LA-UR-13-26270

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Title: Development of a 1D Lagrangian Hydrocode

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Intended for: LANL Presentation

Issued: 2013-08-07

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Development of a 1D Lagrangian Hydrocode

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Overview

Micah

1. Objectives
2. Code Development
3. Time Integration
4. 1st Order Riemann Solver
5. Hydro Methods/Results
 - a. Cell Centered
 - b. Staggered Grid
 - c. Point Centered

Andrew

6. Convergence
7. Curvilinear Coordinates
8. Future Work

Objectives

- ▶ Gain Experience
 - Hydrodynamic equations
 - Numerical methods
 - Software Engineering
- ▶ Develop 1D Hydrocode
 - Explore Hydro methods in 1D
 - Variation in Point Centered Hydro methods
 - 1D Curvilinear Coordinate Methods
 - Future
 - Explore improvement in the advection step for ALE methods

Code Development

Code Flowchart

Time Integration - RK4

$$f^{n+1} \approx f^n + \Delta t \frac{\delta}{\delta t} \left(f^n + \frac{\Delta t}{2} \frac{\delta}{\delta t} \left(f^n + \frac{\Delta t}{3} \frac{\delta}{\delta t} \left(f^n + \frac{\Delta t}{4} \frac{\delta f}{\delta t} \right) \right) \right)$$

Stored
Updated at each stage

1st Order Riemann Solver, A Godunov-like Approach

In 1D:

$$-P^* = -P + q^*$$

$$\mathbf{n}_i q^* = \mathbf{n}_i (-P^* + P_{c,i}) = \mu_i (u^* - u_{c,i})$$

$$\text{Conserve Momentum @ Boundary} \rightarrow \sum_{i=j}^{j+1} \mathbf{n}_i P^* = 0$$

$$u^* = \frac{\sum_{i=j}^{j+1} (\mu_i u_{c,i} + P_{c,i} \mathbf{n}_i)}{\sum_{i=j}^{j+1} (\mu_i)}$$

$$P^* = \mu_{j+1} (u^* - u_{c,j+1}) - P_{c,j+1} \mathbf{n}_{j+1}$$

$$\mu_i = \rho U = \rho_i \left(c_i + \frac{\gamma + 1}{2} \Delta u_i \right)$$

$$\Delta u_i = \left| \frac{u_j + u_{j+1}}{2} - u_i \right|$$

Cell Centered Hydrodynamics

$$\frac{d}{dt} \int_V \rho dV = 0$$

$$\frac{\Delta M}{\Delta t} = 0$$

$$\frac{d}{dt} \int_V \rho \mathbf{u} dV = \oint_A \mathbf{n} \cdot (-P \mathbf{I}) dA$$

$$M \frac{\Delta u_z}{\Delta t} = - \sum_{CV} (\mathbf{n} A P^*)^{n+\frac{1}{2}}$$

$$\frac{d}{dt} \int_V \rho \left(e + \frac{u^2}{2} \right) dV = \oint_A \mathbf{n} \cdot (-P \mathbf{I}) \cdot \mathbf{u} dA$$

$$M \frac{\Delta j_z}{\Delta t} = - \sum_{CV} (\mathbf{n} A P^* u^*)^{n+\frac{1}{2}}$$

Staggered Grid Hydrodynamics

$$\frac{d}{dt} \int_V \rho \mathbf{u} dV = \oint_A \mathbf{n} \cdot (-P \mathbf{I}) dA \quad M_p \frac{\Delta u_p}{\Delta t} = - \sum_{MCV} (\mathbf{n} A P_z^*)^{n+\frac{1}{2}}$$

$$\frac{d}{dt} \int_V \rho e dV = (-P \mathbf{I}) : \oint_A \mathbf{n} \mathbf{u} dA \quad M_z \frac{\Delta e_z}{\Delta t} = -P_z^{*(n+\frac{1}{2})} \sum_{ECV} (\mathbf{n} A u_p)^{n+\frac{1}{2}}$$

Results: Sod

Density

CCH & SGH

$\Delta e = 0.5$ Mbar cc/g

$\Delta \rho = 0.875$ g/cc

$\gamma = 1.4$

$t = 20 \mu s$

$L = 100$ cm

100 zones

Results: Sod

Density

CCH & SGH

$\Delta e = 24.8$ Mbar cc/g

$\Delta \rho = 0.875$ g/cc

$\gamma = 1.4$

$t = 10 \mu s$

$L = 100$ cm

100 zones

Results: Sedov

CCH & SGH

$$E_{ext} = 0.3 \text{ Mbar cc}$$

$$\gamma = 5/3$$

$$t = 1 \text{ } \mu\text{s}$$

$$L = 1.2 \text{ cm}$$

60 zones

Results: Sedov

Viewgraph-norm
Density

CCH & SGH

$E_{ext}=0.3$ Mbar cc

$\gamma=5/3$

$t=1 \mu s$

$L=1.2$ cm

500 zones

Results: Sedov

Viewgraph-norm
Internal Energy

CCH & SGH

$E_{ext} = 0.3$ Mbar cc

$\gamma = 5/3$

$t = 1 \mu s$

$L = 1.2$ cm

500 zones

Point-Centered Hydrodynamics

$$\frac{\Delta u_p}{\Delta t} = \frac{1}{M_p} \sum_{CV} F^* \quad \frac{\Delta j_p}{\Delta t} = \frac{1}{M_p} \sum_{CV} (F_z^* u_{avg})|_z^k$$

Momentum Energy

Where: $F^* = -P^* \mathbf{n} A$ Point movement: $\frac{\Delta x_p}{\Delta t} = u_p^k$

PCHA and MPC

- ▶ Modifications to avoid point crossing
- ▶ Modified PCH – use u^* , shifted CCH

$$\frac{\Delta x_z}{\Delta t} = u^*|_z^k \quad \frac{\Delta V_p}{\Delta t} = \frac{1}{M_p} \sum_{CV} \mathbf{n} \cdot u_z^*|_z^k \quad \frac{\Delta j_p}{\Delta t} = \frac{1}{M_p} \sum_{CV} (F_z^* u_z^*)|_z^k$$

- ▶ PCH Average – update nodes based on CV

$$\frac{\Delta x_z}{\Delta t} = u_{avg}|_p^k \quad \frac{\Delta V_p}{\Delta t} = \frac{1}{M_p} \sum_{CV} \mathbf{n}(u_{avg})|_z^k$$

Piston 1D – 500 zones

PCHA ———
PCH ———
MPC ———

Noh 1D – 500 Zones

PCHA ———
PCH ———
MPC ———

Sod 1D – 500 Zones

No PCH

Convergence – SGH and PCH

Noh

SGH

PCH

L1

L2

Sod

SGH

PCH

Convergence – MPC and CCH

Curvilinear Coordinates

- ▶ Cylindrical – per radian

$$V_z = \frac{\pi(r_{p+1}^2 - r_p^2)}{2\pi} \quad \nabla \cdot \mathbf{u} = \frac{1}{V} \frac{\partial V}{\partial t} = \frac{1}{r} \frac{\partial ru}{\partial r} \quad A_z = \frac{1}{2}(r_{p+1} + r_p)$$

- ▶ Spherical – per steradian

$$V_z = \frac{\frac{4\pi}{3}(r_{p+1}^3 - r_p^3)}{4\pi} \quad \nabla \cdot \mathbf{u} = \frac{1}{r^2} \frac{\partial r^2 u}{\partial r} \quad A_z = \frac{(r_{p+1}^2 + r_{p+1}r_p + r_p^2)}{3}$$

- ▶ Consistent Conservation Equations

$$\frac{\Delta \rho V}{\Delta t} = 0 \quad \underbrace{\rho V \frac{\Delta u}{\Delta t} = \sum_i -P_i^* n_i A}_{\rho \frac{\partial u}{\partial t} = -\nabla P} \quad \underbrace{\rho V \frac{\Delta j}{\Delta t} = \sum_i -P_i^* n_i u_i A}_{\rho \frac{\partial j}{\partial t} = -\nabla P u}$$

Curvilinear Results – Noh Cyl

500 Zones

0.6 μ s

Curvilinear Results – Noh Sph

500 Zones
0.6 μ s

Summary of Findings

- ▶ Planar Coordinates
 - Sod: $SGH > CCH > PCH$
 - Sedov: $SGH > CCH > PCH$
 - Noh: $CCH > SGH > PCH$
 - Piston: $CCH > SGH > PCH$
- ▶ Cylindrical Coordinates
 - Noh: $CCH > SGH > PCH$
- ▶ Spherical Coordinates
 - Noh: $CCH > SGH > PCH$

Future Work

- ▶ Completion of Curvilinear Coordinates
- ▶ Exploration of improvement in the advection step for ALE methods
- ▶ Further Research into Point Centered methods
- ▶ Exploration of different time integration schemes

Special Thanks

- ▶ Dr. Nathaniel Morgan
- ▶ Dr. Vincent Chiravalle – Input Parser Template
- ▶ Dr. Mikhail Shashkov – Curvilinear Coordinates
- ▶ Dr. Scott Runnels – Time Integration & Boundary Conditions

Questions

Curvilinear Improvements

- ▶ Predictor–corrector method tested on SGH
 - Loubere and Shashkov (2008)

$$r_{i,guess}^{k+1} = r_i^n + \frac{(u_i^n + u_i^{k+1})}{2} \Delta t$$

$$u_i^{k+1} = u_i^n + \Delta t \frac{1}{M} \sum_{CV} \mathbf{n} P A_r$$

$$r_i^{k+1} = r_i^n + \frac{(u_i^n + u_i^{k+1})}{2} \Delta t$$

$$if \quad r_i^{k+1} \neq r_{i,guess}^{k+1}$$

- ▶ No significant change in results

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