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Title:	FleCSI: Comprehensive Slide Deck
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What is FleCSI

What problems are we trying to solve?

- **Insulate production code development from uncertainty of evolving runtimes and architectures**
 - Focus: Data Model and Execution Model
- **Allow basic interoperability of multiple physics packages**
 - Composability
- **Allow project decomposition (no heroes...)**
 - Separation of concerns: modularity of design, complementary work
 - Specialization within disciplines: professional roles

What problems are we trying to solve?

- **Provide basic mesh infrastructure (initial focus)**
 - Allow iterability of distributed-parallel collections
 - Allow expression of topological relationships
 - Ability to reason about these relationships statically (compile-time)
- **Provide other infrastructure**
 - Graph and tree-based data structures
 - Coloring and partitioning capabilities
 - Dependency closures

What challenges are we trying to address with our CS development strategy?

Portability

Dynamic Load
Balancing

State-Dependent
Models

In-situ/In-transit
Analysis

Kernel
Specialization

What is FleCSI?

FleCSI is a C++ programming system for developing multi-physics simulation codes...

- **Runtime abstraction layer...**
 - High-level user interface, mid-level static specialization, low-level building blocks, tasking and fine-grained threading back-ends
- **Programming model...**
 - Control, execution, and data models
- **Useful data structure support...**
 - Mesh, N-Tree ($N=3 \rightarrow$ Octree), and KD-Tree

FleCSI: Pure 3D Lagrangian Sedov

FleCSI: 2D/3D Eulerian Sod

Tree Data Structures

- **Tree Topology**

- Hashed n -tree support (dimension independent: octtree, quadtree, etc.)
- Constant-time neighbor look-up
- Refinement and coarsening
- Morton ordering
- Tree traversal predicated with filter function
- Applications: SPH, N-Body, AMR, Complex Flows, Monte Carlo, Molecular Dynamics
- Collaborations with Mesh-Free and Transport teams will help us to mature this capability

How does FleCSI work?

- **Low-Level (FleCSI Core Capability)**
 - FleCSI provides a templated, low-level interface that can be specialized for a particular class of physics packages and application needs
- **Specialization**
 - A specialization provides application developers with a high-level interface that is customized to their nomenclature and data structure requirements
- **Task Abstraction (FleCSI Runtime Abstraction)**
 - Using the FleCSI task abstraction layer and some compile-time techniques, the application developer is given a programming system that is transparently distributed-memory parallel
- **Kernel Abstraction (FleCSI Runtime Abstraction)**
 - Using the FleCSI kernel abstraction layer with compile-time techniques, application developers are given a programming system that is transparently fine-grained, data-parallel
- **Data Abstraction (FleCSI Core + Runtime)**
 - FleCSI provides a data model that integrates with the task and kernel abstractions to provide easy registration and access to various data types with automatic dependency tracking

Where does FleCSI sit in the software stack?

Many people have made significant contributions to FleCSI...

Nick Moss (CCS-7)

Topology types, sparse data, design

Marc Charest (XCP-1)

Mesh specialization, data structures, design

Irina Demeshko (CCS-7)

Legion backend, MPI/Legion interoperability, design

Li-Ta (Ollie) Lo (CCS-7)

Partitioning interface, design

John Wohlbier (formerly CCS-2)

Mesh specialization, I/O

Josh Payne (CCS-7)

Execution model, design

Christoph Junghans (CCS-7)

Build system, continuous integration, Docker packaging

Gary Dilts (CCS-2)

Tree specialization, data structures, design

Christopher Malone (CCS-2)

Remap implementation, design

David Daniel (CCS-7)

Requirements, design

Nathaniel Morgan (XCP-8)

Requirements

Vince Chiravalle (A-2)

Requirements

Joe Schmidt (XTD-NTA)

Requirements

Chris Sewell (CCS-7)

Requirements, design

Rao Garimella (T-7)

Requirements, design

Wes Even (CCS-2)

Requirements, design

Multiblock unstructured meshes

Mixed triangles and quads

Separation of Concerns

Our project structure separates roles and concerns...

FleCSI Control/Execution Model

The FleCSI execution/control model is a reflection of how we (LANL) think about multi-physics...

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FleCSI Data Model Interface

What is the FleCSI data interface?

High-Level

What is the FleCSI data interface?

High-Level

What is the FleCSI data interface?

Direct Interface

```
template<size_t ST, typename T, size_t NS,  
        typename ... Args>  
decltype(auto) register_data(  
    data_client_t & data_client,  
    const const_string_t & key,  
    size_t versions=1,  
    Args && ... args)
```


What is the FleCSI data interface?

Direct Interface

storage
type type namespace

```
template<size_t ST, typename T, size_t NS,  
typename ... Args>  
decltype(auto) register_data(  
data_client_t & data_client, ← topology  
const const_string_t & key, ← identifier  
size_t versions=1, ← versions  
Args && ... args)
```

What is the FleCSI data interface?

Low-Level

```
template<typename T, size_t NS, typename ... Args>
static handle_t<T> register_data(
    data_client_t & data_client,
    data_store_t & data_store, const const_string_t &
    key, size_t versions, size_t index_space, Args && ...
    args)
```

What is the FleCSI data interface?

Low-Level

Mapped to specific storage type

```
template<typename T, size_t NS, typename ... Args>
static handle_t<T> register_data(
    data_client_t & data_client,
    data_store_t & data_store, const const_string_t &
    key, size_t versions, size_t index_space, Args && ...
    args)
```

What is the FleCSI data interface?

Backend

```
FieldSpace fs = runtime->create_field_space(ctx);
```

What is the FleCSI data interface?

Backend

Storage type uses Legion runtime to create appropriate field space(s)

```
FieldSpace fs = runtime->create_field_space(ctx);
```

What storage types do we support?

- **Dense: One dimensional, contiguous array**
 - Use Case: Physics state data
- **Global: Single data instance (there's only one...)**
 - Use Case: Simulation state data
- **Local: One dimensional, contiguous array**
 - Use Case: Scratch data
- **Sparse: Dense index space, sparse population**
 - Use Case: Material data, execution-dependent data, sparse matrices
- **Tuple: Combination of other storage types**
 - Use Case: Provide struct-like support for cleaner task definitions

Sparse Data Storage Type

Managing sparse data representations presents some challenges...

Managing sparse data representations presents some challenges...

How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Generic Compressed Sparse Matrix Insertion: Algorithms and Implementations in MTL4 and FEniCS
POOSC '09 Proceedings of the 8th workshop on Parallel/High-Performance Object-Oriented Scientific Computing

How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Code block to mutate sparse structure →

```
{  
  m = get_mutator(A, 3);  
  m[0][1] = 6;  
} // scope
```

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$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Constructor inserts space for new values →

```
{  
  m = get_mutator(A, 3);  
  m[0][1] = 6;  
} // scope
```

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$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

User specifies
maximum total
slots

Constructor inserts space for new values →

```
{
  m = get_mutator(A, 3);
  m[0][1] = 6;
} // scope
```

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How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Intuitive interface to set non-zeros →

```
{
  m = get_mutator(A, 3);
  m[0][1] = 6;
} // scope
```

How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Intuitive interface to set non-zeros →

(logically, m has a 5x3 dense structure like A)

```
{
  m = get_mutator(A, 3);
  m[0][1] = 6;
} // scope
```

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How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Column order is preserved, single-slot shift, only slot end is incremented

Intuitive interface to set non-zeros →

```
{
  m = get_mutator(A, 3);
  m[0][1] = 6;
} // scope
```


How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

What if we need more than 3 non-zeros? →

```
{
  m = get_mutator(A, 3);
  m[0][1] = 6;
} // scope
```

How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

What if we need more than 3 non-zeros? →

```
{
  m = get_mutator(A, 3);
  m[0][3] = 7;
} // scope
```

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How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

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```
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  m = get_mutator(A, 3);
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How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Destructor recompresses data →

```
{
  m = get_mutator(A, 3);
  m[0][3] = 7;
} // scope
```

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How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

The time complexity for inserting n non-zeros is $O(n)$ for direct insertion and $O(n \log(n))$ for indirect insertion.

How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

The memory complexity depends on the application, but can be quite efficient if good estimates are known.

How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

Implementation will support ELL-like dense number of materials format, i.e., all rows have space for m non-zeros

How does FleCSI handle sparse data?

$$A = \begin{bmatrix} 1 & 6 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 5 \end{bmatrix}$$

We will also support nested sparsity through the use of data handles.

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FleCSI Execution Model Interface

What is the FleCSI execution interface?

```
double my_task(accessor_t & arg0, accessor_t & arg1, double arg2) {  
    // Task logic  
}  
  
register_task(my_task);  
  
void driver(int argc, char ** argv) {  
  
    // Register some data on the cells of the mesh  
    register_data(mesh, solver, pressure, double, dense, 1, cells);  
    register_data(mesh, solver, density, double, dense, 1, cells);  
    double arg2 = 0.0;  
  
    // Get data handles  
    auto arg0 = handle(mesh, solver, pressure, double, read_write);  
    auto arg0 = handle(mesh, solver, density, double, write_discard);  
  
    // Execute a task  
    auto future = execute_task(my_task, arg0, arg1, arg2);  
} // driver
```

Task Execution Flow

`execute_task(user_task, handle args...)`

`flecsi::execute_task`
(static processing)

`flecsi::task_wrapper`
(static processing)

`user_task(accessor args...)`

Legion
Runtime

```
graph TD; A[execute_task(user_task, handle args...)] --> B[flecsi::execute_task (static processing)]; B --> C((Legion Runtime)); C --> D[flecsi::task_wrapper (static processing)]; D --> E[user_task(accessor args...)]
```

The diagram illustrates the task execution flow. It begins with a call to `execute_task(user_task, handle args...)`, which leads to `flecsi::execute_task` (static processing). This function then interacts with the Legion Runtime, represented by a blue cloud. The Legion Runtime then calls `flecsi::task_wrapper` (static processing), which finally executes the `user_task(accessor args...)`.

Processing Data Handle Arguments

```
execute_task(  
    data_handle_t & arg0,  
    data_handle_t & arg1,  
    double arg2  
);
```

```
...  
tuple_for_each(args, [&](auto & arg) {  
    if(std::is_base_of<data_handle_t, decltype(arg)>::value) {  
        launcher.add_region_requirement(/* values from handle */);  
        launcher.add_field(/* one or more field handles */);  
    }  
}  
  
context->execute_task(launcher);  
...
```

Processing Physical Regions

```
...  
tuple_for_each(targs, [&](auto & arg) {  
    if(std::is_base_of<data_handle_t, decltype(arg)>::value) {  
        std::get<el>(targs) = make_accessor(regions[i++]);  
    }  
}
```

```
tuple_function(user_task, targs);
```

```
...
```

```
user_task(  
    accessor_t & arg0,  
    accessor_t & arg1,  
    double arg2  
);
```

Legion Execution Workflow

Legion/MPI Initialization in FleCSI

FleCSI Mesh Description

What is a FleCSI Mesh?

A FleCSI `mesh_topology_t` is a data structure that allows the storage and initialization of topological connectivities.

- A developer uses FleCSI to create mesh interfaces for application developers
- The mesh topology can be statically specialized to use different entity types **and** to store different connectivity information
- *Clarification (definition): mesh domains separate mesh topologies that would otherwise have incompatible types, e.g., edges and corners have the same topological dimension and cannot reside in the same topology.*

FleCSI meshes are composed of mesh entities...

FleCSI meshes are composed of mesh entities...

Vertices

FleCSI meshes are composed of mesh entities...

Edges

FleCSI meshes are composed of mesh entities...

Cells

FleCSI meshes are composed of mesh entities...

Corners

Example mesh specialization types...

```
// Definitions of mesh entities
using entity_types =
    std::tuple<
        std::pair<domain_<0>, vertex_t>,
        std::pair<domain_<0>, edge_t>,
        std::pair<domain_<0>, cell_t>,
        std::pair<domain_<1>, corner_t>
    >;

// Connectivities are adjacencies of entities within a single domain
using connectivities =
    std::tuple<
        std::tuple<domain_<0>, vertex_t, edge_t>,
        std::tuple<domain_<0>, vertex_t, cell_t>,
        std::tuple<domain_<0>, cell_t, vertex_t>
    >;

// Bindings are adjacencies of entities across two domains
using bindings =
    std::tuple<
        std::tuple<domain_<0>, domain_<1>, cell_t, corner_t>,
        std::tuple<domain_<0>, domain_<1>, vertex_t, corner_t>
    >;
```


FleCSI mesh entities can be enumerated to create Index Spaces

FleCSI mesh entities can be enumerated to create Index Spaces

Vertices

$\{ 0, 1, 2, 3, 4, 5, 6, 7, 8 \}$

FleCSI mesh entities can be enumerated to create Index Spaces

Edges

$\{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \}$

FleCSI mesh entities can be enumerated to create Index Spaces

Cells
 $\{ 0, 1, 2, 3 \}$

FleCSI mesh entities can be enumerated to create Index Spaces

Corners

{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 }

The FleCSI interface separates access from data layout...

- The entities of the mesh define iterable collections:

```
foreach(auto c: mesh.cells())...
```

- The index spaces of the mesh define data attachment sites:

```
register_data(m, hydro, pressure, double, dense, 1, cells);
```

- Index spaces and data model map well to modern programming efforts like Legion

High-level Example

```
void driver(int argc, char ** argv) {  
  
    // Mesh object  
    minimal_mesh_t mesh;  
  
    // Initialize the mesh  
    init_mesh(mesh);  
  
    // Register some data on the cells of the mesh  
    register_data(mesh, solver, pressure, double, dense, 1, cells);  
  
    // Iterate over the interior cells  
    {  
        auto p = get_accessor(mesh, solver, pressure, double, dense, 0);  
  
        for(auto c: mesh.cells(interior)) {  
            p(c) = 1.0;  
        } // for  
    } // scope  
  
} // driver
```

The minimal_mesh_t uses FleCSI to implement the mesh interface...

```
class minimal_mesh_t
: public topology::mesh_topology_t<minimal_types_t> {
public:

    // Initialize the mesh topology. This is called after the specialization
    // adds the mesh entities.
    void init() {

        // Initialize domain 0 of the mesh topology.
        topology::mesh_topology_t<minimal_types_t>::init<0>();

        // Other initialization things here, e.g. create special static index spaces...

    } // init

}; // minimal_mesh_t
```


The minimal_mesh_t uses FleCSI to implement the mesh interface...

```
class minimal_mesh_t
: public topology::mesh_topology_t<minimal_types_t> {
public:
    // Lots of code stuff here...

    // Return vertex iterator. A vertex has topological dimension 0, the second '0' is the domain
    auto vertices() {
        return topology::mesh_topology_t<minimal_types_t>::entities<0, 0>();
    } // vertices

    // Return iterator to the vertices of an entity 'e'
    template<typename E>
    auto vertices(E * e) {
        return topology::mesh_topology_t<minimal_types_t>::entities<0, 0>(e);
    } // vertices

    // Return cell iterator
    auto cells() {
        return topology::mesh_topology_t<minimal_types_t>::entities<dimension, 0>();
    } // cells

}; // minimal_mesh_t
```

The minimal_mesh_t uses FleCSI to implement the mesh interface...

```
class minimal_mesh_t
: public topology::mesh_topology_t<minimal_types_t> {
public:
    // Lots of code stuff here...

    // Return an iterator to a static subset of the cells (created in init())
    auto cells(size_t is) {
        switch(is) {
            case minimal::interior:
                return interior_cells_;
            case minimal::boundary:
                return boundary_cells_;
            default:
                assert(false && "unknown index space");
        } // switch
    } // cells

}; // minimal_mesh_t
```

The `minimal_types_t` type provides static specialization information to `mesh_topology_t`

```
// Definitions of mesh entities
using entity_types =
    std::tuple<
        std::pair<domain_<0>, vertex_t>,
        std::pair<domain_<0>, edge_t>,
        std::pair<domain_<0>, cell_t>,
        std::pair<domain_<1>, corner_t>
    >;

// Connectivities are adjacencies of entities within a single domain
using connectivities =
    std::tuple<
        std::tuple<domain_<0>, vertex_t, edge_t>,
        std::tuple<domain_<0>, vertex_t, cell_t>,
        std::tuple<domain_<0>, cell_t, vertex_t>
    >;

// Bindings are adjacencies of entities across two domains
using bindings =
    std::tuple<
        std::tuple<domain_<0>, domain_<1>, cell_t, corner_t>,
        std::tuple<domain_<0>, domain_<1>, vertex_t, corner_t>
    >;
```

Mesh Partitioning

How does FleCSI handle distributed memory?

Mesh

How does FleCSI handle distributed memory?

Mesh

The mesh is partitioned by entities in one of the topological dimensions, e.g., cells.

How does FleCSI handle distributed memory?

Mesh

The mesh is partitioned by entities in one of the topological dimensions, e.g., cells.

The primary partitioning splits the topology into contiguous sets of cells.

How does FleCSI handle distributed memory?

Mesh

Using a strategy defined by the specialization, the dependency closure of the mesh is formed, creating several sets of indices (index spaces).

How does FleCSI handle distributed memory?

Mesh

Using a strategy defined by the specialization, the dependency closure of the mesh is formed, creating several sets of indices (index spaces).

Exclusive: I own them, and nobody else cares about them

Shared: I own them, and some other people care about them

Ghost: I don't own them, but I care about them

- exclusive 1
- exclusive 2
- exclusive 3
- shared 1
- shared 2
- shared 3
- ghost 1
- ghost 2
- ghost 3

How does FleCSI handle distributed memory?

Mesh

Multiple partitionings and partition closure strategies can be employed within the same specialization.

How does FleCSI handle distributed memory?

Mesh

On each SPMD task (rank), the closure forms a set of virtual index spaces that represent a complete set of dependency data.

How does FleCSI handle distributed memory?

Mesh

The virtual index spaces can be iterated using *foreach* semantics.

```
{  
  auto m = get_accessor("materials", material_t, cells, sparse, rw);  
  
  foreach(auto c: mesh.cells()) {  
    foreach(auto i: c.materials()) {  
      m[i] = 1.0;  
    } // for  
  } // for  
} // scope
```

How does FleCSI handle distributed memory?

Processor 1

Mesh

The virtual index spaces can be iterated using *foreach* semantics.

exclusive 1
shared 1
ghost 1

```
{  
  auto m = get_accessor("materials", material_t, cells, sparse, rw);  
  
  foreach(auto c: mesh.cells()) { // traverse all cells (union of sets )  
    foreach(auto i: c.materials()) {  
      m[i] = 1.0;  
    } // for  
  } // for  
} // scope
```

How does FleCSI handle distributed memory?

How does FleCSI handle distributed memory?

Mesh

The virtual index spaces can be iterated using *foreach* semantics.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

```
{
  auto m = get_accessor("materials", material_t, cells, sparse, rw);

  foreach(auto c: mesh.cells(ghost)) { // traverse cells in the ghost set
    foreach(auto i: c.materials()) {
      m[i] = 1.0;
    } // for
  } // for
} // scope
```

How does FleCSI handle distributed memory?

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

Mesh

This provides a clean interface for complex data access and execution that handles dependency updates using permissions specified for the Legion task and logical regions.

```
{
  auto m = get_accessor("materials", material_t, cells, sparse, rw);

  foreach(auto c: mesh.cells(local)) {
    foreach(auto i: c.materials()) {
      m[i] = 1.0;
    } // for
  } // for
} // scope
```


How does FleCSI use Legion for dependency updates?

Mesh

The local mesh stores the partition information as several index spaces using an *IndexPartition* (a Legion C++ type).

How do the topology and data models work together for Legion?

How do the topology and data models work together for Legion?

Index
Space(s)

When a user registers data, the data manager and topology cooperate to create several logical regions...

```
{  
  register_data("pressure", double, cells, dense);  
} // scope
```

The Legion backend will use *exclusive 1*, *shared 1*, and *ghost 1* with new field spaces of type double to create three new logical regions.

How do the topology and data models work together for Legion?

Index
Space(s)

FleCSI tasks operate transparently on Legion logical regions using compile-time data handles that are available through the FleCSI interface:

```
{  
  auto r = get_handle("density", double, cells, local, dense, ro);  
  auto e = get_handle("internal energy", double, cells, local, dense, ro);  
  auto p = get_handle("pressure", double, cells, local, dense, rw);  
  
  auto p = execute_task(eos, r, e);  
} // scope
```

Field
Space(s)

Legion Partitioning (old interface)

Cells Index Space

Mesh

exclusive 1

exclusive 2

exclusive 3

shared 1

shared 2

shared 3

ghost 1

ghost 2

ghost 3

Mesh Cells IS → Global Cells IS

{ 4, 9, 10, 11, 16, 17, 18, 19, 24, 28, 1, 2, 5, 6, 12, 13, 20, 25, 29, 3, 7, 8, 14, 15, 21, 22, 23, 26, 27 }
{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28 }

Mesh Cells IS → Global Cells IS

primary 1: { 4, 9, 10, 11, 16, 17, 18, 19, 24, 28 }
actual 1: { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

primary 2: { 1, 2, 5, 6, 12, 13, 20, 25, 29 }
actual 2: { 10, 11, 12, 13, 14, 15, 16, 17, 18 }

primary 3: { 3, 7, 8, 14, 15, 21, 22, 23, 26, 27 }
actual 3: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28 }

ghost 1: { 1, 5, 8, 23 }
actual 1: { 10, 12, 21, 26 }

ghost 2: { 3, 4, 7, 11 }
actual 2: { 19, 0, 20, 3 }

ghost 3: { 1, 6, 16 }
actual 3: { 10, 13, 4 }

Index Partitions (Global IS indices)

exclusive 1: { 1, 2, 5, 6, 7, 8, 9 }
shared 1: { 0, 3, 4 }
ghost 1: { 10, 12, 21, 26 }

exclusive 2: { 11, 14, 15, 16, 17, 18 }
shared 2: { 10, 12, 13 }
ghost 2: { 19, 0, 20, 3 }

exclusive 3: { 22, 23, 24, 25, 27, 28 }
shared 3: { 19, 20, 21, 26 }
ghost 3: { 10, 13, 4 }

Vertices Index Space

Mesh

exclusive 1
exclusive 2
exclusive 3

shared 1
shared 2
shared 3

ghost 1
ghost 2
ghost 3

Mesh Vertices IS → Global Vertices IS

{ 8, 9, 10, 11, 12, 13, 21, 22, 23, 1, 2, 3, 14, 15, 16, 17, 4, 5, 6, 7, 18, 19, 20}
{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22}

Mesh Vertices IS → Global Vertices IS

primary 1: { 8, 9, 10, 11, 12, 13, 21, 22, 23 }
actual 1: { 0, 1, 2, 3, 4, 5, 6, 7, 8 }

primary 2: { 1, 2, 3, 14, 15, 16, 17 }
actual 2: { 9, 10, 11, 12, 13, 14, 15 }

primary 3: { 4, 5, 6, 7, 18, 19, 20 }
actual 3: { 16, 17, 18, 19, 20, 21, 22 }

ghost 1: { 14, 16, 17, 18, 7 }
actual 1: { 12, 14, 15, 20, 19 }

ghost 2: { 13, 22, 23, 21, 18, 4 }
actual 2: { 5, 7, 8, 6, 20, 16 }

ghost 3: { 16, 17, 23, 21, 8 }
actual 3: { 14, 15, 8, 6, 0 }

Index Partitions (Global IS indices)

exclusive 1: { 1, 2, 3, 4 }
shared 1: { 0, 5, 6, 7, 8 }
ghost 1: { 12, 14, 15, 19, 20 }

exclusive 2: { 9, 10, 11, 13 }
shared 2: { 12, 14, 15 }
ghost 2: { 5, 6, 7, 8, 16, 20 }

exclusive 3: { 17, 18, 19 }
shared 3: { 16, 19, 20, 22 }
ghost 3: { 0, 6, 8, 14, 15 }

Representing connectivity in a dependent partitioning friendly way

- These are homogenous polygons so we could simply have a field for each vertex, but imagine if these polygons are heterogeneous or we are representing vertex to edge connectivity where a single vertex could be incident on many edges.

Mesh Cells IS → Global Cells IS

{ 4, 9, 10, 11, 16, 17, 18, 19, 24, }
{ 0, 1, 2, 3, 4, 5, 6, 7, 8, }

We add a field over the Global Cells IS which references ranges in another index space “D”

{ 0:2, 3:5, 6:8, 9:11, 12:14, 15:17, 18:20, 21:23, 24:26 } IS C x Field<Range> Vertices

We create an index space D and add a field over this IS that points to elements in the Global Vertices IS

{ 6,7,8, 1,7,6, 5,7,8, 5,8,12, 0,6,22, 0,1,6, 1,2,7, 4,5,7, 2,4,7, } IS D x Field<ptr_t> vertices

Mesh Vertices IS → Global Vertices IS

{ 8, 9, 10, 11, 12, 13, 21, 22, 23, 1, 2, 3, 14, 15, 16, 17, 4, 5, 6, 7, 18, 19, 20 }
{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 }

Simple 8x8 mesh

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Creating the primary partition...

```
// Create a graph definition from file. The simple_definition_t type maps the
// information in the file, but does not read the mesh definition into memory.
flecsi::topology::simple_definition_t gd("simple2d-8x8.msh");

// Create the dCRS representation for the distributed partitioner.
auto dcrs = flecsi::dmp::make_dcrs(gd);

// Create a partitioner instance to generate the primary partition.
auto partitioner = std::make_shared<flecsi::dmp::parmetis_partitioner_t>();

// Create the primary partition.
auto primary = partitioner->partition(dcrs);
```

Cell 0 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 1 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 2 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 3 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 4 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 5 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 6 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
5 7 14 6 15 0 9 16 1 8 10 17
2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 7 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 8 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 9 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 10 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell 11 Adjacencies

dCRS for rank 0

offsets: 0 2 5 8 11 14 17 20 22 25 29 33 37
indices: 1 8 0 2 9 1 3 10 2 4 11 3 5 12 4 6 13
 5 7 14 6 15 0 9 16 1 8 10 17
 2 9 11 18 3 10 12 19
distribution: 0 12 25 38 51 64

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Creating the primary partition...

```
// Create the dCRS representation for the distributed partitioner.  
auto dcrs = flecsi::dmp::make_dcrs(gd);  
  
// Create a partitioner instance to generate the primary partition.  
auto partitioner = std::make_shared<flecsi::dmp::parmetis_partitioner_t>();  
  
// Create the primary partition.  
auto primary = partitioner->partition(dcrs);
```

What ParMETIS gives you back...

- rank 0
- rank 1
- rank 2
- rank 3
- rank 4

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Simple 8x8 mesh partitioned with 5 ranks

- rank 0
- rank 1
- rank 2
- rank 3
- rank 4

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Forming the dependency closure...

```
// Compute the dependency closure of the primary cell partition
// through vertex intersections (specified by last template argument "0").
// First two template arguments are "2" (cell) to "2" (cell) in 2D.
// To specify edge or face intersections, use 1 (edges) or 2 (faces in 3D).
auto closure = flecsi::topology::entity_closure<2,2,0>(gd, primary);

// Subtracting out the initial set leaves just the nearest
// neighbors. This is similar to the image of the adjacency
// graph of the initial indices.
auto nearest_neighbors = flecsi::utils::set_difference(closure, primary);
```

Rank 0: primary + nearest neighbors

- rank 0
- rank 1
- rank 2
- rank 3
- rank 4

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Rank 0: nearest neighbors

- rank 0
- rank 1
- rank 2
- rank 3
- rank 4

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Index Spaces

- exclusive
- shared
- ghost

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Cell Partition Information

exclusive cells

0 0 0 []
1 0 1 []
8 0 4 []
9 0 5 []
16 0 7 []
17 0 8 []

shared cells

2 0 2 [1]
3 0 3 [1]
10 0 6 [1 2]
18 0 9 [1 2]
24 0 10 [3]
25 0 11 [3]
26 0 12 [2 3]

ghost cells

4 1 0 []
11 1 4 []
12 1 5 []
19 2 0 []
27 2 1 []
32 3 0 []
33 3 1 []
34 3 2 []
35 2 5 []

Forming the vertex dependency closure...

```
// We can iteratively add halos of nearest neighbors, e.g.,  
// here we add the next nearest neighbors. For most mesh types  
// we actually need information about the ownership of these indices  
// so that we can deterministically assign rank ownership to vertices.  
auto nearest_neighbor_closure =  
    flecsi::topology::entity_closure<2,2,0>(gd, nearest_neighbors);  
  
// The union of the nearest and next-nearest neighbors gives us all  
// of the cells that might reference a vertex that we need.  
auto all_neighbors = flecsi::utils::set_union(nearest_neighbors,  
    next_nearest_neighbors);
```

Rank 0: nearest neighbors + next nearest neighbors

- rank 0
- rank 1
- rank 2
- rank 3
- rank 4

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Rank 0: primary + nearest neighbors + next nearest neighbors

- rank 0
- rank 1
- rank 2
- rank 3
- rank 4

56	57	58	59	60	61	62	63
48	49	50	51	52	53	54	55
40	41	42	43	44	45	46	47
32	33	34	35	36	37	38	39
24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

Rank 0 vertex closure

- rank 0
- rank 1
- rank 2
- rank 3
- rank 4

		56	57	58	59	60	61	62	63
		48	49	50	51	52	53	54	55
		40	41	42	43	44	45	46	47
45		46 32	47 33	48 34	49 35	36	37	38	39
36		37 24	38 25	39 26	40 27	28	29	30	31
27		28 16	29 17	30 18	31 19	20	21	22	23
18		19 8	20 9	21 10	22 11	23 12	13	14	15
9		10 0	11 1	12 2	13 3	14 4	5	6	7
	0	1	2	3	4	5			

Rank 0 vertex closure partitioning

- exclusive
- shared
- ghost

	56	57	58	59	60	61	62	63
	48	49	50	51	52	53	54	55
	40	41	42	43	44	45	46	47
45	46 32	47 33	48 34	49 35	36	37	38	39
36	37 24	38 25	39 26	40 27	28	29	30	31
27	28 16	29 17	30 18	31 19	20	21	22	23
18	19 8	20 9	21 10	22 11	23 12	13	14	15
9	10 0	11 1	12 2	13 3	14 4	5	6	7
0	1	2	3	4	5			

Vertex Partition Information

exclusive vertices	shared vertices	ghost vertices
0 0 0 []	2 0 2 [1]	5 1 0 []
1 0 1 []	3 0 3 [1]	14 1 4 []
9 0 5 []	4 0 4 [1]	22 1 8 []
10 0 6 []	11 0 7 [1 2]	23 1 9 []
18 0 10 []	12 0 8 [1 2]	31 1 13 []
19 0 11 []	13 0 9 [1]	40 2 0 []
	20 0 12 [1 2]	45 3 0 []
	21 0 13 [1 2]	46 3 1 []
	27 0 14 [3]	47 3 2 []
	28 0 15 [3]	48 2 4 []
	29 0 16 [1 2 3]	49 2 5 []
	30 0 17 [1 2 3]	
	36 0 18 [3]	
	37 0 19 [3]	
	38 0 20 [2 3]	
	39 0 21 [2 3]	

Simple Partition Examples

45	46	47	48	49			
	32	33	34	35			
36	37	38	39	40			
	24	25	26	27			
27	28	29	30	31			
	16	17	18	19			
18	19	20	21	22	23		
	8	9	10	11	12		
9	10	11	12	13	14		
0	1	2	3	4	5		

			39	40	41	42	43	44
			27	28	29	30	31	
		29	30	31	32	33	34	35
		18	19	20	21	22	23	
	20	21	22	23	24	25	26	
	10	11	12	13	14	15	16	
	11	12	13	14	15	16	17	
	2	3	4	5	6	7	8	

	74	75	76	77	78		
	65	58	59	60	61		
		50	51	52	53		
	56	42	43	44	45	46	47
	47	34	35	36	37	38	39
	38	26	27	28	29	30	31
	29	30	31	32	33	34	35
	18	19	20	21	22	23	
	20	21	22	23	24	25	26
	10	11	12				
	11	12	13	14			

72	73	74	75	76			
	56	57	58	59			
63	64	65	66	67			
	48	49	50	51			
54	55	56	57	58			
	40	41	42	43			
45	46	47	48	49			
	32	33	34	35			
36	37	38	39	40			
	24	25	26	27			
27	28	29	30	31			

	74	75	76	77	78	79	80
	65	58	59	60	61	62	63
		50	51	52	53	54	55
	56	57	58	59	60	61	62
			49	50	51	52	53
			36	37	38	39	
			40	41	42	43	44
					33	34	35
					22	23	
					24	25	26

72	73	74	75	76	77			
	56	57	58	59	60			
63	64	65	66	67	68			
	48	49	50	51	52			
54	55	56	57	58	59			
	40	41	42	43	44			
45	46	47	48	49	50			
	32	33	34	35	36			
36	37	38	39	40	41			
	24	25	26	27	28			
27	28	29	30	31	32			

		75	76	77	78	79	80
		59	60	61	62	63	
		66	67	68	69	70	71
		51	52	53	54	55	
		57	58	59	60	61	62
		43	44	45	46	47	
		48	49	50	51	52	53
		35	36	37	38	39	
		39	40	41	42	43	44
		27	28	29	30	31	
		30	31	32	33	34	35

45	46	47	48	49	50		
	32	33	34	35	36		
36	37	38	39	40	41		
	24	25	26	27	28		
27	28	29	30	31	32		
	16	17	18	19	20		
18	19	20	21	22	23		
	8	9	10	11	12		
9	10	11	12	13	14		
	0	1	2	3	4		
0	1	2	3	4	5		

		48	49	50	51	52	53
		35	36	37	38	39	
		39	40	41	42	43	44
		27	28	29	30	31	
		30	31	32	33	34	35
		19	20	21	22	23	
		21	22	23	24	25	26
		11	12	13	14	15	
		12	13	14	15	16	17
		3	4	5	6	7	8
		3	4	5	6	7	8