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Los Alamos National Laboratory
Los Alamos, New Mexico 87545

memorandum

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FROM: Hugh Murphy, Zora Dash and George MAIL STOP/TELEPHONE 7-4318/J981
Zyvoloski, ESS-4, Andrew White, CDO, and Milton Wing, C-3

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SUBJECT: FLUID FLOW IN DEFORMABLE ROCK JOINTS. PART I

INTRODUCTION

If fluid is injected into joints in rock masses, several types of joint deformations can take place. At first the pressure rise in the joint is small enough that the joint does not actually open. Nevertheless, the effective closure stress, that is, the difference between the total earth stress acting normal to the joint plane and the fluid pressure, is reduced. Consequently the tightness of joint closure is lessened, resulting in a small increase of the effective open space, or aperture, of the joint (Snow, 1968). If the fluid pressure rise is small enough, the aperture can still be treated as nearly constant, and the pressure response therefore follows the usual laws of linear diffusion. But if the pressure increase is large, aperture increases must be accounted for, and the flow will be affected by nonlinear diffusion due to pressure-dependent aperture, as well as a new storage coefficient due to joint compressibility.

Eventually the fluid pressure may attain a value equal to, and even slightly greater than, the original total earth stress, and the opposing surfaces of the rock that meet at the joint can actually part. We refer to this behavior as joint "liftoff," or "jacking." During liftoff, the changes in joint aperture and compressibility are very large compared with changes while the joint is still in roughness-to-roughness contact along the joint surfaces, and the flow equation becomes so highly nonlinear that pressure pulses are no longer transmitted in a smooth, diffusive manner but as a traveling contact discontinuity, analogous to a propagating hydrodynamic shock wave.

The joint deformations just discussed are those due to normal opening, that is, opening only in the direction normal to the joint plane. However, there also is the possibility that shearing may occur at a pressure somewhat lower than that required for liftoff. This is particularly true for those joints that are not oriented either parallel to or perpendicular to the axes of the principal earth stresses. Shearing results in yet another type of aperture variation in which not just pressure but shear slippage is important. Modeling of shearing deformation requires extensive knowledge of the state of stress and the shear stress acting on a joint, as well as a knowledge of the joint roughness topology and friction characteristics. Shearing deformation per se will not be considered here, but nevertheless the results we obtain below for the simpler case of normal joint opening are general enough that once a constitutive shearing law for a particular joint is determined, the following analyses could be applicable.

Part I of this report presents the theoretical development, while Part II, in preparation, applies the results to Fenton Hill geothermal field tests.

JOINT DEFORMATION BEHAVIOR

Physical Insight Model For Joints In Point-to-Point Contact

Figure 1 shows a highly idealized joint network model, and the inset shows another idealized model, this time of the joint surface roughness. The joint network consists of an infinite array of parallel joints, all of which have planar dimensions which are very much larger than the joint spacing, L . Acting normal to the joints is the total earth stress S .

The aperture and spacing of all the joints are assumed to be the same, hence we need only consider one representative joint of the infinite array. This is an exceedingly simple model, but it serves as a starting point. Later we will consider the opposite extreme, just one joint. While this also may seem too extreme a case, Tom Dey (personal communication) has presented calculations that indicate that when fluid pressure becomes equal

Figure 1. Idealized Model for Flow In Joints.

to or slightly greater than S , only one joint will open, while its neighbors remain closed. In other words, parallel fractures are inherently unstable at high pressure; the joint that initially was slightly more open than its neighbors, or slightly more perpendicular to the minimum earth stress, will preferentially accept fluid. However, at low pressures the parallel joint model may be more appropriate, and so theories for both models will be presented, but first we consider the parallel joint model.

The idealized joint roughness shown in the inset may be recognized as a simplified version of the "Bed of Nails" model of Gangi, (1978). Once again, we use such a simple model first, simply to develop the fundamental physics of the situation; later we will resort to actual experiments for quantitative modeling. In the idealized joint roughness the aperture w is constant, and the fluid pressure P is exerted in the open portions. In between, the joint is closed at the idealized surface roughness asperity, and the asperity stress, or contact stress, is S_c . If the total joint area is A and the contact area is A_c , then a simple force balance gives

$$SA = S_c A_c + P (A - A_c) \quad (1)$$

Static equilibrium requires that the midpoints of the rock blocks remain stationary, so the displacements of the dashed lines in Fig. 1 are zero. Consequently, while the fluid pressure may deform the joint, that is, the aperture may increase by dw , this deformation must be equal and opposite to the deformation, dL , of the rock block between the joint and the stationary midpoints. Thus

$$dw = -dL \quad (2)$$

Deformations stem from the stress changes, dS and dS_c , resulting from the fluid pressure change, dP , caused by fluid injection. Assuming Hookean elastic behavior and noting that displacements and strains are zero in the two dimensions parallel to the joint planes the relationships are

$$dL/L = dS/\bar{E} \quad (3)$$

and

$$dw/w = dS_c/\bar{E} \quad (4)$$

where $\bar{E} = E(1-\nu)/(1-\nu-2\nu^2)$,

E is Young's elastic modulus of the rock and ν is Poisson's ratio. A typical value of ν is 0.25 and then $\bar{E} = 1.2 E$. The stress distribution in an asperity is complicated and depends upon the irregular geometry of contact. Hence, Eq. (4) is a simple approximation, and to further simplify matters we have assumed that: (1) the asperities and joint filling material, if any, have a modulus equal to that of the rock, and, (2) even though plane strain is not strictly appropriate for an asperity, in view of the small difference between the two, $\bar{E}$ will be used in Eq. (4) instead of the theoretically more appropriate E .

Combining Eqs. (2) - (4) results in $dS_c/dS = -L/w$ and substitution in (1) results in

$$(1 + fL/w) dS = (1-f) dP + (S-P) df/f \quad (5)$$

where f is the fractional area of contact, A_c/A . Equation (5) allows for the possibility that f can change with pressure. This is particularly true when a joint is just poised to lift off. However, for a closed joint, df is normally small, so Eq. (5) simplifies to

$$dS/dP = (1-f)/(1+fL/w) \quad (6)$$

A similar result was obtained by Pine (1985). The joint spacing, L , is typically one to tens of meters, whereas w is measured in micrometers or at maximum, a few millimeters. Hence L/w is typically of the order of 10^3 to 10^7 . The fractional contact area, f , also varies widely, but 0.1 to 0.5 is a typical range for joints still in contact. For example, Bandis (1980)

found that $f = 0.4$ to 0.7 when joints were compressed to about one half their compressive strength.

With these estimates of L/w and f , it is readily observed that dS/dP must be very small when joints are still in contact, that is, before liftoff. For example, even if L is only 1 m, w as much as 1 mm, and f reduced to 1%, $dS/dP = 0.01$. In other words, the in situ earth stresses can be only insignificantly affected when joints are still in contact, even though fluid injections can change pressure by tens of MPa (thousands of psi).

A little more algebra shows that a quantity defined as the joint compressibility, β_j , is very closely approximated as

$$\beta_j \equiv (1/w) \partial w / \partial P \approx (1-f)/f \bar{E} \quad (7)$$

When we derive the fluid flow equation later, it will be found that the storage coefficient, the ability to transiently store water in a joint due to pressure increases, has two components: The first is due to the compressibility of the fluid itself, and the fluid compressibility is $\beta_f = 1/K_f$, where K_f is the bulk elastic modulus of the fluid. The second component is due to joint compressibility, β_j . For water, K_f is about 2 GPa, and for crystalline rocks, E is of the order of 30 GPa, so the ratio of compressibilities is approximately

$$\frac{\beta_j}{\beta_f} \approx \frac{K_f}{\bar{E}} \frac{1-f}{f} \approx 0.07 \frac{1-f}{f} \quad (8)$$

When joints are tightly closed, say $f \approx 0.5$, one may be able to ignore β_j ; but when $f \approx 0.1$, as for a moderately stressed joint, β_j is nearly as important as β_f ; and if liftoff is approached, say $f = 0.01$ or less, then β_j is far more important.

A More Realistic Deformation Model for Joints in Contact

To develop a more quantitatively realistic model of joint opening and closure, several authors (Goodman 1974; Bandis et al. 1983) have reviewed available data and proposed "the hyperbolic closure law"

$$\sigma = \frac{(w_i - w)}{a - b(w_i - w)} \quad , \quad (9)$$

where the effective stress, σ , is S-P; a and b are experimentally determined coefficients; $w_i - w$ is the closure, and w_i is the aperture when $\sigma=0$.

Civil and geotechnical engineers are usually interested in near-surface applications where σ is normally small and so is the closure. In this case the joint stiffness, $-d\sigma/dw$, a parameter of great importance in their computational models, is approximated as $1/a$. However, in our geothermal work the joints are buried under kilometers of overburden, so we must consider the opposite extreme, where σ is very large. In fact Eq. (9) predicts that the stress will approach infinity when the aperture attains its minimum value given by

$$a = b(w_i - w_{\min}) \quad , \quad (10)$$

where $w_{\min}$ is the smallest possible aperture that could ever be attained. While $w_{\min}$ is usually very small, measured in micrometers, it can never be truly zero, except in the limiting cases of two perfectly matching surfaces or infinitely weak joint strength. Despite the fact that it is nearly impossible to actually attain $w_{\min} = 0$, nevertheless the voids that do remain are small and increasingly isolated (it may be recalled that Bandis found that the fraction of area in contact could approach 0.7). Indeed, at very high stress Kranz et al. (1979), Tsang and Witherspoon (1981), and Barton et al. (1984) have found that the mechanical measure of the joint aperture, for example, the average of many feeler gauge measurements, while itself very small, was still many times the value which would be computed from the permeability of the joint if a flow test was conducted. Because

it is the flow aperture, not the mechanical aperture, that is important in this work, we will assume that $w_{\min} = 0$.

With this simplification, substitution of Eq. (10) in Eq. (9) results in

$$\sigma = S-P = \frac{(w_i - w)}{b w} \quad (11)$$

Earlier we showed that $dS/dP \ll 1$ for joints still in contact, so differentiation of Eq. (11) results in a more realistic estimate of joint compressibility

$$\beta_j = \frac{1}{w} \frac{\partial w}{\partial P} = b w/w_i \quad (12)$$

Thus the joint compressibility can vary tremendously because w/w_i varies approximately from 10^{-3} to 1. Values of the coefficient b must be evaluated for each particular situation, but as shown below, its variability is quite limited for hard crystalline rocks, even for joint surfaces varying greatly in topology. A recasting of Eq. (11), but without the necessity of taking $w_{\min} = 0$, results in

$$1/b = \sigma \left[\frac{w_i - w_{\min}}{w_i - w} - 1 \right] \quad (13)$$

If one determines the effective stress, $\sigma_{0.9}$, required to attain a joint closure, $w_i - w$, equal to say, 0.9, of the maximum closure, $w_i - w_{\min}$, then $b = 10/\sigma_{0.9}$. The value of $\sigma_{0.9}$ varies surprisingly little. For example, Goodman (1976) provides closure data for a matched, and then a deliberately mismatched, joint in a granodiorite specimen. Even though $(w_i - w)_{0.9}$ and

$(w_i - w_{\min})$ were three times higher for the mismatched joint, $\sigma_{0.9}$ varied only 20%, and the mean value was 10 MPa. Likewise, Barton et al., (1984) summarized considerable data for matched and mismatched joints, and from these data it can be shown that $\sigma_{0.9}$ ranges only from 10 to 40 MPa. In summary then,

$$\beta_j = 10(w/w_i) / \sigma_{0.9} = 10^{-6} w/w_i, \text{ Pa}^{-1}. \quad (14)$$

Recall that for water $\beta_f = 5 \times 10^{-10} \text{ Pa}^{-1}$. Comparison with Eq. (14) shows that $\beta_j \geq \beta_f$ when $w/w_i \geq 5 \times 10^{-4}$. The zero stress aperture, w_i , depends upon joint roughness, but it is typically of the order of a fraction to a full millimeter, so as a rough rule β_j exceeds β_f whenever w exceeds only $1 \mu\text{m}$!

Incidentally, comparison with Eq. (7) indicates that f is approximately related to w as

$$\frac{1}{1-f} = 0.1 \frac{w_i}{w} \frac{\sigma_{0.9}}{E} \quad (15)$$

Post-Liftoff Behavior

Liftoff implies no interfacial contact, $A_c = 0$ and $f = 0$, so the force balance analogous to Eq. (1) is simply

$$S = P, \quad (16)$$

so

$$dS = dP. \quad (17)$$

Equations (2) and (3) still apply, so the joint compressibility is now

$$\beta_j = L/wE. \quad (18)$$

Equation (17) indicates that $dS/dP=1$ when liftoff occurs, whereas earlier, for contacting joints, $dS/dP \ll 1$. By itself Eq. (16) implies that, once liftoff occurs, large changes can be induced in the in situ earth stress by fluid injection. However, the new relationship for joint compressibility, Eq. (18), indicates that β_j becomes many orders of magnitude larger than it was for joints in contact. In other words, when lift off occurs, the rate of formation of void space is so large that the injected fluid is easily transmitted and stored, and the fluid pressure can no longer continue to rise as easily as when the joints were in contact. The formation of void space due to increasing pressure is large for liftoff because then $dS = dP$, so that the full force of pressure increase is felt by the rock blocks and they compress, thus opening the joints. Before liftoff $dS/dP < 1$, so little deformation of the blocks occurred.

The whole earth stress and rock joint system is in a marvelous balance that results in nearly constant in situ stress: When joints are in contact, large pressure changes can be induced, but $dS/dP \ll 1$; when joints are lifted off, $dS/dP = 1$, but dP is very small because of tremendously increased storage and permeability.

JOINT FLUID FLOW BEHAVIOR

Armed with the simple approximate methods outlined above for relating the aperture of joints to fluid pressure, we now relate pressures to fluid injection conditions. As usual the starting point is the classic situation of laminar flow between two parallel smooth plates separated by the aperture w . For the present purposes, we will ignore roughness and tortuosity effects, except for noting, as we did earlier, that the aperture derived from flow tests is usually smaller than that measured mechanically. Elementary fluid dynamics shows that the equivalent laminar flow permeability of a joint is $w^2/12$. The fluid velocity, V , in the joint is related to its permeability, the fluid viscosity, μ , and pressure gradient $\partial P/\partial x$ as

$$V = \frac{-w^2}{12\mu} \frac{\partial P}{\partial x} \quad , \quad (19)$$

where x is the direction of flow. In this work we do not consider radial flow but will consider one-dimensional rectilinear flow, as shown in Fig. 1, and the flow rate per unit height* of the joint will be

$$q = wV = \frac{-w^3}{12\mu} \frac{\partial P}{\partial x} \quad . \quad (20)$$

The rate of mass flow per unit height is the density, ρ , times q , and conservation of mass requires

$$\frac{\partial(\rho q)}{\partial x} = \frac{-\partial(\rho w)}{\partial t} \quad , \quad \text{where } t \text{ is time.}$$

The right-hand side represents the local storage of mass in a joint, either as a consequence of the fluid density increasing or the aperture increasing. We have assumed that the rock blocks adjoining the joint are impermeable and that the joint under consideration does not intersect other joints. In other words, the flow injected into a joint remains in that joint.

The complete fluid flow equation can then be expressed as

$$\frac{\partial}{\partial x} \left(\frac{\rho w^3}{12\mu} \frac{\partial P}{\partial x} \right) = \left(\frac{\rho \partial w}{\partial P} + \frac{w \partial \rho}{\partial P} \right) \frac{\partial P}{\partial t} \quad . \quad (21)$$

*While height implies that the joint is vertical, for more generality H is the joint dimension in the direction perpendicular to both x and w .

Normally ρ and μ can be assumed to be constant, so factoring them, and dividing both sides by w results in

$$\frac{1}{12 \mu w} \frac{\partial}{\partial x} \left(w^3 \frac{\partial P}{\partial x} \right) = \beta \frac{\partial P}{\partial t} \quad (22)$$

where $\beta = \beta_j + \beta_f$, and where it will be recalled that

$$\beta_j = (1/w) (\partial w / \partial P)$$

and

$$\beta_f = (1/\rho) (\partial \rho / \partial P) = 1/K_f$$

Linear Diffusion

Equation (22) is often simplified by neglecting the variation of $w^3/\partial x$. This simplification yields the well-known linear diffusion equation

$$\kappa \frac{\partial^2 P}{\partial x^2} = \frac{\partial P}{\partial t} \quad (23)$$

where the hydraulic diffusivity is

$$\kappa = \frac{w^2}{12 \mu \beta} \quad (24)$$

The linear diffusion approximation holds only so long as w^3 is reasonably constant, or, more precisely, so long as

$$\frac{\partial w^3}{w^3} \ll 1$$

or

$$\frac{1}{w^3} \frac{\partial(w^3)}{\partial P} dP \ll 1 ,$$

which, using the definition of β_j , is equivalent to

$$3\beta_j dP \ll 1 .$$

For joints in contact, $\beta_j \approx 10^{-6} w/w_i \text{ Pa}^{-1}$, so that one requires

$$\Delta P(\text{MPa}) \ll \frac{w_i}{3w} . \quad (25)$$

For example, if one introduces a small pressure disturbance in a reservoir, say 1 MPa (145 psi), then the ratio of the maximum aperture, w_i , to the in situ aperture, w , must be much greater than 3, say 30, to be on the safe side. If $w_i = 0.3 \text{ mm}$, then w must be $10 \text{ } \mu\text{m}$ or less. If ΔP is instead 10 MPa (1450 psi) w must be $1 \text{ } \mu\text{m}$ or less. We estimate in situ joint apertures for the Fenton Hill reservoirs to be about $10 \text{ } \mu\text{m}$, so nonlinear diffusion behavior may be apparent for experiments in which the pressure rises by a few MPa. Nonlinear behavior should be expected for most of our Fenton Hill experiments, as the pressure rise is often tens of MPa.

Nonlinear Diffusion

Above it was shown that for many cases of practical importance the linear diffusion approximation will not hold. Furthermore, in our discussion following Eq. (14) it was pointed out that when the injected fluid is water, β_j exceeds β_f when w exceeds about $1 \text{ } \mu\text{m}$. Estimates of the

in situ aperture vary considerably, but as indicated earlier, values of $10\mu\text{m}$ and greater appear realistic. Consequently $\beta_j \gg \beta_f$, and we can ignore water compressibility. Furthermore, since

$$\beta_j = \frac{1}{w} \frac{\partial w}{\partial P} ,$$

then

$$\frac{\partial P}{\partial x} = \frac{1}{\beta_j w} \frac{\partial w}{\partial x} ,$$

and therefore Eq. (21) can be recast (still taking μ and ρ as constant) as

$$\frac{1}{12\mu} \frac{\partial}{\partial x} \left(\frac{w^2}{\beta_j} \frac{\partial w}{\partial x} \right) = \frac{\partial w}{\partial t} .$$

Recalling that for a joint in contact, $\beta_j = b w/w_i$, the result is

$$\frac{w_i}{12\mu b} \frac{\partial}{\partial x} \left(w \frac{\partial w}{\partial x} \right) = \frac{\partial w}{\partial t} . \quad (26)$$

Conversely, for a lifted-off joint,

$$\beta_j = L/(wE) \quad (18) ,$$

so

$$\frac{E}{12\mu L} \frac{\partial}{\partial x} \left(w^3 \frac{\partial w}{\partial x} \right) = \frac{\partial w}{\partial t} . \quad (27)$$

When w varies only slightly, we can recast Eq. (26) as

$$\frac{w w_i}{12\mu b} \frac{\partial^2 w}{\partial x^2} = \frac{\partial w}{\partial t} , \quad (28)$$

which is just another variation of the linear diffusion equation, only this time in terms of w , not P . (Given one variable, we can always compute the other using the appropriate aperture-pressure relationship). In summary, the equation for flow in any kind of joint can be expressed in the general form

$$a \frac{\partial}{\partial x} \left(w^n \frac{\partial w}{\partial x} \right) = \frac{\partial w}{\partial t} , \quad (29)$$

where a represents the grouping of coefficients on the left-hand side of Eqs. (26)-(28), and $n = 0, 1$, or 3 depending upon whether the joint aperture is nearly constant, the joint is in point-to-point contact, or lifted off.

SOME SOLUTIONS

Equation (29) is a parabolic partial differential equation known as the diffusion equation. When $n = 0$, the equation is linear, and its solution is well known for a variety of boundary and initial conditions (Carslaw and Jaeger, 1959). A unique characteristic of linear diffusion is its infinite velocity of disturbance propagation. Examples will be given later, but in brief, if a change in aperture is applied at a boundary, say $x = 0$, then an infinitesimally small time later this disturbance will be propagated, although very much attenuated, to an infinitely large distance.

Nonlinear diffusion has also been studied (Barenblatt, 1953; Oleinik et al., 1958; and Knerr, 1977). Solutions applicable to our situation are rarely provided, but the expected features of the anticipated solutions are known, and are particularly well presented by Zel'dovich and Raizer (1967). They show that, unlike linear diffusion where the disturbance monotonically and asymptotically declines as $x \rightarrow \infty$, nonlinear diffusion exhibits a sharp boundary at a finite value of x which separates regions of zero and nonzero disturbance. This boundary, call it an interface or a front, moves at a finite velocity, so the solution can be thought of as analogous to the passage of a hydrodynamic shock wave - ahead of the shock the medium does not sense the disturbance because it is moving faster than the acoustic velocity of the medium.

To further understand how aperture and pressure waves travel through joints, consider the semi infinite case where, at $x \rightarrow \infty$, the pressure and aperture disturbance is zero. For the initial condition, take the disturbance to again be zero. For the second boundary condition, consider first that a fixed aperture (equivalent to fixed pressure) is imposed at $x = 0$; and second, that a fixed fluid injection rate is imposed at $x = 0$.

Constant Aperture

Recapping this situation: $w(\infty, t) = 0$, $w(x, 0) = 0$ and $w(0, t) = w_0$, where w_0 is a constant aperture. For a similarity solution let $\theta = w/w_0$ and $\zeta = x/\delta$, where $\delta = (\kappa t)^{1/2}$ is the propagation distance (the distance in the x direction over which the aperture is changed) and now $\kappa = a w_0^n$. Application of the chain rule for partial differentiation results in the conversion of the original partial differential equation, (29), to the ordinary differential equation:

$$\frac{d}{d\zeta} \left(\theta^n \frac{d\theta}{d\zeta} \right) + \frac{\zeta}{2} \frac{d\theta}{d\zeta} = 0 \quad , \quad (30)$$

with boundary conditions

$$\begin{aligned} \theta &= 0 & \zeta &\rightarrow \infty \\ \theta &= 1 & \zeta &= 0 \end{aligned} \quad (31)$$

Solutions to Eqs. (30) and (31) were obtained numerically, using finite differences, and are presented in Fig. 2. When $n = 0$, linear diffusion prevails and the well-known exact solution is (Carslaw and Jaeger, 1959)

$$\theta = w/w_0 = \operatorname{erfc}(\zeta/2) \quad (32)$$

The numerical result in Fig. 2 for $n = 0$ agrees with Eq. (32) with errors less than 1% whenever θ is 0.2 or greater. When θ attains a very small value, 0.005, a more significant numerical error, 6%, crept in, but this was due to the smallness of differences. For $n = 1$, Crank (1975) provides a numerical solution which differs less than one percent from our results. Consequently, we regard our numerical solutions as very satisfactory.

Note the distinct change in character of the aperture profile (variation of $\theta = w/w_0$ with ζ) in Fig. 2 as n increases. When $n = 0$, the previously discussed linear diffusion behavior is apparent: θ approaches zero very slowly and never actually attains it, except at $\zeta = \infty$. For very small times θ is finite at great distances, so the disturbance propagation velocity is infinite. However, when $n = 3$, the aperture disturbance is zero when ζ exceeds approximately 1^* . Note also the sharp decrease in θ near the discontinuity: as $\theta \rightarrow 0$, $d\theta/d\zeta \rightarrow -\infty$. The aperture profile appears like a shock

*

Indicating that the propagation distance $\delta = (\kappa t)^{1/2}$ was an excellent choice as a scaling parameter for x .

Figure 2. Dimensionless Aperture Profiles for Fixed Aperture at $x = 0$.

front, which moves at the propagation speed of $d\delta/dt = 0.5 (\kappa/t)^{1/2}$. Similar findings, but with slightly different numerical coefficients, are found for $n = 1$, but for reduced n the steepness of the front is lessened. A final note, concerning δ , is that while the nonlinearities do not affect the usual diffusion relationship, that is $\delta = (\kappa t)^{1/2}$, κ is no longer constant but depends upon the value of the imposed aperture at $x = 0$, $\kappa = a w_0^n$.

Constant Flow Boundary Conditions

Once again we require that $w(\infty, t) = 0$ and $w(x, 0) = 0$, but now the injection flow per unit joint height, q , must be constant, or

$$q = -a \left[w^n \frac{\partial w}{\partial x} \right]_{x=0} \quad (33)$$

Introduction of the similarity variables used successfully for the constant aperture case no longer works because Eq. (33) would result in an inhomogenous systems of equations. To avoid this problem, define the new similarity variables

$$\theta = w / (q^2 t/a)^{\frac{1}{n+2}} \quad (34)$$

$$\zeta = x/\delta, \quad (35)$$

where now

$$\delta = (a q^n t^{n+1})^{1/(n+2)}. \quad (36)$$

The transformed equations become

$$\frac{d}{d\zeta} \left(\theta^n \frac{d\theta}{d\zeta} \right) - \frac{1}{n+2} \left[\theta - (n+1) \zeta \frac{d\theta}{d\zeta} \right] = 0 \quad (37)$$

with

$$\theta^n(0) \frac{d\theta(0)}{d\zeta} = -1 \quad \text{and} \quad \theta(\infty) = 0 \quad (38)$$

Numerical solutions to Eqs. (37) and (38) are presented in Fig. 3. The calculations were checked in three ways:

- (1) Solutions were obtained by finite difference and by Runge-Kutta integrations and the differences were less than two percent in every case.
- (2) If Eq. (37) is analytically integrated from 0 to ∞ , using the boundary conditions (38), it can be shown that $\int_0^\infty \theta d\zeta = 1$, which is the mathematical statement that all the fluid injected at $\zeta=0$ must be accounted for by its storage in the dilating joint. All our numerical solutions satisfy this criterion with error less than one percent.
- (3) The final check is provided by the analytical solution for linear diffusion, $n=0$. In this case the exact solution (Carslaw and Jaeger, 1959) for $\theta(0)$ is 1.128, whereas we compute 1.137, only an 0.8% error.

While the above numerical solutions are obviously useful, the fundamental findings were already embodied in the similarity transformations Eqs. (34) and (36). Examining first Eq. (36), the propagation distance for $n = 0$ is $\delta = (at)^{1/2}$, the familiar result for linear diffusion. However for $n > 0$, δ varies more strongly with time, and

Figure 3. Dimensionless Aperture Profiles For Constant Flow Rate At $x = 0$.

furthermore, it varies with q ; whereas, for linear diffusion, δ does not depend upon the injection flow rate at all. For liftoff conditions $n = 3$ and $a = E/(12 \mu L)$, so

$$\delta = (\bar{E} q^3 t / 12 \mu L)^{1/5}. \quad (39)$$

Thus the propagation velocity is $d\delta/dt \propto t^{-1/5}$, where $\propto$ indicates proportionality. The propagation velocity varies only slightly with time, indicating a behavior approaching that of the ordinary wave equation,

$$c \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial t^2} \quad \text{that governs propagation of constant velocity waves.}$$

The approximate value of the aperture behind the propagating aperture wave is given by the denominator of Eq. (34), that is,

$$w \propto \left[\frac{q^2 t}{a} \right]^{\frac{1}{n+2}}. \quad (40)$$

For joints still in contact, $n = 1$ and $a = w_i / (12 \mu b)$, so

$$w \propto \left[\frac{\mu b q^2 t}{w_i} \right]^{1/3}. \quad (41)$$

Consequently w varies reasonably strongly with flow rate but is relatively insensitive to the other parameters. Notice also that, apart from w_i , other aspects of joint geometry, particularly the spacing L , have no effect when joints are in contact. For a computational example, take $\mu = 10^{-3} \text{ Pa s}$ (corresponding to room temperature water), $b = 10/\sigma_{0.9} \approx 10^{-6} \text{ Pa}^{-1}$, $q = 2.5 \times 10^{-5} \text{ m}^2/\text{s}$ (corresponding to a total rate of only 1 bpm in a joint 100 m high), $t = 86000 \text{ s}$ (1 day), and $w_i \approx 0.3 \text{ mm}$. Then Eq. (41) predicts that $w \approx 0.3 \text{ mm}$, the same as w_i . Thus the conditions specified

result in an aperture (and pressure) such that the joint is just poised to liftoff. Increasing μ , b , q , or t would do so. When liftoff does occur, $n = 3$ and $a = E/(12\mu L)$. Consequently

$$w \propto \frac{\mu L q^2 t}{E}^{1/5} \quad (42)$$

Notice that now w varies only slightly with most variables. As an example, take $\mu = 10^{-3}$ Pa-s again, $L = 10$ m, $q = 2.5 \times 10^{-4}$ m²/s (corresponding to 10 bpm in a single joint 100 m high), $t = 86,000$ s (1 day), and $E = 30$ GPa. Then w is about 1 mm. It may be recalled that for liftoff,

$$\beta_j = \frac{L}{wE} \quad (18)$$

so simple integration yields $\Delta P = \bar{E} \Delta w/L$, where ΔP is the fluid pressure excess over S , and Δw is closely approximated as w ($= 1$ mm), because the aperture before pressurization was negligibly small, typically 1 to 10 μ m. Consequently, after attaining the value $P = S_i$ for liftoff, the additional pressure required to open the joint to 1 mm is Ew/L , or 0.3 MPa. This additional pressure must be matched by an increase of the in situ stress, because for lifted-off joints

$$dS = dP \quad (17)$$

For the Fenton Hill phase II reservoir, the initial in situ earth stress normal to the opening joints is about 70 MPa, so a 0.3-MPa increase is hardly noticeable. These rough computations support the earlier discussion that it is difficult to change in situ stresses by fluid injection, even when, as in this case, many joints as closely spaced as

just 10 m apart are lifted off. Having shown that $P-S_i^*$ is small for one example of a lifted-off joint, we can show that this conclusion more generally holds for other examples by combining Eqs. (18), (39), and (42):

$$P-S_i \propto \left[\frac{E^3 \mu q \delta}{L^3} \right]^{1/4} \quad (43)$$

So, except for the rock modulus and joint separation distance, the variation of ΔP ($=P-S_i$) with other parameters is very weak. For example, to double ΔP requires a 16-fold increase of either fluid viscosity, flow rate, or propagation distance. But propagation distance scales with $t^{4/5}$, so as a practical matter, one can effectively control ΔP primarily by increasing viscosity, because the addition of gelling agents can result in fluids as much as 10^4 times more viscous than water. Even such a drastic viscosity increase would increase ΔP in the above example 10 fold to 3 MPa, which is still fairly small in comparison to $S_i = 70$ MPa.

ANALOGY WITH FRACTURING BEHAVIOR

The liftoff of a joint is analogous to conventional hydraulic fracturing behavior, except that for natural joints there are effects due to geometry, and unlike intact rock, the jointed rock considered here exhibits no tensile strength. For the lifted off joint presented above, the characteristic dimension was L , the joint spacing. The other dimensions of the joint were assumed to be large in comparison to L . However, for hydraulic fracturing the usual theories assume that only a single fracture occurs in an infinite rock mass and that the characteristic

* The subscript i on S indicates that S_i is the initial, pre-injection value of the earth stress normal to the joint. As soon as $P > S_i$, S rises in value to P , as required by equilibrium, Eq. (17).

Figure 4. Rectangular Hydraulic Fracture.

dimension is its vertical height, H , shown in Figure 4. The length of the fracture in the x direction is usually much larger than H . If Q is the total flow rate into both sides of the rectangular fracture, then in analogy with the previous definition of the flow rate per unit height, $q = Q/(2H)$. Using our nomenclature, Perkins and Kern (1961) and Nordgren (1972) found that the fracture propagation distance, corresponding to our joint aperture propagation distance, was

$$\delta \propto \left[\frac{\bar{E} q^3 t^4}{\mu H} \right]^{1/5} \quad (44)$$

In functional form the fracture aperture was

$$w \propto \left[\frac{\mu H q^2 t}{\bar{E}} \right]^{1/5} \quad (45)$$

and the pressure excess was of the order

$$P - S_i \propto \left[\frac{\bar{E}^3}{H^3} \mu q \delta \right]^{1/4} \quad (46)$$

The agreement of these functional forms with Eqs. (39), (42), and (43) is readily apparent, so except for the change in characteristic length, H vs L , it is clear, as intuitively expected, that propagation of a single fracture and liftoff of an infinite array of parallel joints are similar processes. The effect of rock strength is not more apparent for fracturing theory because the extra pressure required to propagate a fracture in intact rock diminishes with δ . For most rocks, as soon as δ is 1 to 10 m, one may neglect strength. However, a very localized effect that should be considered in fracturing intact rock is the smooth closure criterion (Barenblatt 1962). This criterion, that $\partial w / \partial x = 0$ at the fracture tip, is required to avoid stress singularities at the tip. Unfortunately, the present results do not consider strength or stress singularities, so this

criterion is not met, but once again, this is important only in a very small region at the fracture tip.

A conceptually different fracturing model was developed by Geertsma and de Klerk (1969). To simply illustrate the difference, let us digress a moment to introduce the elastic stiffness, which is defined as the stress required to induce a unit displacement. It has the same units as elastic modulus divided by a dimension, and is particularly easy to define for the case we discussed earlier, parallel joints separated by L . To induce an opening, w , in the direction normal to the plane of the joint results in a strain of w/L , and the extra stress induced in the rock is simply Ew/L . Hence the stiffness in this case is readily seen to be E/L . Now in the case of the single hydraulic fracture in a very large rock mass, there is no characteristic dimension in the direction, z , normal to the fracture plane - the rock just extends to infinity. However, a generalized St. Venant principle is that stresses fall off as soon as the distance in any direction greatly exceeds the minimum of the fracture planar dimensions. In the case of the Perkins and Kern fracture, it was assumed that its height was smaller than δ , thus the stresses should fall off rapidly when $z > H$. In other words, the z dimension of that amount of rock mobilized into elastic resistance is roughly the same as H , so that the effective stiffness is E/H .

Suppose now that a single vertical fracture has just begun to grow or is so vertically large that $H > \delta$. In this case the mobilized rock stiffness is proportional to the smaller dimension, the propagation distance. Thus the stiffness is E/δ . Consequently the aperture of this type of fracture will depend on the ever-changing value of δ , not the previously used L or H , which was constant. The case of variable stiffness was considered by Geertsma and de Klerk and their results were

$$w \propto \left[\frac{\mu q^3 t^2}{E} \right]^{1/6} \quad (47)$$

$$\delta \propto \left[\frac{E q^3 t^4}{\mu} \right]^{1/6} \quad (48)$$

and

$$(P - S_i) \propto \left[\frac{E^3 \mu q}{\delta^2} \right]^{1/4} \quad (49)$$

Equations (47-49) are expected to be more appropriate for use than the previous equation sets whenever $\delta < H$ or $\delta < L$. Once δ exceeds the other characteristic dimension, the stiffness should be dominated by the smaller dimension. It is thus clear that the Geertsma and de Klerk model will always be superior to the others for at least some period of time. Furthermore, the reader is reminded of the earlier discussion regarding instabilities of parallel arrays of multiple joints. It is quite likely that when $P \geq S_i$, only one joint will open. Such a single joint would exhibit the more complicated stiffness behavior discussed above, and in the next section we provide solutions for this case.

SOLUTIONS FOR FLOW IN A SINGLE JOINT WITH VARIABLE STIFFNESS

The characteristic length is no longer the constant joint spacing, L , but the time varying distance, δ . We continue to maintain generality, and solve

$$\frac{a'}{\delta} \frac{\partial}{\partial x} \left(w^n \frac{\partial w}{\partial x} \right) = \frac{\partial w}{\partial t} \quad (50)$$

The a' in Eq. (50) is the same as the a previously used, except that now L has been deleted. For $n = 3$, a' is $E/(12 \mu)$, whereas a was $E/(12 \mu L)$.

Again consider two broad families of solutions for semi-infinite joints: either constant w_0 or q at $x = 0$. For the first case, again let $\theta = w/w_0$ and $\zeta = x/\delta$. Now let

$$\zeta = (\kappa t)^{1/3}, \quad (51)$$

where $\kappa = a' w_0^n$. Notice that because the stiffness now depends upon δ , the diffusion distance no longer scales with $t^{1/2}$ but $t^{1/3}$. The similarity equation becomes

$$\frac{d}{d\zeta} \left(\theta^n \frac{d\theta}{d\zeta} \right) + \frac{1}{3} \zeta \frac{d\theta}{d\zeta} = 0, \quad (52)$$

$$\theta(0) = 1, \quad \theta(\infty) = 0. \quad (53)$$

Equation (52) is similar to Eq. (30), but the coefficient is now $1/3$ rather than $1/2$. Solutions were obtained numerically and are presented in Fig. 5. Once again note the transition to nearly shock-like behavior as n increases. There is little point in further belaboring these profiles as the important features were already elicited by the similarity variables - particularly that $\delta \propto (a' w_0^n t)^{1/3}$.

For the constant flow case, the similarity transformations are

$$w = \left[q^3 t^2 / a' \right]^{\frac{1}{n+3}} \theta \quad (54)$$

and

$$\zeta = x / (a' q^n t^{n+1})^{\frac{1}{n+3}}, \quad (55)$$

Figure 5. Dimensionless Aperture Profiles for Fixed Aperture at $x = 0$,
Single Joint with Stiffness $= E/\delta$.

which results in

$$\frac{d}{d\zeta} \left(\theta^n \frac{d\theta}{d\zeta} \right) = \frac{1}{n+3} \left[2\theta - (n+1) \zeta \frac{d\theta}{d\zeta} \right], \quad (56)$$

$$\theta^n(0) d\theta(0)/d\zeta = -1, \quad \theta(\infty) = 0.$$

Numerical solutions are presented in Fig. 6. Evaluating these at $\zeta = 0$ and for $n = 3$:

$$w(0, t) = 1.98 \left[\frac{\mu q^3 t^2}{E} \right]^{1/6} \quad (57)$$

The joint propagation distance is the denominator of Eq. (55), and for $n = 3$:

$$\delta = 0.66 \left[\frac{E q^3 t^4}{\mu} \right]^{1/6} \quad (58)$$

A comparison of Eq. (58) with Eq. (39) indicates that δ varies slightly more weakly with E , μ , q , and t when the stiffness is variable rather than constant. This is to be expected, because as the stiffness diminishes with time w increases, so more flow is diverted to joint dilatation, and δ cannot increase as fast.

Eqs. (57) and (58) are in the same functional form as those of Geertsma and de Klerk, Eqs. (47) and (48). Furthermore, evaluating their results for $\nu = 0.25$, a reasonable value for rock, and using our nomenclature, their numerical coefficient for $w(0, t)$ was 2.14 instead of our 1.98, a seven percent difference, whereas for δ the coefficient was 0.59 instead of 0.66, a 9 percent difference. When rock strength is negligible, Spence and Turcotte (1985) found that the coefficients for

Figure 6. Dimensionless Aperture Profiles for Constant Flow Rate at $x = 0$,
Single Joint With Stiffness $= E/\delta$.

$w(o,t)$ and δ were 1.41 and 0.68 respectively. Considering the fact that we assumed a rather simple stiffness relation, $\bar{E}/\delta$, to describe the elastic response of the rock, rather than solve the complete stress - strain equations as approximated by Geertsma and de Klerk, and Spence and Turcotte, we are pleased that our results are intermediate to, and in fair agreement with these other approaches.

SUMMARY

We have described simple rock mechanics models for treating deformations of closed, constant aperture joints; opening-but-still-in-contact joints, and joints that lifted off when $P = S_i$. For joints still in contact we showed that the joint compressibility could be simply, but reasonably accurately, described as $\beta_j = \left[1/w\right] \partial w / \partial P = b w / w_i$ where w_i is the joint opening when the closure stress is zero, and b can be approximated as $10/\sigma_{0.9}$ where $\sigma_{0.9}$ is the effective stress required to obtain a closure of 90% of the maximum possible. It was shown that when w exceeds a few micrometers the fluid compressibility, at least of water, should be negligible. For lifted-off joints the joint compressibility is L/wE where L is the joint spacing. The stiffness in this case is E/L , but for a single joint or a hydraulic fracture, where the only characteristic dimension is either height or propagation distance, we modified the stiffness, by an ad hoc extension of St. Venant's principle, to either $\bar{E}/H$ or E/δ . The latter stiffness diminishes with time, so modified sets of equations had to be solved to predict joint behavior for variable stiffness.

In any event, it was shown that it was difficult to affect the earth stresses. For $P < S$, it is easy to increase P by injection, but $dS/dP \ll 1$. For $P = S$, $dS/dP = 1$, but the rapid opening of joint apertures almost precludes the buildup of pressure excess over the initial earth stress.

With these rock mechanics models, we then examined laminar fluid dynamics in semi-infinite joints with pressure dependent aperture. It was found that a simpler, more direct way of describing fluid dynamics was to

write the equation in terms of aperture rather than pressure. We solved the equations for two boundary conditions of great practical importance, either constant aperture at $x = 0$ or constant fluid injection flow rate. The equations were generalized, by the exponent n , so that linear diffusion behavior was recaptured for $n = 0$; joint-in-contact behavior was predicted for $n = 1$; and liftoff conditions for $n = 3$. For the constant aperture boundary condition, the similarity variables and equations are

$$\theta = w/w_0 ,$$

$$\zeta = x/\delta , \quad \delta = \left[a w_0^n t \right]^{\frac{1}{k+2}} ,$$

and

$$\frac{d}{d\zeta} \left(\theta^n \frac{d\theta}{d\zeta} \right) + \frac{\zeta}{k+2} \frac{d\theta}{d\zeta} = 0 ,$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 .$$

The integer k is zero for constant stiffness, or $k = 1$ when stiffness varies as E/δ .

For the constant flow rate condition, the similarity variables and equations are

$$\theta = w / (q^{2+k} t^{1+k} / a)^{\frac{1}{n+k+2}} ,$$

$$\zeta = x/\delta, \quad \delta = \left[a q^n t^{n+1} \right]^{\frac{1}{n+k+2}} ,$$

$$\frac{d}{dz} \left(\theta^n \frac{d\theta}{dz} \right) - \frac{1}{n+k+2} \left[(1+k) \theta^{-(n+1)} z \frac{d\theta}{dz} \right] = 0 \quad ,$$

$$\theta^n(0) \frac{d\theta(0)}{dz} = -1, \quad \theta(\infty) = 0 \quad .$$

When $k = 1$, a' should be used in place of a , and the pertinent definitions of a and a' are given in the table below. Numerical solutions to these equations were given in the text, but the essential physics are contained in the transformed variables themselves. For example, when the flow rate is constant, the joint aperture behind the wave front varies as

$$w \propto (q^{k+2} t^{k+1} / a) \frac{1}{n+k+2} \quad ,$$

and the propagation distance is

$$\delta \propto \left[a q^n t^{n+1} \right] \frac{1}{n+k+2} \quad ,$$

where again a is used if $k = 0$, but a' is used if $k = 1$.

For $n = 3$, joint liftoff, our results were in good agreement with hydraulic fracturing theories. Perhaps the most important aspect of this work is that entire families of solutions are provided for joints under several deformation conditions. The only serious limitation to their applicability is that the solutions provided are for one-dimensional rectangular flow. A useful extension would be to consider radial flow next.

TABLE I.
Definitions Of Coefficients a And a'

n	Type Of Flow	k = 0 (Constant Stiffness, infinite array of parallel joints)	k = 1 (Variable Stiffness, only one joint and $P \geq S_i$)
0	Linear Diffusion	$w_i w / (12\mu b)$	No Phys. Appl.
1	Joint In Contact	$w_i / (12\mu b)$	No Phys. Appl.
2	—	No Phys. Appl.	No Phys. Appl.
3	Liftoff	$E / (12\mu L)$	$E / (12 \mu)$

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