LA-UR-12-24269

Approved for public release; distribution is unlimited.

Title:	Development and Implementation of Radiation-Hydrodynamics Verification Test Problems
Author(s):	Marcath, Matthew J. Wang, Matthew Y. Ramsey, Scott D.
Intended for:	Report

Disclaimer:

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the Los Alamos National Security, LLC for the National Nuclear Security Administration of the U.S. Department of Energy under contract DE-AC52-06NA25396. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

Development and Implementation of Radiation-Hydrodynamics Verification Test Problems

Computational Physics Student Summer Workshop

Matthew J. Marcath, Matthew Y. Wang, Scott D. Ramsey

7/20/2012

Analytic solutions to the radiation-hydrodynamic equations are useful for verifying any large-scale numerical simulation software that solves the same set of equations. The one-dimensional, spherically symmetric Coggeshall #9 and #11 analytic solutions, cell-averaged over a uniform-grid have been developed to analyze the corresponding solutions from the Los Alamos National Laboratory Eulerian Applications Project radiation-hydrodynamics code xRAGE. These Coggeshall solutions have been shown to be independent of heat conduction, providing a unique opportunity for comparison with xRAGE solutions with and without the heat conduction module. Solution convergence was analyzed based on radial step size. Since no shocks are involved in either problem and the solutions are smooth, second-order convergence was expected for both cases. The global L1 errors were used to estimate the convergence rates with and without the heat conduction module implemented.

Contents

Introduction.....	3
Verification	4
Initialization	5
Radiation-Hydrodynamic Equations.....	5
Coggeshall Equations.....	6
Discretization of Cell Quantities.....	10
Analysis and Data	10
Conclusions.....	12
Acknowledgements.....	13
References.....	14
Appendices.....	15
A: The Radiation Flux Expression.....	15
B: Heat Conduction Invariance.....	17
C: Initialization Discrepancies in xRAGE.....	19
D: Coggeshall Solution 9	20
Viewgraph Norms for Conserved Cell-Averages with and without the Heat Conduction Module Implemented	20
Viewgraph Norms for Unconserved Cell-Averages with and without the Heat Conduction Module Implemented	22
Viewgraph Norms with the Heat Conduction Module Not Implemented with and without Conserved Cell-Averages	24
Viewgraph Norms with the Heat Conduction Module Implemented with and without Conserved Cell-Averages	26
Conserved, Heat Conduction On	28
Non-Conserved, Heat Conduction On	32
Conserved, Heat Conduction Off.....	36
Non-Conserved, Heat Conduction Off.....	40
E: Coggeshall Solution 11	44
Viewgraph Norms for Conserved Cell-Averages with and without the Heat Conduction Module Implemented	44

Viewgraph Norms for Unconserved Cell-Averages with and without the Heat Conduction Module Implemented	46
Viewgraph Norms with the Heat Conduction Module Not Implemented with and without Conserved Cell-Averages	48
Viewgraph Norms with the Heat Conduction Module Implemented with and without Conserved Cell-Averages	50
Conserved, Heat Conduction On	52
Non-conserved, Heat Conduction On	56
Conserved, Heat Conduction Off.....	60
Non-conserved, Heat Conduction Off	64

Introduction

The Los Alamos National Laboratory Eulerian Applications Project radiation-hydrodynamics code xRAGE [1] solves a version of the radiation-hydrodynamics equations numerically [2]. Since the code solution is numerically derived, there are expected discretization errors associated with any solution found through iterating from initial conditions through time for a given spatial mesh. Since the spatial mesh and time stepping can be varied, the convergence rate of the code solution to an exact solution can easily be found. While the code does have adaptive mesh refinement and adaptive time stepping capabilities, only fixed time step and meshes for individual solutions will be used for ease of analysis.

Convergence rate analysis allows for quantified verification of the code. Verifying the code's numerical solution method ensures that the code is solving the intended equations correctly and as expected. Verification, however, does not insist that the code is producing solutions in agreement with experimental data. This analysis focuses on convergence rates as a function of spatial mesh size. Convergence rate refers to the rate at which the code approaches the exact solution over a series of spatial mesh reductions. Convergence can also be thought of as the rate at which error in the solution is reduced for each refinement of spatial mesh. Typically, the analytic solution used in convergence analysis is derived from the same set of equations that the code's numerical method is derived. There are expected rates of convergence for each solution method, and enabling different modules of the xRAGE code can affect these rates. In finding these rates, aspects of the code may be verified as functioning correctly. For smooth one-dimensional problems second-order convergence rates are expected of xRAGE's default hydrodynamics solver, which is a Godunov type solver. Shocks in the problem reduce the convergence rate to first order.

To analyze convergence of the xRAGE code it is convenient to use an analytically derived solution of the governing equations as the reference solution. The Coggeshall solutions nine and eleven [3] are a good choice for convergence analysis because they are smooth and physically meaningful. In the past, xRAGE has been successfully verified using solution eight, which has also been implemented in the regression test problem modules [4]. This analysis uses solutions nine and eleven. Both solutions are spherically symmetric and invariant with respect to radiation heat conduction, providing a unique opportunity to analyze heat conduction modules of xRAGE. Coggeshall created a closed-form radiation heat flux equation. The methods and assumptions that Coggeshall used to create a closed form of the radiation-hydrodynamics equations were scrutinized and confirmed to be reasonable in this analysis. Lie group theory was used to find analytic solutions to the set of equations. There are many possible analytic solutions, however only a fraction are physically meaningful.

xRAGE solves the radiation-hydrodynamics equations for a given discretization, meaning it produces discrete values for volume elements. These discrete values are analogous to cell-averaged quantities. It is necessary for practical analysis of xRAGE solutions to compare the xRAGE solutions to some form of cell-averaged quantities derived from the reference solution. There are many methods to find the cell-averages, however only the standard mean and conserved quantity weighting are included in this analysis. Conserved quantity weighting is thought to be most analogous to the modes with which xRAGE forms solutions. The algorithms that the code uses conserve mass, momentum, and energy. It has been proposed that computing cell-averages using the three conserved quantities will produce averages that most accurately show the convergence behavior of the code. The analysis presented compares the two methods of producing cell-averages.

Verification

Code verification is the process of confirming that the code solves the prescribed equations as intended. xRAGE solves the radiation-hydrodynamics equations. Verification of xRAGE will involve determining whether xRAGE solves these equations properly and at the appropriate convergence rate. The convergence rate, n , of the spatial mesh is the exponential rate at which the code more accurately approximates the solution for subsequently smaller spatial mesh sizes with respect to the error metric, E . Convergence rate varies based on the numerical method used to solve the equations. For smooth problems with no shocks, second order convergence is expected for discretized spatial mesh refinements. However, slight deviations from this value are possible because of numerical errors.

$$E(\Delta x) \approx C \Delta x^n$$

$$E = \text{error metric}$$

$$C = \text{convergence coefficient}$$

$$(1) \quad \log(E) \approx n \log(\Delta x) + \log(C)$$

$$\Delta x = \text{spatial mesh size}$$

$$n = \text{order of convergence}$$

To verify a code, it is most useful to have an analytic solution to the equations to which the code produces an approximate solution. The analytic solution often must be found by making assumptions or simplifications in the radiation heat flux term and problem geometry. While the benefit of a numerical code package is to produce solutions to complicated problems, simple problems are useful for code verification purposes. The Coggeshall solutions [3] come from a few symmetry assumptions and the postulated closed-form of the radiation heat flux. The assumptions and simplifications must be justified and done appropriately to produce an analytic solution that is consistent with a numerical approximation.

The first step of verification is to compare the code solution to the expected analytic solution. Visualization of the code and analytic solutions is known as finding the “viewgraph norm;” while not a rigorous analysis of the code, this shows if the code solution is similar to the analytic solution. Once it is confirmed that the viewgraph norm is satisfactory, the convergence rate analysis should be performed. The convergence rate should perform as well as the solution algorithm predicts but can be different for each observed quantity and for each problem type. For Coggeshall solutions nine and eleven, the algorithm should produce solutions that converge at a second order rate in space.

Convergence rate is not constant for all mesh spacings. There are regions for very small and very large mesh sizes that converge irregularly or very little. In the case of very small spatial meshes, the convergence becomes dominated by the time step and not the spatial mesh. The case of very large spatial meshes has irregular and unpredictable convergence rates. Only between these two regions does a nearly constant spatial convergence rate occur. These regions occur at different size meshes for each numerical method and each test problem [5].

Initialization

xRAGE presents difficulties at boundaries especially where problems that extend infinitely in space must be restricted to a finite region that is manageable by the code. The code treats the problem as symmetric about the origin. For infinite valued quantities near the origin, large numeric discrepancies are often observed. Usually these discrepancies dampen quickly to the analytic solution away from the origin, but numeric error in initialization can potentially propagate throughout a problem in time and space. At the outer boundary of the problem, frozen conditions are applied. A frozen outer boundary cell condition fixes the initial conditions at those outer cells for all time steps. This is a necessary approximation, but it is evident even in a viewgraph norm that it does not agree well with the analytic solution. However, in most cases the solution quickly approaches the analytic away from the boundary.

The outer boundary condition is non-physical and a result of simplifications that must be made to initialize the problem. The Coggeshall solutions are defined for all space, but must be restricted to a finite space in xRAGE. This restriction artificially imposes a false boundary condition that does not exist in the exact solution. Since the code solution is physically applicable in all ranges excluding near the outer boundary, it is appropriate to compare only these portions of the analytic and code solution.

Radiation-Hydrodynamic Equations

xRAGE and Coggeshall solve some form of the non-relativistic radiation-hydrodynamic equations. The equations consist of two sets of terms: the material state terms and the radiation heat terms. The non-relativistic radiation-hydrodynamic equations are as follows.

Conservation of Mass

$$(2) \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

Conservation of Momentum

$$(3) \quad \frac{\partial}{\partial t} \left(\rho \vec{u} + \frac{\vec{F}}{c^2} \right) + \nabla P_{mat} + \nabla \cdot (\rho \vec{u} \vec{u} + P_{rad}) = 0$$

Conservation of Energy

$$(4) \quad \frac{\partial}{\partial t} \left(\frac{1}{2} \rho u^2 + E_{mat} + E_{rad} \right) + \nabla \cdot \left[\left(\frac{1}{2} \rho u^2 + E_{mat} + P_{mat} \right) \vec{u} + \vec{F} \right] = 0$$

Where the subscript “mat” refers to the gas material and subscript “rad” refers to the radiation.

$c = \text{speed of light}$

$\rho = \rho(\vec{r}, t) \equiv \text{material density}$

$P = P(\vec{r}, t) \equiv \text{pressure}$

$\vec{u} = \vec{u}(\vec{r}, t) \equiv \text{flow velocity}$

$$\vec{F} = \vec{F}(\vec{r}, t) \equiv \text{heat flux}$$

$$E = E(\vec{r}, t) \equiv \text{energy density}$$

The radiation-hydrodynamic equations can be reduced to the Euler hydrodynamic equations [6]. The equations are reduced to (5), (6), and (7) by neglecting the radiation energy density and radiation pressure. Also $\frac{\vec{F}}{c^2} \ll 1$ is assumed in (3), so this term is neglected. Conversely, the Coggeshall solutions used in this analysis have the property of heat conduction invariance which ultimately reduces the set of radiation-hydrodynamic equations to the hydrodynamic equations.

Conservation of Mass

$$(5) \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

Conservation of Momentum

$$(6) \quad \frac{\partial}{\partial t}(\rho \vec{u}) + \nabla P_{mat} + \nabla \cdot (\rho \vec{u} \vec{u}) = 0$$

Conservation of Energy

$$(7) \quad \frac{\partial}{\partial t} \left(\frac{1}{2} \rho u^2 + E_{mat} \right) + \nabla \cdot \left[\left(\frac{1}{2} \rho u^2 + E_{mat} + P_{mat} \right) \vec{u} + \vec{F} \right] = 0$$

$$\nabla \cdot \vec{F} = 0$$

Coggeshall Equations

The radiation-hydrodynamic equations can be reduced to the form used by Coggeshall to find analytic solutions for a one-dimensional case [7]. Coggeshall assumes the ideal gas law.

$$P = \Gamma \rho T$$

$$\Gamma = \text{gas constant}$$

$$\gamma = \text{adiabatic exponent}$$

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\nabla \cdot f = \frac{1}{r} \frac{\partial(rf)}{\partial r} \text{ for one-dimensional cylindrical coordinates}$$

$$\nabla \cdot f = \frac{1}{r^2} \frac{\partial(r^2 f)}{\partial r} \text{ for one-dimensional spherical coordinates}$$

The following will be carried out in spherical symmetry.

$$(8) \quad \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 \rho \vec{u})}{\partial r} = 0$$

$$\rho_t + \frac{2\rho u}{r^2} + \rho_r u + \rho u_r = 0$$

Conservation of Momentum

$$\frac{\partial}{\partial t}(\rho \vec{u}) + \nabla P + \nabla \cdot \rho \vec{u} \vec{u} - \vec{u} \left[\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{u}) \right] = 0$$

$$\vec{u} \rho_t + \rho \vec{u}_t + \nabla P_{mat} + \vec{u}(\nabla \rho \cdot \vec{u}) + 2\vec{u} \rho(\nabla \cdot \vec{u}) - \vec{u} \rho_t - \vec{u}(\rho \nabla \cdot \vec{u}) - \vec{u}(\vec{u} \cdot \nabla \rho) = 0$$

$$\rho \vec{u}_t + \nabla P + \vec{u} \rho(\nabla \cdot \vec{u}) = 0$$

$$\rho \left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right) \vec{u} + \nabla P_{mat} = 0$$

$$\rho \vec{u}_t + \rho \vec{u} \vec{u}_r + \frac{\partial}{\partial r}(\Gamma \rho T) = 0$$

$$(9) \quad \vec{u}_t + \vec{u} \vec{u}_r + \frac{\Gamma \rho_r T}{\rho} + \Gamma T_r = 0$$

Conservation of Energy

$$\frac{\delta E}{\delta t} + \vec{u} \cdot \nabla E + \frac{1}{\rho} P \nabla \cdot \vec{u} + \frac{1}{\rho} \nabla \cdot \vec{F} = 0$$

$$E = \frac{\Gamma}{\gamma - 1} T$$

$$\frac{\Gamma}{\gamma - 1} T_t + \vec{u} \frac{\partial}{\partial r} \left(\frac{\Gamma}{\gamma - 1} T \right) + \frac{\Gamma T}{\rho r^2} \frac{\partial}{\partial r} (r^2 \vec{u}) + \frac{1}{\rho r^2} \frac{\partial}{\partial r} (r^2 \vec{F}) = 0$$

$$(10) \quad \frac{\Gamma}{\gamma - 1} (T_t + \vec{u} T_r) + \Gamma T \vec{u}_r + \frac{2\Gamma T \vec{u}}{r} + \frac{\vec{F}_r}{\rho} + \frac{2\vec{F}}{\rho r} = 0$$

From these equations and a closure model for the radiation heat flux, $\vec{F} = -K(\rho, T) \nabla T$ (appendix A), Coggeshall uses Lie group theory to find a set of analytic solutions. The general forms of the equations contain a geometry factor k ($k = 1$ for cylindrical and $k = 2$ for spherical), the adiabatic exponent γ , and conduction constants α and β . The specific heat at constant volume, C_V , was set to unity in the code input deck, so specific internal energy equal to temperature.

$$(11) \quad e = C_V T$$

Solution #9

$$\rho(r, t) = \rho_0 r^{-(2\beta+k+7)/\alpha} t^{-2[\alpha(k+1)-2\beta-k-7]/\alpha[2+(\gamma-1)(k+1)]}$$

$$u(r, t) = \frac{2r}{t[2 + (\gamma - 1)(k + 1)]}$$

$$T(r, t) = \frac{2\alpha(\gamma - 1)(k + 1)r^2}{t^2\Gamma[2 + (\gamma - 1)(k + 1)]^2(2\alpha - 2\beta - k - 7)}$$

For the analysis presented here, let $\alpha = -1$, $\beta = 2$, $\gamma = 5/3$, and $k = 2$. α and β are experimentally derived quantities [3]. $\gamma=5/3$ indicates a monatomic gas.

$$(12) \quad \rho(r, t) = \rho_0 r^{13} t^{-8}$$

$$(13) \quad u(r, t) = \frac{r}{2t}$$

$$(14) \quad T(r, t) = \frac{r^2}{60\Gamma t^2}$$

$$(15) \quad P(r, t) = \frac{\rho_0}{60} r^{15} t^{-10}$$

Figure 1: Coggeshall nine normalized solutions for key flow variables.

Solution #11

$$\rho(r, t) = \rho_0 r^{(\gamma-1)(k+1)-2} t^{1-k-(\gamma-1)(k+1)}$$

$$u(r, t) = r/t$$

$$T(r, t) = T_0 r^{2-(\gamma-1)(k+1)} t^{-2}$$

With the constraint, $\alpha = \beta + 4 + (k - 1)/[2 - (\gamma - 1)(k + 1)]$

For the analysis presented here, let $\alpha = -1$, $\beta = 2$, and $k = 2$ then $\gamma = 12/7$.

$$(16) \quad \rho(r, t) = \rho_0 r^{1/7} t^{-22/7}$$

$$(17) \quad u(r, t) = r/t$$

$$(18) \quad T(r, t) = T_0 r^{-\frac{1}{7}} t^{-2}$$

$$(19) \quad P(r, t) = \Gamma T_0 \rho_0 t^{-36/7}$$

Figure 2: Coggeshall eleven normalized solutions for key flow variables.

With appropriate constants applied, these solutions are physically meaningful. The velocity solutions increase linearly through space and decrease inversely with time. This velocity behavior, with the density and temperature behaviors, are consistent with an expanding and cooling system for both solutions. Solution nine pressure increases with power law variation

with respect to radius, but decreases inversely with time. Whereas, solution eleven pressure decreases with power law variation with respect to time and is invariant in space. These solutions have the feature of being invariant with respect to heat conduction as indicated by the $\nabla \cdot \vec{F} = 0$ condition that Coggeshall assumes. The heat flux into a volume is equal to the heat out of a volume, so there is no net effect on the volume by heat (appendix B).

Discretization of Cell Quantities

Given a spatial discretization, xRAGE produces discrete solutions: one zone-averaged value for each flow variable for each mesh zone. Since the code solution is discretized, a method must be introduced to compose corresponding discrete values from the analytic solutions. Using zone center flow variable values directly from the reference solution can produce misleading convergence results [4]. It is more useful to compare a zone-averaged quantity with the code solution, because it is more analogous to the code solution output. This analysis uses cell volume-averaged quantities to compare with the code solution. Cell-averages used in this analysis were produced by weighting the average with conserved quantities and by a standard mean. The conserved quantities refer to the quantities that xRAGE conserves, which are mass, momentum, and energy. Cell-averaged density is conserved since it is already a function of a conserved quantity, mass.

Table 1: Conserved and non-conserved cell-averaging equations over individual mesh zones used in the analysis.

Cell-Average	Non-conserved	Conserved
Density	$\bar{\rho} = \frac{\int r^2 dr \rho(r)}{\int r^2 dr}$	$\bar{\rho} = \frac{\int r^2 dr \rho(r)}{\int r^2 dr}$
Velocity	$\bar{u} = \frac{\int r^2 dr u(r)}{\int r^2 dr}$	$\bar{u} = \frac{\int r^2 dr \rho(r) u(r)}{\int r^2 dr \rho(r)}$
Specific Internal Energy	$\bar{e} = \frac{\int r^2 dr e(r)}{\int r^2 dr}$	$TE = 4\pi \int r^2 dr \rho(r) [e(r) + \frac{1}{2} u^2(r)]$ $\bar{e} = \frac{(\frac{TE}{\bar{\rho}} - \frac{1}{2} \bar{u}^2)}{\int r^2 dr}$ [8]
Pressure	$\bar{P} = \frac{\int r^2 dr \Gamma \rho(r) T(r)}{\int r^2 dr}$	$\bar{P} = \Gamma \bar{\rho}_{cons} e_{cons}$

The cell-averaged quantities are used to provide initial conditions to input into xRAGE for each mesh cell. The reason for using cell-averaged quantities for initialization is to provide a quantity that is most similar to a spatially discretized quantity that the code would produce. Also, the cell averages are used as the exact solution to compare with the code solution and form error quantities. The cell-average method used for the initial conditions must be used in analysis of the code solution.

Analysis and Data

xRAGE solutions for the quantities of interest were compared with their analytically derived counterparts to produce a volume weighted global L1 norm (21) for each spatial mesh size for a given time step. The global L1 norm is derived from the continuous L1 norm (20).

Since the outer boundary condition is not consistent with the exact solution of the outer boundary, points beyond a radius of 2.75 are not included in the convergence analysis. The plots for the L1 error norms at each time step as well as plots for convergence analysis at 3.0 s are attached in appendix D and E. The linear curve fits in the convergence analysis plots provide a slope that is used to represent the convergence rate for the given quantity at a given time in the solution. It was found that 50, 100, and 200 mesh zones appear to be in the convergence region dominated by the spatial mesh size for most state quantities.

$$(20) \quad L_1 = \frac{\int_{x_{min}}^{x_{max}} dV |f_{exact} - f_{xRAGE}|}{\int_{x_{min}}^{x_{max}} dV}$$

$$(21) \quad Error \equiv L_1 \approx \frac{\sum_{i=1}^N |f_{exact,i} - f_{xRAGE,i}| (x_{max,i}^3 - x_{min,i}^3)}{\sum_{i=1}^N (x_{max,i}^3 - x_{min,i}^3)}$$

Table 2: Convergence rates at time 2.0 s for key quantities

t = 2.0 s	Heat Conduction Module	Cell-Averages	Density	Velocity	Internal Energy	Pressure
Coggeshall #9	On	Conserved	2.19734	2.2009	1.94221	2.14571
		Unconserved	2.1924	2.19378	1.93846	2.14514
	Off	Conserved	2.19734	2.20084	1.94225	2.14571
		Unconserved	2.1924	2.19372	1.9385	2.14514
Coggeshall #11	On	Conserved	3.51083	2.60838	1.22401	2.09167
		Unconserved	3.19004	0.94538	1.9552	2.495
	Off	Conserved	3.76259	2.71418	1.23488	2.09128
		Unconserved	3.18902	0.96388	1.95858	2.48846

Table 3: Convergence rates at time 3.0 s for key quantities

t = 3.0 s	Heat Conduction Module	Cell-Averages	Density	Velocity	Internal Energy	Pressure
Coggeshall #9	On	Conserved	2.22078	2.9966	1.8986	2.17192
		Unconserved	2.2165	2.28098	1.89169	2.17158
	Off	Conserved	2.22078	2.29951	1.89866	2.17192
		Unconserved	2.2165	2.28084	1.89176	2.17158
Coggeshall #11	On	Conserved	2.80453	2.43544	1.86782	2.01101
		Unconserved	2.65554	0.82368	2.1264	1.62433
	Off	Conserved	3.1982	2.65904	1.8974	2.0056
		Unconserved	2.60006	0.905	2.14088	1.60959

Table 2 shows that at time 2.0 s the convergence rate of the Coggeshall nine solution is affected minimally by the use of the heat conduction module for both conserved and unconserved cell-averages. However, the Coggeshall eleven solution convergence rate is reduced by the use of the heat conduction module for both conserved and unconserved cell-averages for the majority of the quantities. The slight reduction in convergence rate suggests that the heat conduction module may have a negative effect on convergence. This effect is likely due

to the propagation of errors through the problem in space and time, caused by wall heating-like phenomenon near the origin.

In almost all cases the convergence rate is higher when using conserved cell-averages. Also, the convergence rates are usually second-order when using conserved cell averages. This is likely a result of the method being more consistent with the code methods; thus, it is more consistent with the order of accuracy. Coggeshall eleven velocity solutions with unconserved cell-averages exhibit less than first-order convergence, while the conserved counterparts exhibit second-order or higher rates. However, in Table 3 for Coggeshall eleven specific internal energy solutions at time 3.0 s the conserved cell-averages converge just below second-order, but the unconserved converge slightly above second order. The difference in convergence rates at this time dump for the specific internal energy solutions is minimal and one can reasonably accept that the convergence rate is still second-order for conserved averages. The only exception to the appearance of second-order convergence for conserved averages occurs at time 2.0 s for Coggeshall eleven specific internal energy solution. Both the higher convergence and the second-order behavior suggest that using conserved cell-averages is the best method to analyze convergence.

Conclusions

xRAGE displayed second-order spatial convergence for both Coggeshall nine where $\alpha = -1$, $\beta = 2$, $\gamma = 5/3$, and $k = 2$ and eleven problems where $\alpha = -1$, $\beta = 2$, $k = 2$, and $\gamma = 12/7$. Convergence analysis is best performed using conserved cell-averages for the majority of state quantities. There is a small reduction in convergence rate for Coggeshall solution eleven. The analysis of xRAGE convergence using Coggeshall nine and eleven solutions extended and reinforced existing results from the Coggeshall eight solution.

This work could be refined by improving the initialization of the flow variables in Coggeshall problem eleven near the origin. It would be especially useful for the initialization to closely match the analytic solution when the heat conduction module is being used. Investigation into the few flow variable convergence rates that were either far below or above second order is necessary. Convergence analysis with respect to the time step should be done. Also, extension of the analysis to two and three dimensions would be useful.

Acknowledgements

This work was performed under the auspices of the United States Department of Energy by Los Alamos National Security, LLC, at Los Alamos National Laboratory under contract DE-AC52-06NA25396. The authors acknowledge the support of the US Department of Energy Advanced Strategic Computing Program Verification Project under project leader S. Doebling, mentor S. Ramsey, and the support of the Los Alamos National Laboratory Computational Physics Student Summer Workshop under the direction of S. Runnels. The authors thank R.C. Hendon for valuable insights on these topics.

References

- [1] M. Gittings, R. Weaver, M. Clover, T. Betlach, N. Byrne, R. Coker, E. Dendy, R. Hueckstaedt, K. New, W. Oakes, D. Ranta and R. Stefan, *The RAGE radiation-hydrodynamics code*, Computational Science Disc. 1, 015005, 2008.
- [2] Los Alamos National Laboratory Eulerian Applications Project, "xRAGE Users Manual (v1109.03)," LA-CP-11-00643, 2012.
- [3] S. Coggeshall, "Analytic Solutions of Hydrodynamics equations," vol. A, no. 3, 1991.
- [4] F. X. Timmes, M. R. Clover, J. R. Kamm and S. D. Ramsey, "On a Cell-Averaged Solution of the Coggeshall #8 Problem," LA-UR-09-004438, 2009.
- [5] R. Hendon and S. Ramsey, "Radiation Hydrodynamics Test Problems with Linear Velocity Profiles," Los Alamos National Laboratory, 2012.
- [6] R. Drake, *High-Energy-Density Physics: Fundamentals, Inertial Fusion, and Experimental Astrophysics*, Berlin: Springer, 2006.
- [7] S. Coggeshall and R. A. Axford, "Lie group invariance properties of radiation hydrodynamics equations and their associated similarity solutions," *American Institute of Physics: Physics of Fluids*, vol. 29, 1986.
- [8] S. D. Ramsey, J. R. Kamm and J. H. Bolstad, "The Guderley Problem Revisited," Los Alamos National Laboratory, 2006.
- [9] D. Mihalas and B. Weibel-Mihalas, *Foundations of Radiation Hydrodynamics*, Mineola, New York: Dover Publications, Inc., 1984.

Appendices

A: The Radiation Flux Expression

Coggeshall states that “heat flux F can be represented through a radiation diffusion approximation” [3]. The radiation diffusion approximation is an extension of Fick’s law.

$$\text{Fick's Law } \vec{F} = -D\nabla\phi$$

$$\vec{F} = \text{diffusion flux}$$

$$D = \text{diffusion coefficient}$$

$$\phi = \text{Concentration}$$

Fick’s law is used in conjunction with the Stefan-Boltzmann law which states that the power of the radiation emitted from a black-body is proportional to its temperature, T , to the fourth power [9]. In this case $\phi \equiv \text{power of the radiation emitted} \propto T^4$.

$$\vec{F} = -D\nabla T^4$$

At this point only the diffusion coefficient, D , remains to be rationalized. The diffusion coefficient should have units $[Wm^{-2}K^{-4}]$. Classically the diffusion coefficient is composed of three main parts that are then multiplied: a factor that originates from classical electrodynamics where the radiation pressure is proportional to the internal energy density of a black-body; the characteristic velocity of the radiation, c ; and the characteristic length scale of the radiation, λ , or its mean free path in a given material. The energy and temperature units are accounted for by use of a radiation constant, a .

$$D = \frac{1}{3}c\lambda a$$

$$(22) \quad \vec{F} = -\frac{1}{3}c\lambda a\nabla T^4$$

The radiation mean free path λ , a function of density and temperature, is related to the Rosseland mean opacity, κ_v [9], and the material density, ρ .

$$\lambda = (\kappa_v\rho)^{-1}$$

The Rosseland mean opacity is also a function of density and temperature. A power-law fit of it in these variables is then assumed yielding the following. α and β were determined experimentally [7].

$$(23) \quad \lambda(\rho, T) = \lambda_0\rho^\alpha T^\beta$$

Coggeshall uses (22) and (23) to obtain a form analogous to Fourier's law in one dimensional spherical coordinates [3].

$$\vec{F} = -\frac{1}{3}c\lambda a \frac{\partial}{\partial r} T^4$$

If the problem is spherically symmetric then the equation only has radial variance.

$$\vec{F} = -\frac{4}{3}c\lambda\alpha T^3 \frac{\partial T}{\partial r} \hat{r}$$

Otherwise direction must be retained.

$$\vec{F} = -\frac{4}{3}c\lambda\alpha T^3 \nabla T$$

The prefactors can be combined to obtain a form like Fourier's law where $K(\rho, T)$ is the conductivity of the material.

$$\vec{F} = -K(\rho, T) \nabla T$$

B: Heat Conduction Invariance

The Coggleshall eight solution for a spherical geometry, $\alpha=-1$, $\beta=2$, and $\gamma=5/3$ has been stated to be independent of conduction in literature [4], “The heat conduction is such that the area-weighted flux on each cell face is equal, that is, conduction moves as much energy into a cell as it removes.” A discussion of this feature of the Coggleshall eight solution follows.

To prove the statement, the area-weighted heat flux through a cell face in spherical symmetry must be shown to equal to the area-weighted heat flux outward through the opposing cell face. This implies that the heat deposited by conduction to the cell through the one face is removed through the other face by conduction at a particular instance in time, shown in Figure 1.

Figure 3: Area-weighted heat flux into and out of a cell is equal

Show that $Q_{in} = Q_{out}$.

$Q \equiv \text{heat conducted}$

$A \equiv \text{Cell face area}$

$$F \equiv \text{heat flux} = \frac{Q}{A}$$

$$FA \equiv \text{area-weighted flux} = Q$$

$$F_{in}A_{in} = Q_{in} = Q_{out} = F_{out}A_{out}$$

For spherical symmetry $A = 4\pi r^2$.

$$F_{in}r_{in}^2 = F_{out}r_{out}^2$$

$$F = -\left(\frac{c\lambda}{3}\right)\nabla aT^4 \quad [3]$$

$$r_{in}^2\lambda_{in}\nabla T_{in}^4 = r_{out}^2\lambda_{out}\nabla T_{out}^4$$

$$\lambda(\rho, T) = \lambda_0 \rho^\alpha T^\beta \quad [3]$$

$$\lambda(\rho, T) = \lambda_0 \rho^{-1} T^2$$

$$\rho(r, t) = \rho_0 \frac{r^{\frac{1}{7}}}{t^{\frac{22}{7}}} \quad T(r, t) = T_0 (r^{-\frac{1}{7}}) (t^{-\frac{13}{7}}) \quad [4]$$

$$\lambda = \lambda_0 (\rho_0^{-1} r^{-\frac{1}{7}} t^{\frac{22}{7}}) \left(T_0^2 r^{-\frac{2}{7}} t^{\frac{13}{7}} \right)$$

$$\nabla T^4 = -\frac{4}{7} T_0^4 r^{-\frac{11}{7}} t^{-\frac{13}{7}}$$

$$\begin{aligned} r_{in}^2 \lambda_0 (\rho_0^{-1} r_{in}^{-\frac{1}{7}} t^{\frac{22}{7}}) \left(T_0^2 r_{in}^{-\frac{2}{7}} t^{\frac{13}{7}} \right) \left(-\frac{4}{7} T_0^4 r_{in}^{-\frac{11}{7}} t^{-\frac{13}{7}} \right) \\ = r_{out}^2 \lambda_0 (\rho_0^{-1} r_{out}^{-\frac{1}{7}} t^{\frac{22}{7}}) \left(T_0^2 r_{out}^{-\frac{2}{7}} t^{\frac{13}{7}} \right) \left(-\frac{4}{7} T_0^4 r_{out}^{-\frac{11}{7}} t^{-\frac{13}{7}} \right) \end{aligned}$$

$$r_{in}^2 \left(r_{in}^{-\frac{1}{7}} \right) \left(r_{in}^{-\frac{2}{7}} \right) \left(r_{in}^{-\frac{11}{7}} \right) = r_{out}^2 \left(r_{out}^{-\frac{1}{7}} \right) \left(r_{out}^{-\frac{2}{7}} \right) \left(r_{out}^{-\frac{11}{7}} \right)$$

$$1 = 1$$

The statement is true.

This method was also used to show that both Coggeshall solutions #9 and #11 are invariant to heat conduction.

C: Initialization Discrepancies in xRAGE

There are three options for initialization parameters for the xRAGE code for material regions (computational zones):

1. rhoreg and siereg (Density and Specific Internal Energy)
2. rhoreg and tevreg (Density and Temperature)
3. prsreg and tevreg (Pressure and Temperature)

All initializations of xRAGE also require the initialization of velocity. xRAGE then uses equations of state and the provided parameters to calculate the rest the initialization values. Although the input values are provided by the user, it has been observed that xRAGE uses the equation of state to recalculate temperature and pressure when given as initialization parameters. The recalculated values do not exactly match the original input values. The order of accuracy is of order 10^{-10} . Although the difference is very small, it does create errors in the simulation.

The xRAGE code manual [2] recognizes and discusses an analytic equation of state implementation error that may be responsible for these phenomena.

D: Coggeshall Solution 9

Viewgraph Norms for Conserved Cell-Averages with and without the Heat Conduction Module Implemented

Figure 4: Viewgraph norm of the xRAGE specific internal energy solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Figure 5: Viewgraph norm of the xRAGE pressure solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Figure 6: Viewgraph norm of the xRAGE density solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Figure 7: Viewgraph norm of the xRAGE velocity solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Viewgraph Norms for Unconserved Cell-Averages with and without the Heat Conduction Module Implemented

Figure 8: Viewgraph norm of the xRAGE specific internal energy solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Figure 9: Viewgraph norm of the xRAGE pressure solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Figure 10: Viewgraph norm of the xRAGE density solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Figure 11: Viewgraph norm of the xRAGE velocity solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Viewgraph Norms with the Heat Conduction Module Not Implemented with and without Conserved Cell-Averages

Figure 12: Viewgraph norm of the xRAGE pressure solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Figure 13: Viewgraph norm of the xRAGE density solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Figure 14: Viewgraph norm of the xRAGE velocity solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Figure 15: Viewgraph norm of the xRAGE specific internal energy solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Viewgraph Norms with the Heat Conduction Module Implemented with and without Conserved Cell-Averages

Figure 16: Viewgraph norm of the xRAGE specific internal energy solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Figure 17: Viewgraph norm of the xRAGE pressure solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Figure 18: Viewgraph norm of the xRAGE density solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Figure 19: Viewgraph norm of the xRAGE velocity solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Conserved, Heat Conduction On

Figure 20: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Figure 21: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Figure 22: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Figure 23: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Figure 24: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Figure 25: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Figure 26: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Figure 27: Log-log error plot with a fitted curve as a function of cell size for xRAGE specific internal energy using the Coggeshall 9 solution with conserved values and the heat conduction module implemented

Non-Conserved, Heat Conduction On

Figure 28: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Figure 29: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Figure 30: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Figure 31: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Figure 32: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Figure 33: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Figure 34: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Figure 35: Log-log error plot with a fitted curve as a function of cell size for xRAGE specific internal energy using the Coggeshall 9 solution with non-conserved values and the heat conduction module implemented

Conserved, Heat Conduction Off

Figure 36: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Figure 37: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Figure 38: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Figure 39: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Figure 40: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Figure 41: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Figure 42: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Figure 43: Log-log error plot with a fitted curve as a function of cell size for xRAGE energy using the Coggeshall 9 solution with conserved values and the heat conduction module not implemented

Non-Conserved, Heat Conduction Off

Figure 44: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

Figure 45: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

Figure 46: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

Figure 47: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

Figure 48: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

Figure 49: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

Figure 50: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

Figure 51: Log-log error plot with a fitted curve as a function of cell size for xRAGE specific internal energy using the Coggeshall 9 solution with non-conserved values and the heat conduction module not implemented

E: Coggeshall Solution 11

Viewgraph Norms for Conserved Cell-Averages with and without the Heat Conduction Module Implemented

Figure 52: Viewgraph norm of the xRAGE pressure solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Figure 53: Viewgraph norm of the xRAGE specific internal energy solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Figure 54: Viewgraph norm of the xRAGE density solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Figure 55: Viewgraph norm of the xRAGE velocity solution with and without the heat conduction module implemented for conserved cell-average values at $t=3.0$ s

Viewgraph Norms for Unconserved Cell-Averages with and without the Heat Conduction Module Implemented

Figure 56: Viewgraph norm of the xRAGE velocity solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Figure 57: Viewgraph norm of the xRAGE pressure solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Figure 58: Viewgraph norm of the xRAGE density solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Figure 59: Viewgraph norm of the xRAGE velocity solution with and without the heat conduction module implemented for unconserved cell-average values at $t=3.0$ s

Viewgraph Norms with the Heat Conduction Module Not Implemented with and without Conserved Cell-Averages

Figure 60: Viewgraph norm of the xRAGE specific internal energy solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Figure 61: Viewgraph norm of the xRAGE pressure solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Figure 62: Viewgraph norm of the xRAGE density solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Figure 63: Viewgraph norm of the xRAGE velocity solution with and without conserved cell-averages with the heat conduction module not implemented at $t=3.0$ s

Viewgraph Norms with the Heat Conduction Module Implemented with and without Conserved Cell-Averages

Figure 64: Viewgraph norm of the xRAGE specific internal energy solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Figure 65: Viewgraph norm of the xRAGE pressure solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Figure 66: Viewgraph norm of the xRAGE density solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Figure 67: Viewgraph norm of the xRAGE velocity solution with and without conserved cell-averages with the heat conduction module implemented at $t=3.0$ s

Conserved, Heat Conduction On

Figure 68: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with conserved values and the heat conduction module implemented

Figure 69: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 11 solution with conserved values and the heat conduction module implemented

Figure 70: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 11 solution with conserved values and the heat conduction module implemented

Figure 71: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 11 solution with conserved values and the heat conduction module implemented

Figure 72: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with conserved values and the heat conduction module implemented

Figure 73: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 11 solution with conserved values and the heat conduction module implemented

Figure 74: Log-log error plot with a fitted curve as a function of cell size for xRAGE specific internal energy using the Coggihall 11 solution with conserved values and the heat conduction module implemented

Figure 75: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggihall 11 solution with conserved values and the heat conduction module implemented

Non-conserved, Heat Conduction On

Figure 76: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Figure 77: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Figure 78: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Figure 79: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Figure 80: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Figure 81: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Figure 82: Log-log error plot with a fitted curve as a function of cell size for xRAGE specific internal energy using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Figure 83: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggeshall 11 solution with non-conserved values and the heat conduction module implemented

Conserved, Heat Conduction Off

Figure 84: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Figure 85: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Figure 86: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Figure 87: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Figure 88: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Figure 89: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Figure 90: Log-log error plot with a fitted curve as a function of cell size for xRAGE specific internal energy using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Figure 91: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggeshall 11 solution with conserved values and the heat conduction module not implemented

Non-conserved, Heat Conduction Off

Figure 92: Log-log error plot for all times as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented

Figure 93: Log-log error plot for all times as a function of cell size for xRAGE density using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented

Figure 94: Log-log error plot for all times as a function of cell size for xRAGE specific internal energy using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented

Figure 95: Log-log error plot for all times as a function of cell size for xRAGE velocity using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented

Figure 96: Log-log error plot with a fitted curve as a function of cell size for xRAGE pressure using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented

Figure 97: Log-log error plot with a fitted curve as a function of cell size for xRAGE density using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented

Figure 98: Log-log error plot with a fitted curve as a function of cell size for xRAGE specific internal energy using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented

Figure 99: Log-log error plot with a fitted curve as a function of cell size for xRAGE velocity using the Coggeshall 11 solution with non-conserved values and the heat conduction module not implemented