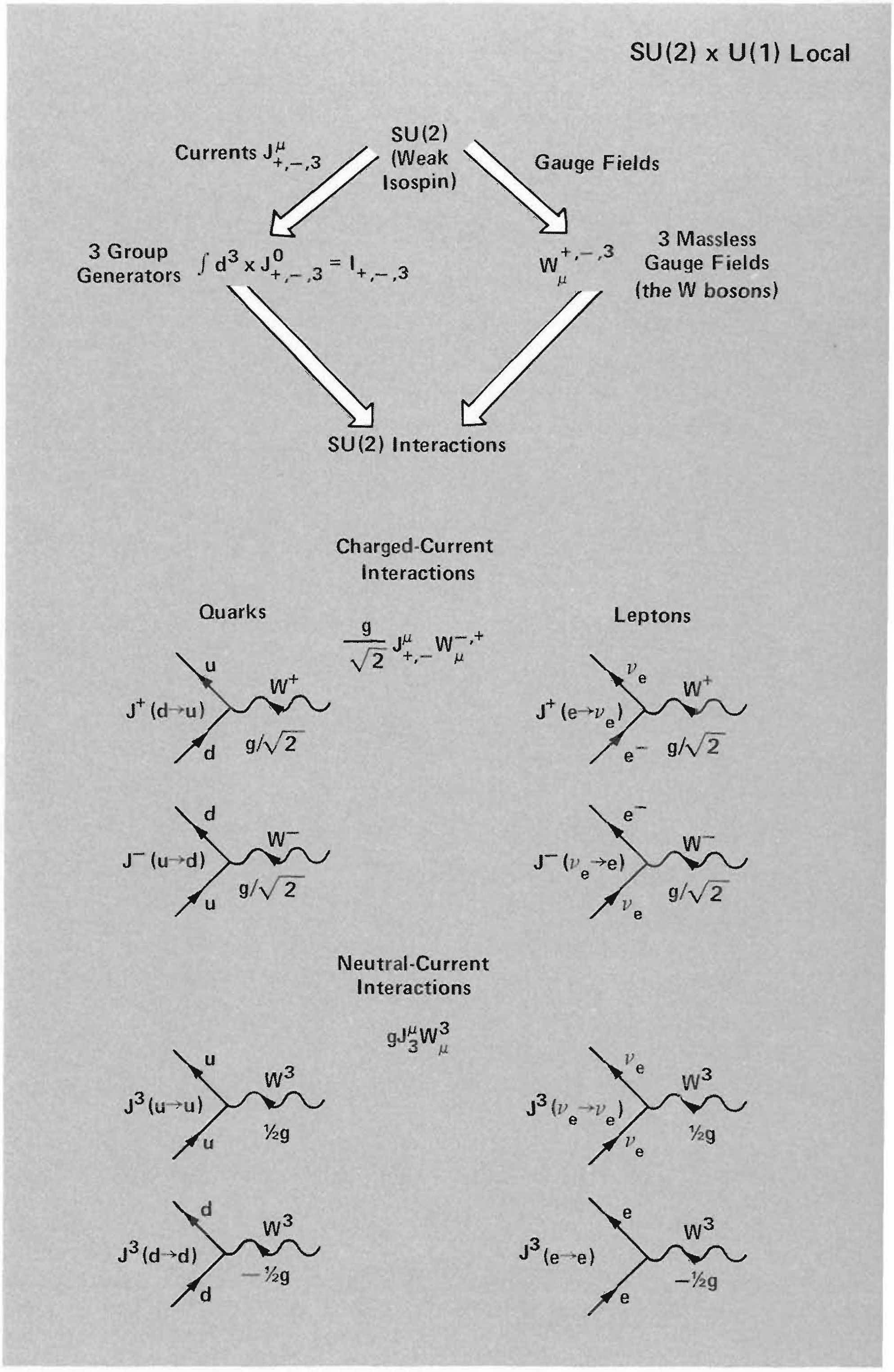
To get the correct charge assignments, we can either put quarks and leptons into SU(2) triplets (or larger multiplets) instead of doublets, or we can enlarge the local symmetry group. The first possibility requires the introduction of new heavy fermions to fill the multiplets. The second possibility requires the introduction of at least one new U(1) symmetry (let's call it weak hypercharge Y), which yields the correct electric charge assignments if we define

$$Q = I_3 + \tfrac{1}{2}\,Y\,.$$

This is exactly the possibility that has been confirmed experimentally. Indeed, the electroweak theory of Glashow, Salam, and Weinberg is a local gauge theory with the symmetry group SU(2) $\times$ U(1). Table 2 gives the quark and lepton multiplets and their associated quantum numbers under SU(2) $\times$ U(1), and Fig. 13 displays the interactions defined by this local symmetry. There is one coupling associated with each factor of SU(2) $\times$ U(1), a coupling g for SU(2) and a coupling $g'/2$ for U(1).

The addition of the local U(1) symmetry introduces a new uncharged gauge particle into the theory that gives rise to the so-called neutral-current interactions. This new type of weak interaction, which allows a neutrino to interact with matter without changing its identity, had not been observed when the neutral weak boson was first proposed in 1961 by Glashow. Not until 1973, after all the technical problems with the SU(2) $\times$ U(1) theory had been worked out, were these interactions observed in data taken at CERN in 1969 (see Fig. 14).

The physical particle that mediates the weak interaction between neutral currents is the massive Z^0. The electromagnetic interaction between neutral currents is mediated by the familiar massless photon. These two



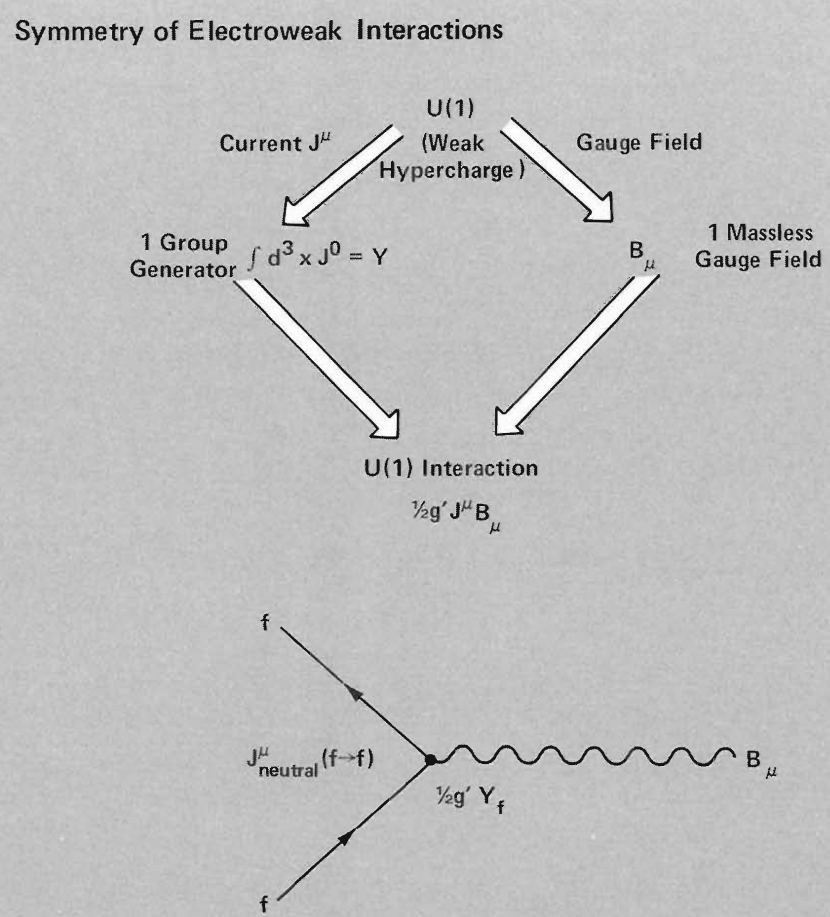


Fig. 13. The unbroken SU(2) × U(1) local symmetry of the electroweak theory has associated with it gauge fields, currents, and interactions analogous to those of QED and QCD (see Figs. 5 and 8). The figure shows the lowest order interactions between the fermion fields and the gauge fields. The SU(2) interaction involves left-handed quark and lepton fields only. The f in the U(1) interaction stands for both left- and right-handed fermion fields with charge Y_f. (Y_f differs for the left- and right-handed components.) Although the gauge fields are self-interacting as in the case of QCD, the SU(2) × U(1) symmetry is broken and the gauge fields are massive so that their self-interactions contribute only very small corrections to the lowest order diagrams and are not shown.

physical particles are different from the two neutral gauge particles (B and W_3) associated with the unbroken SU(2) × U(1) symmetry shown in Fig. 13. In fact, the photon and the Z^0 are linear combinations of the neutral gauge particles W_3 and B:

$$A = B \cos \theta_W + W_3 \sin \theta_W$$

and

$$Z^0 = B \sin \theta_W - W_3 \cos \theta_W .$$

The mixing of SU(2) and U(1) gauge particles to give the physical particles is one result of the fact that the SU(2) × U(1) symmetry must be a broken symmetry.

Spontaneous Symmetry Breaking. The astute reader may well be wondering how a local gauge theory, which in QED required the photon to be massless, can allow the mediator of the weak interactions to be massive, especially since the two forces are to be unified. The solution to this paradox lies in the curious way in which the SU(2) × U(1) symmetry is broken.

As Nambu described so well, this breaking is very much analogous to the symmetry breaking that occurs in a superconductor. A superconductor has a local U(1) symmetry, namely, electromagnetism. The ground state, however, is not invariant under this symmetry since it is an ordered state of bound electron-electron pairs (the so-called Cooper pairs) and therefore has a nonzero electric charge distribution. As a result of this asymmetry, photons inside the superconductor acquire an effective mass, which is responsible for the Meissner effect. (A magnetic field cannot penetrate into a superconductor; at the surface it decreases exponentially at a rate proportional to the effective mass of the photon.)

In the weak interactions the symmetry is also assumed to be broken by an asymmetry of the ground state, which in this case is the "vacuum." The asymmetry is due to an ordered state of electrically neutral bosons that carry the weak charge, the so-called Higgs bosons. They break the SU(2) × U(1) symmetry to give the U(1) of electromagnetism in such a way that the $W^{\pm}$ and the Z^0 obtain masses and the photon remains massless. As a result the charges I_3 and Y associated with SU(2) $\times$ U(1) are not conserved in weak processes because the vacuum can absorb these quantum numbers. The electric charge Q associated with U(1) of electromagnetism remains conserved.

The asymmetry of the ground state is frequently referred to as spontaneous symmetry breaking; it does not destroy the symmetry of the Lagrangian but destroys only the symmetry of the states. This symmetry breaking mechanism allows the electroweak Lagrangian to remain invariant under the local symmetry transformations while the gauge particles become massive (see Lecture Notes 3, 6, and 8 for details).

In the spontaneously broken theory the electromagnetic coupling e is given by the expression $e = g\sin\theta_W$, where

$$\sin^2\theta_W \equiv g'^2/(g^2 + g'^2) .$$

Thus, e and θ_W are an alternative way of expressing the couplings g and g', and just as e is not determined in QED, the equally important mixing angle θ_W is not determined by the electroweak theory. It is, however, measured in the neutral-current interactions. The experimental value is $\sin^2\theta_W = 0.224 \pm 0.015$. The theory predicts that

$$M_W/M_Z = \cos\theta_W$$

and

$$M_W = \left(\frac{\pi\alpha}{\sqrt{2}G_F}\right)^{1/2}\frac{1}{\sin\theta_W} .$$

These relations (which are changed only slightly by small quantum corrections) and the experimental value for the weak angle θ_W predict masses for the $W^{\pm}$ and Z^0 that are in very good agreement with the 1983 observations of the $W^{\pm}$ and Z^0 at CERN.

In the electroweak theory quarks and leptons also obtain mass by interacting with the ordered vacuum state. However, the values of their masses are not predicted by the

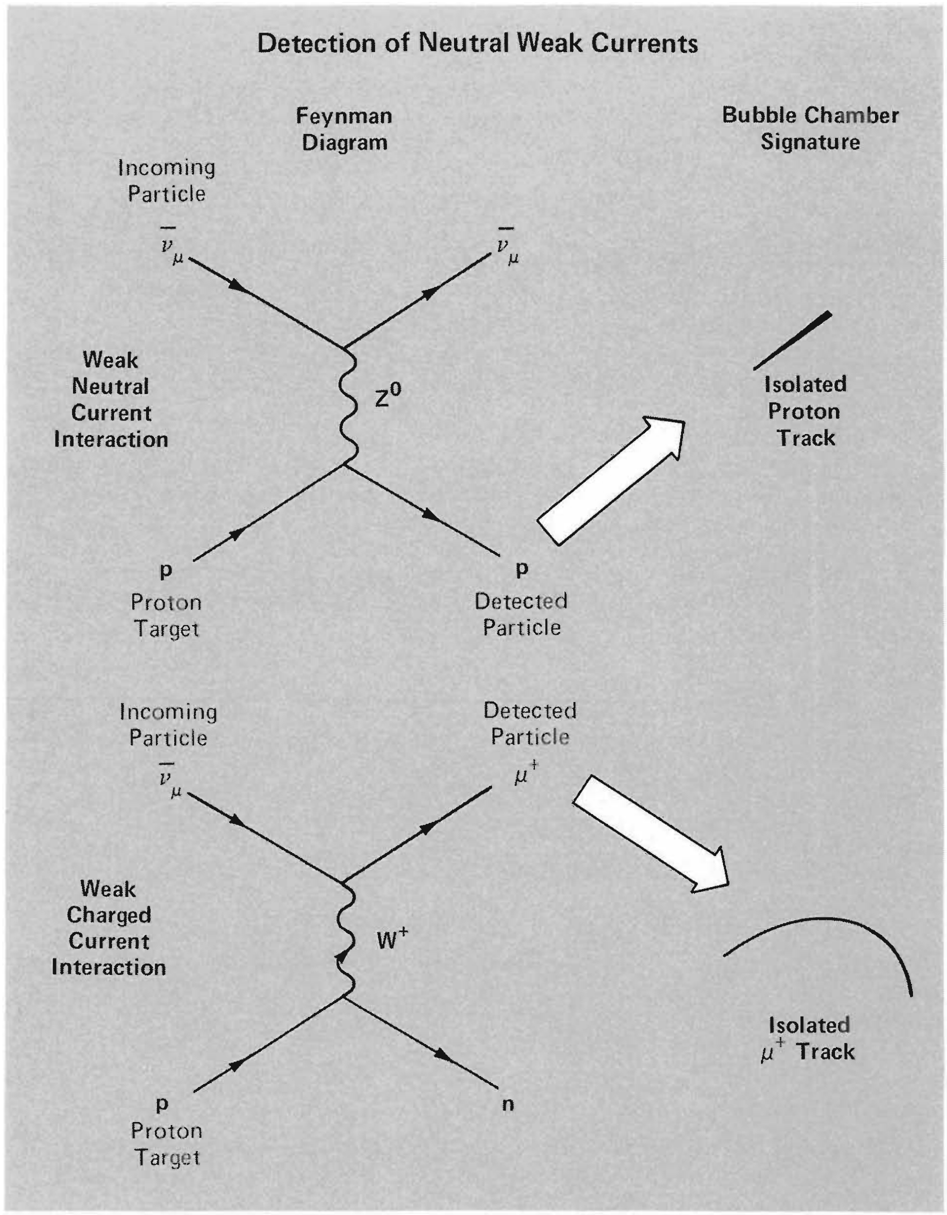
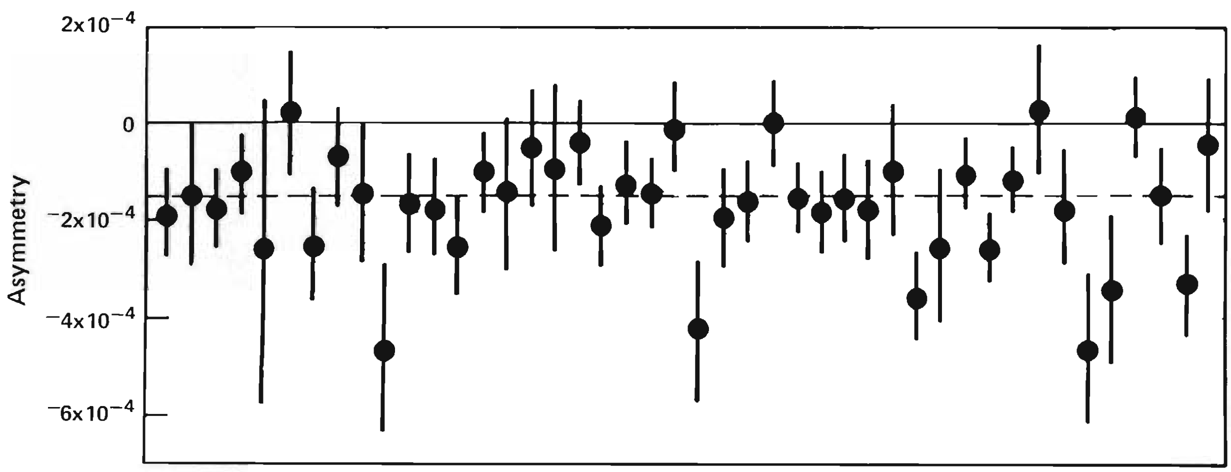


Fig. 14. Neutral-current interactions were first identified in 1973 in photographs taken with the CERN Gargamelle bubble chamber. The figure illustrates the difference between neutral-current and charged-current interactions and shows the bubble-chamber signature of each. The bubble tracks are created by charged particles moving through superheated liquid freon. The incoming antineutrinos interact with protons in the liquid. A neutral-current interaction leaves no track from a lepton, only a track from the positivley charged proton and perhaps some tracks from pions. A charged-current interaction leaves a track from a positively charged muon only.

Electronic Weak Neutral Current

Nonconservation of parity was first proposed by C. N. Yang and T. D. Lee in 1956 as a solution to the so-called τ-θ puzzle: the decay products of the τ meson (three π mesons) differed in parity from the decay products of the θ meson (two π mesons), yet in all other respects the two mesons (now known as K^0_L and K^0_S) appeared identical. Yang and Lee's heretical suggestion was proved correct only months later by the cobalt-60 experiment of C. S. Wu and E. Ambler. This experiment, which revealed a decided asymmetry in the direction of emission of beta particles from spin-aligned cobalt-60 nuclei, established parity violation as a feature of charged-current weak interactions and thus of the τ and θ decays. According to the Glashow-Weinberg-Salam theory unifying electromagnetic and weak interactions, parity violation should be a feature also of neutral-current weak interactions but at a low level because of competing electromagnetic interactions. In 1978 a group of twenty physicists headed by C. Prescott observed a parity violation of almost exactly the predicted magnitude in a beautifully executed experiment at the Stanford Linear Accelerator Center. The experiment clearly revealed a small difference (of order 1 part in 10,000) between the cross sections for scattering of right- and left-handed longitudinally polarized electrons by deuterons or protons. ■



theory but are proportional to arbitrary parameters related to the strength of the coupling of the quarks and leptons to the Higgs boson.

The Higgs Boson. In the simplest version of the spontaneously broken electroweak model, the Higgs boson is a complex SU(2) doublet consisting of four real fields (see Table 2). These four fields are needed to transform massless gauge fields into massive ones. A massless gauge boson such as the photon has only two orthogonal spin components (both transverse to the direction of motion), whereas a massive gauge boson has three (two transverse and one longitudinal, that is, in the direction of motion). In the electroweak theory the W^+, the W^-, and the Z^0 absorb three of the four real Higgs fields to form their longitudinal spin components and in so doing become massive. In more picturesque language, the gauge bosons "eat" the Higgs boson and become massive from the feast. The remaining neutral Higgs field is not used up in this magic transformation from massless to massive gauge bosons and therefore should be observable as a particle in its own right. Unfortunately, its mass is not fixed by the theory. However, it can decay into quarks and leptons with a definite signature. It is certainly a necessary component of the theory and is presently being looked for in high-energy experiments at CERN. Its absence is a crucial missing link in the confirmation of the standard model.

Open Problems. Our review of the standard model would not be complete without mention of some questions that it leaves unanswered. We discussed above how the three charged leptons (e, μ, and τ) may form a triplet under some broken symmetry. This is only part of the story. There are, in fact, three quark-lepton families (Table 3), and these three families may form a triplet under such a broken symmetry. (There is a missing state in this picture: conclusive evidence for the top quark t has yet to be presented. The bottom quark b has been observed in e^+e^-annihilation experiments at SLAC and

W^-, W^+, Z^0

In January 1983 two groups announced the results of separate searches at the CERN proton-antiproton collider for the W^- and W^+ vector bosons of the electroweak model. One group, headed by C. Rubbia and A. Astbury, reported definite identification, from among about a billion proton-antiproton collisions, of four W^- decays and one W^+ decay. The mass reported by this group (81 ± 5 GeV/c^2) agrees well with that predicted by the electroweak model (82 ± 2.4 GeV/c^2). The other group, headed by P. Darriulat and using a different detector, reported identification of four possible $W^\pm$ decays, again from among a billion events. The charged vector bosons were produced by annihilation of a quark inside a proton (uud) with an antiquark inside an antiproton ($\bar{u}\bar{u}\bar{d}$):

$$d + \bar{u} \rightarrow W^-$$

and

$$u + \bar{d} \rightarrow W^+ .$$

Since these reactions have a threshold energy equal to the mass of the charged bosons, the colliding proton and antiproton beams were each accelerated to about 270 GeV to provide the quarks with an average center-of-mass energy slightly above the threshold energy. (Only one-half of the energy of a proton or antiproton is carried by its three quark constituents; the other half is carried by the gluons.) Rubbia's group distinguished the two-body decay of the bosons (into a charged and neutral lepton pair such as $e^+\nu_e$) by two methods: selection of events in which the charged lepton possessed a large momentum transverse to the axis of the colliding beams, and selection of events in which a large amount of energy appeared to be missing, presumably carried off by the (undetected) neutrino. Both methods converged on the same events.

By mid 1983 each of the two groups had succeeded also in finding Z^0, the neutral vector boson of the electroweak model. They reported slightly different mass values (96.5 ± 1.5 and 91.2 ± 1.7 GeV/c^2), both in agreement with the predicted value of 94.0 $\pm$ 2.5 GeV/c^2. For Z^0 the production and decay processes are given by

$$u + \bar{u}\ (\text{or } d + \bar{d}) \rightarrow Z^0 \rightarrow e^- + e^+\ (\text{or } \mu^- + \mu^+) .$$

In addition, both groups reported an asymmetry in the angular distribution of charged leptons from the many more decays of W^- and W^+ that had been seen since their discovery. This parity violation confirmed that the particles observed are truly electroweak vector bosons. ■

Table 3

The three families of quarks and leptons and their masses. The subscripts R and L denote right- and left-handed particles as defined in Fig. 12.

Quark Mass (MeV/c^2)	Quarks			Leptons			Lepton Mass (MeV/c^2)
			First Family				
5	up	u_L	u_R	$(\nu_e)_L$		electron neutrino	0
8	down	d_L	d_R	e_L	e_R	electron	0.511
			Second Family				
1270	charm	c_L	c_R	$(\nu_\mu)_L$		muon neutrino	0
175	strange	s_L	s_R	μ_L	μ_R	muon	105.7
			Third Family				
45000 (?)	top	t_L	t_R	$(\nu_\tau)_L$		tau neutrino	0
4250	bottom	b_L	b_R	τ_L	τ_R	tau	1784

Cornell.) The standard model says nothing about why three identical families of quarks and leptons should exist, nor does it give any clue about the hierarchical pattern of their masses (the τ family is heavier than the μ family, which is heavier than the e family). This hierarchy is both puzzling and intriguing. Perhaps there are even more undiscovered families connected to the broken family symmetry. The symmetry could be global or local, and either case would predict new, weaker interactions among quarks and leptons.

Table 3 brings up two other open questions. First, we have listed the neutrinos as being massless. Experimentally, however, there exist only upper limits on their possible masses. The most restrictive limit comes from cosmology, which requires the sum of neutrino masses to be less then 100 eV. It is known from astrophysical observations that most of the energy in the universe is in a form that does not radiate electromagnetically. If neutrinos have mass, they could, in fact, be the dominant form of energy in the universe today.

Second, we have listed u and d, c and s, and t and b as doublets under weak SU(2). This is, however, only approximately true. As a result of the *broken* family symmetry, states with the same electric charge (the d, s, and b quarks or the u, c, and t quarks) can mix, and the weak doublets that couple to the $W^{\pm}$ bosons are actually given by u and d', c and s', and t and b'. A 3×3 unitary matrix known as the Kobayashi-Maskawa (K-M) matrix rotates the mass eigenstates (states of definite mass) d, s, and b into the weak doublet states d', s', and b'. The K-M matrix is conventionally written in terms of three mixing angles and an arbitrary phase. The largest mixing is between the d and s quarks and is characterized by the Cabibbo angle θ_C (see Lecture Note 9), which is named for the man who studied strangeness-changing weak decays such as $\Sigma^0 \rightarrow p + e^- + \bar{\nu}_e$. The observed value of $\sin\theta_C$ is about 0.22. The other mixing angles are all at least an order of magnitude smaller. The structure of the K-M matrix, like the masses of the quarks and leptons, is a complete mystery.

Conclusions

Although many mysteries remain, the standard model represents an intriguing and compelling theoretical framework for our present-day knowledge of the elementary particles. Its great virtue is that all of the known forces can be described as local gauge theories in which the interactions are generated from the single unifying principle of local gauge invariance. The fact that in quantum field theory interactions can drastically change their character with scale is crucial to

J/ψ

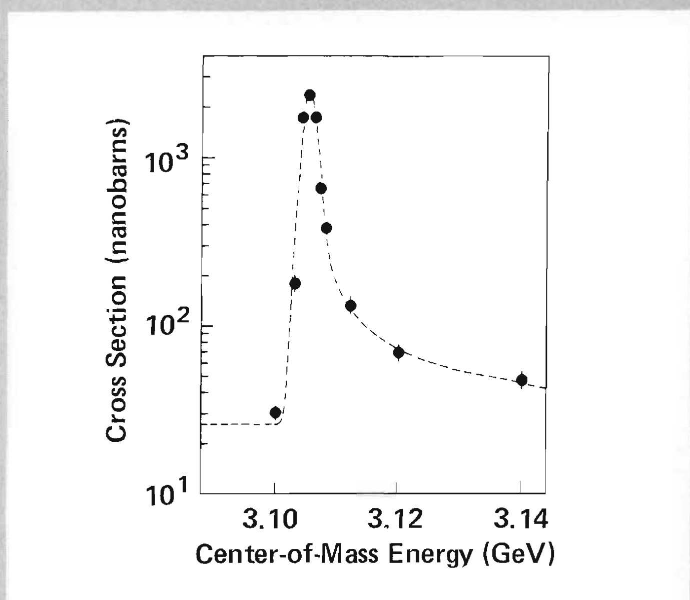
In 1974 two experimental groups pursuing completely different lines of research at different laboratories simultaneously discovered the same particle. (In deference to the different names adopted by the two groups, the particle is now referred to as J/ψ.) At SPEAR, the electron-positron storage ring at the Stanford Linear Accelerator Center, a group led by B. Richter was investigating, as a function of incident energy, the process of electron-positron annihilation to hadrons. They found an enormous and very narrow resonance at a collision energy of about 3.1 GeV and attributed it to the formation of a new particle ψ. Meanwhile, at the Brookhaven AGS, a group led by S. Ting was investigating essentially the inverse process, the formation of electron-positron pairs in collisions of protons with nucleons. They determined the number of pair-producing events as a function of the mass of the parent particle (as deduced from the energy and angular separation of each electron-positron pair) and found a very large, well-defined increase at a mass of about 3.1 GeV/c^2. This resonance also was attributed to the formation of a new particle J.

The surprisingly long lifetime of J/ψ, as indicated by the narrowness of the resonance, implied that its decay to lighter hadrons (all, according to the original quark model, composed of the up, down, and strange quarks) was somehow inhibited. This inhibition was given two possible interpretations: J/ψ was perhaps a form of matter exhibiting a net "color" (a quantum property of quarks), or it was perhaps a meson containing the postulated charmed and anticharmed quarks. The latter interpretation was soon adopted, and in those terms the production of J/ψ in the two experiments can be written

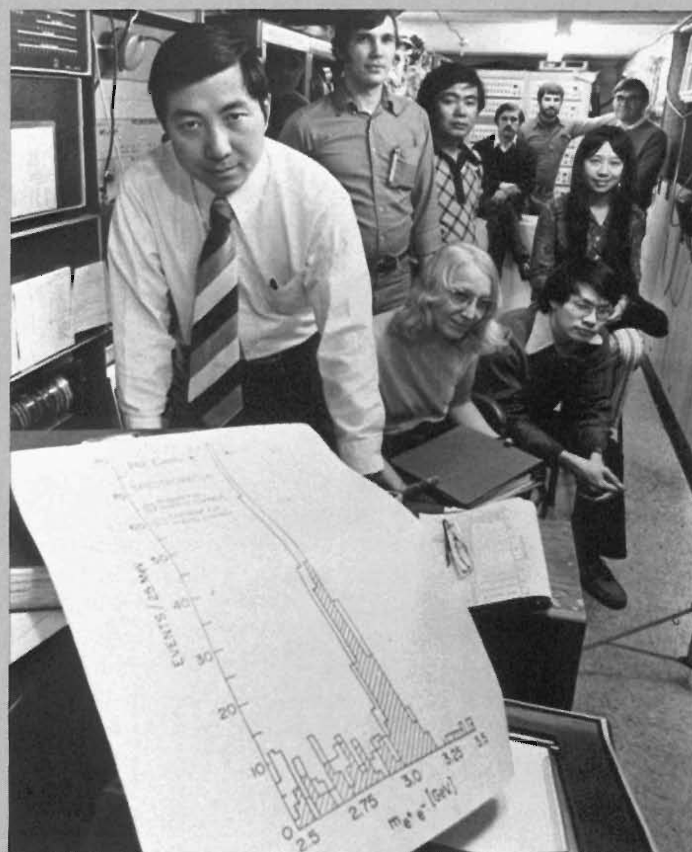
$$e^+ + e^- \leftrightarrow c + \bar{c}\,.$$

For further elucidation of the J/ψ system, electron-positron annihilation proved more fruitful than the hadronic production process.

This discovery of a fourth quark (which had been postulated by S. Glashow and J. Bjorken in 1964 to achieve a symmetry between the number of quarks and the known number of leptons and again by Glashow, J. Iliopoulos, and L. Maiani in 1970 to reconcile the weak-interaction selection rules and the electroweak model) convinced theorists that renormalizable gauge field theories, in conjunction with spontaneous symmetry breaking, were the right tool for understanding the fundamental interactions of nature. ■



Graph of the evidence for formation of J/ψ in electron-positron annihilations at SPEAR. (Adapted from SLAC Beam Line, *Volume 7, Number 11, November 1976.)*



The group from M.I.T. and Brookhaven that discovered J/ψ in proton-nucleon collisions at the AGS, together with a graph of their evidence. (Photo courtesy of the Niels Bohr Library of the American Institute of Physics and Brookhaven National Laboratory.)

Υ

In 1977 a group led by L. Lederman provided evidence for a fifth, or bottom, quark with the discovery of Υ, a long-lived particle three times more massive than J/ψ. In an experiment similar to that of Ting and coworkers and performed at the Fermilab proton accelerator, the group determined the number of events giving rise to muon-antimuon pairs as a function of the mass of the parent particle and found a sharp increase at about 9.5 GeV/c^2. Like the J/ψ system, the Υ system has been elucidated in detail from experiments involving electron-positron collisions rather than proton collisions, in this case at Cornell's electron storage ring, CESR.

The existence of the bottom quark, and of a sixth, or top, quark, was expected on the basis of the discovery of the tau lepton at SPEAR in 1975 and Glashow and Bjorken's 1964 argument of quark-lepton symmetry. Recent results from high-energy proton-antiproton collision experiments at CERN have been interpreted as possible evidence for the top quark with a mass somewhere between 30 and 50 GeV/c^2. ■

this approach. The essence of the standard model is to put the physics of the apparently separate strong, weak, and electromagnetic interactions in the single language of local gauge field theories, much as Maxwell put the apparently separate physics of Coulomb's, Ampère's, and Faraday's laws into the single language of classical field theory.

It is very tempting to speculate that, because of the chameleon-like behavior of quantum field theory, all the interactions are simply manifestations of a *single* field theory. Just as the "undetermined parameters" ε_0 and μ_0 were related to the velocity of light through Maxwell's unification of electricity and magnetism, so the undetermined parameters of the standard model (such as quark and lepton masses and mixing angles) might be fixed by embedding the standard model in some grand unified theory.

A great deal of effort has been focused on this question during the past few years, and some of the problems and successes are discussed in "Toward a Unified Theory" and "Supersymmetry at 100 GeV." Although hints of a solution have emerged, it is fair to say that we are still a long way from formulating an ultimate synthesis of all physical laws. Perhaps one of the reasons for this is that the role of gravitation still remains mysterious. This weakest of all the forces, whose effects are so dramatic in the macroscopic world, may well hold the key to a truly deep understanding of the physical world. Many particle physicists are therefore turning their attention to the Einsteinian view in which geometry becomes the language of expression. This has led to many weird and wonderful speculations concerning higher dimensions, complex manifolds, and other arcane subjects.

An alternative approach to these questions has been to peel yet another skin off the onion and suggest that the quarks and leptons are themselves composite objects made of still more elementary objects called preons. After all, the proliferation of quarks, leptons, gauge bosons, and Higgs particles is beginning to resemble the situation in the early 1960s when the proliferation of the observed hadronic states made way for the introduction of quarks. Maybe introducing preons can account for the mystery of flavor: e, μ, and τ, for example, may simply be bound states of such objects.

Regardless of whether the ultimate understanding of the structure of matter, should there be one, lies in the realm of preons, some single primitive group, higher dimensions, or whatever, the standard model represents the first great step in that direction. The situation appears ripe for some kind of grand unification. Where are you, Maxwell? ■

Further Reading

Gerard 't Hooft. "Gauge Theories of the Forces Between Elementary Particles." *Scientific American*, June 1980, pp. 104-137.

Howard Georgi. "A Unified Theory of Elementary Particles and Forces." *Scientific American*, April 1981, pp. 48-63.