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Title: Topology and Geometry Effects in Electronic Systems

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TOPOLOGY AND GEOMETRY EFFECTS IN ELECTRONIC SYSTEMS

Avadh Saxena (Los Alamos National Lab)

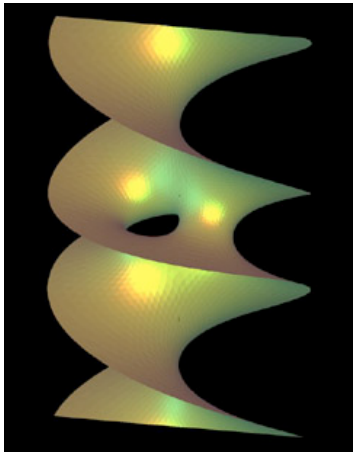
1. Importance of Geometry and Topology in materials.
2. Helicoidal Graphene Nanoribbons and effects.
3. Geometry induced Hall effect of chiral electrons.
4. Topological defects in chiral magnets: Skyrmions.
5. Model: Exchange with Spin-Orbit (or DM) interaction.
6. Novel applications in low-power Spintronics.

Collaborators: V.A. Atanasov (Univ. Sofia), S.-Z. Lin (LANL)

CHIRALTRONICS: HELICOIDAL GRAPHENE NANORIBBONS

PART - 1

Victor Atanasov (Univ. Sofia), **Avadh Saxena (Los Alamos)**



1. Experimental observation of helicoidal GNRs.

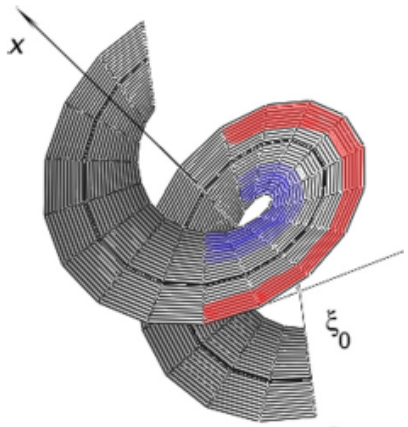
2. Twist acts as an effective transverse electric field.

3. Separation of two species of chiral electrons.

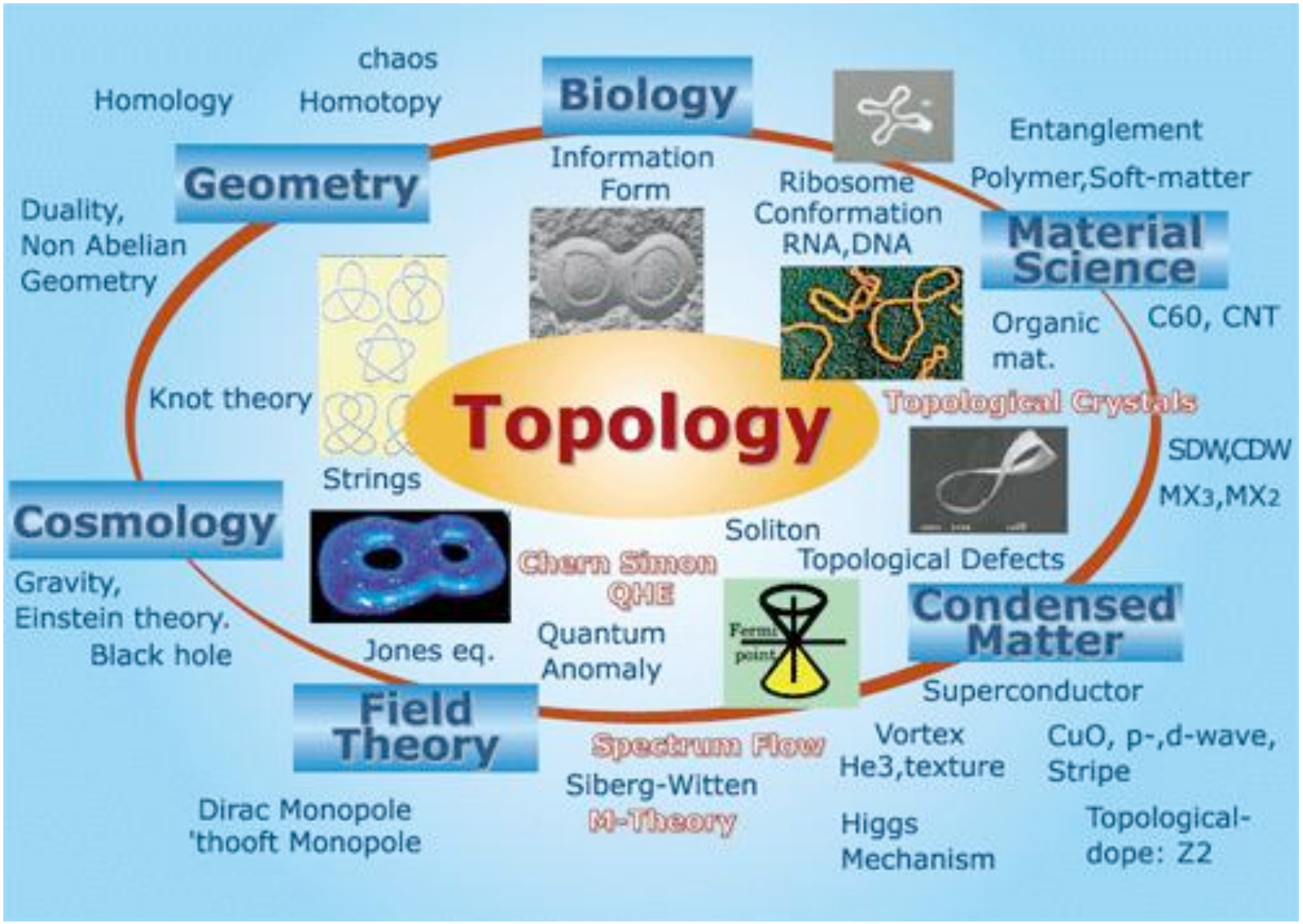
4. Dirac equation for confined quantum particles.

5. Two iso-spin states move to the two rims.

6. Conclusions: geometry-induced Hall effect.



Interplay of real & momentum space topology (Satoshi Tanda)



Helicoid: Minimal Surface

Plane and catenoid are the other two.

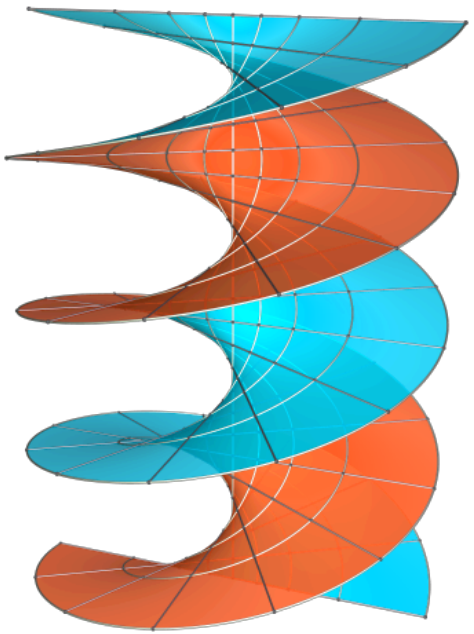
Discovered by Jean Baptiste Meusnier: 1776

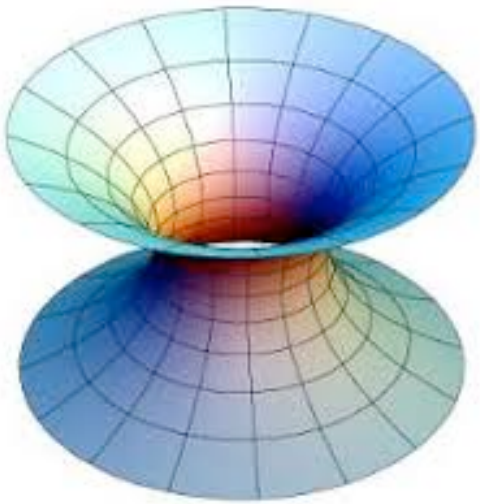
Principal curvatures: $$H_{1,2} = \pm \frac{1}{1+r^2}$$

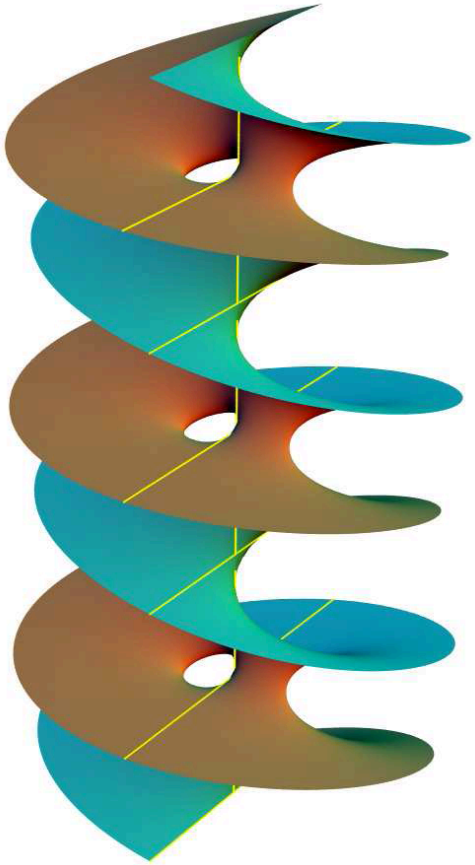
Mean Curvature: $$M = H_1 + H_2 = 0$$

Gaussian Curvature: $$K = H_1 H_2 = -\frac{1}{(1+r^2)^2}$$

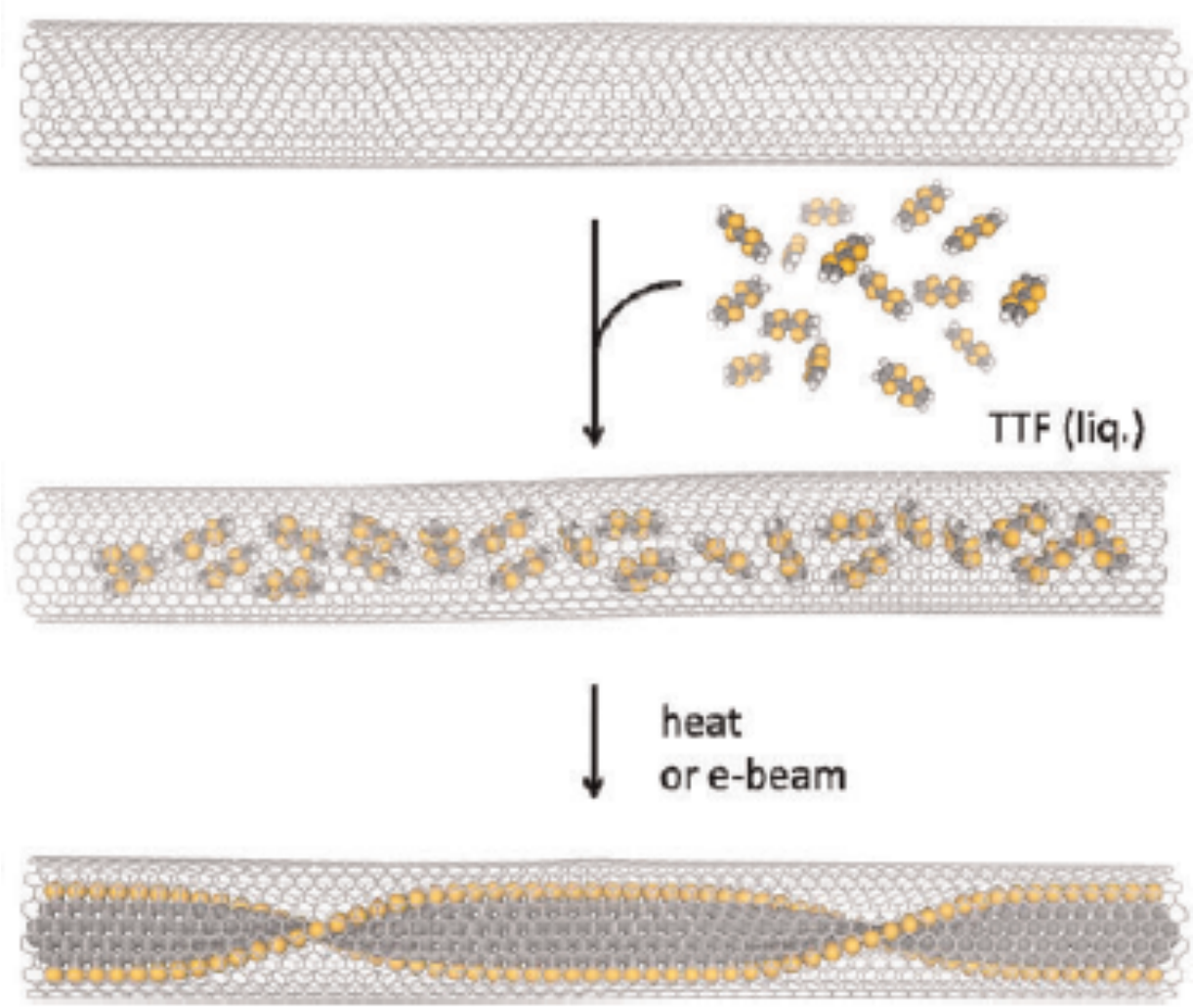
Minimal surfaces: Helicoid, Catenoid
Hoffman's surface





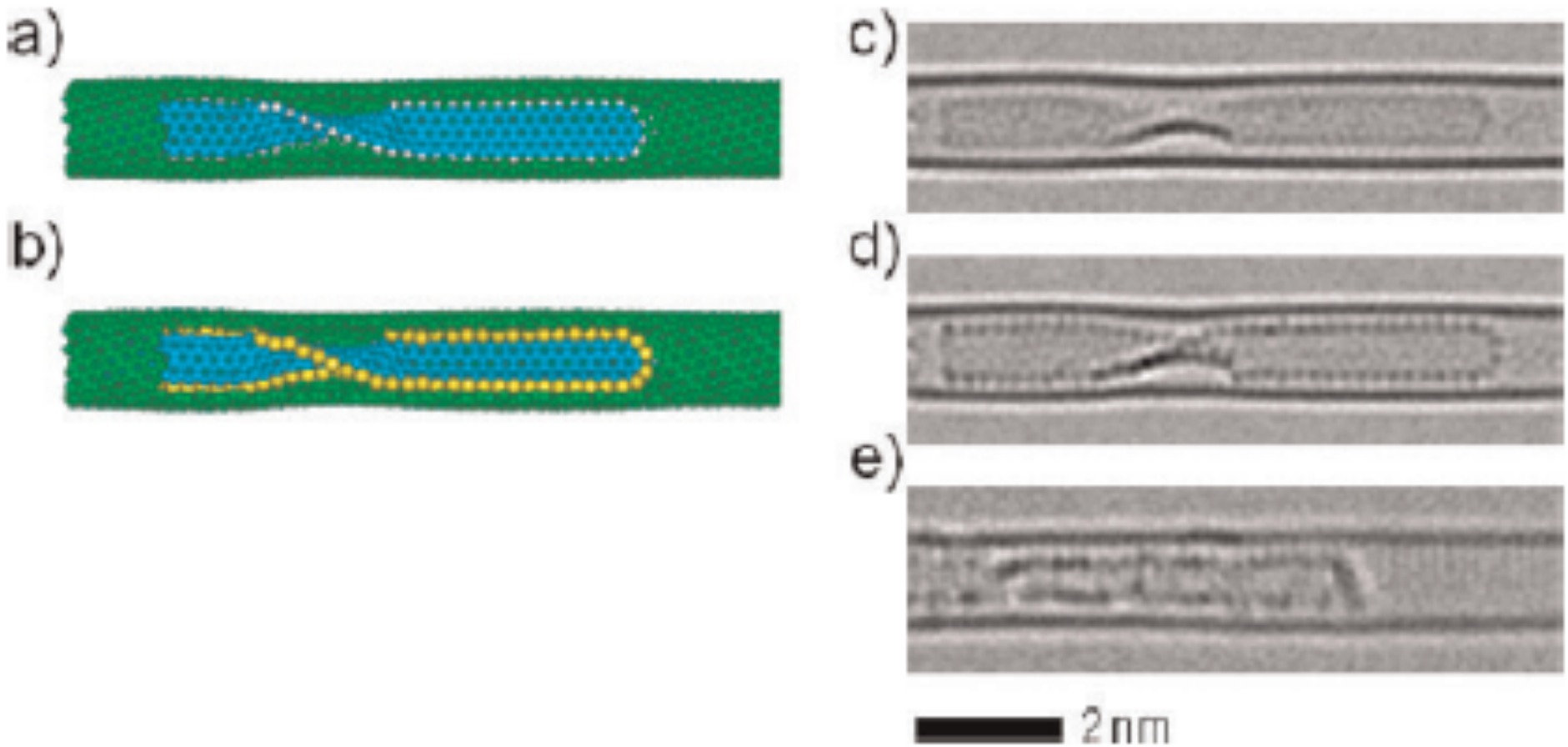


Templating of GNRs in SWNT

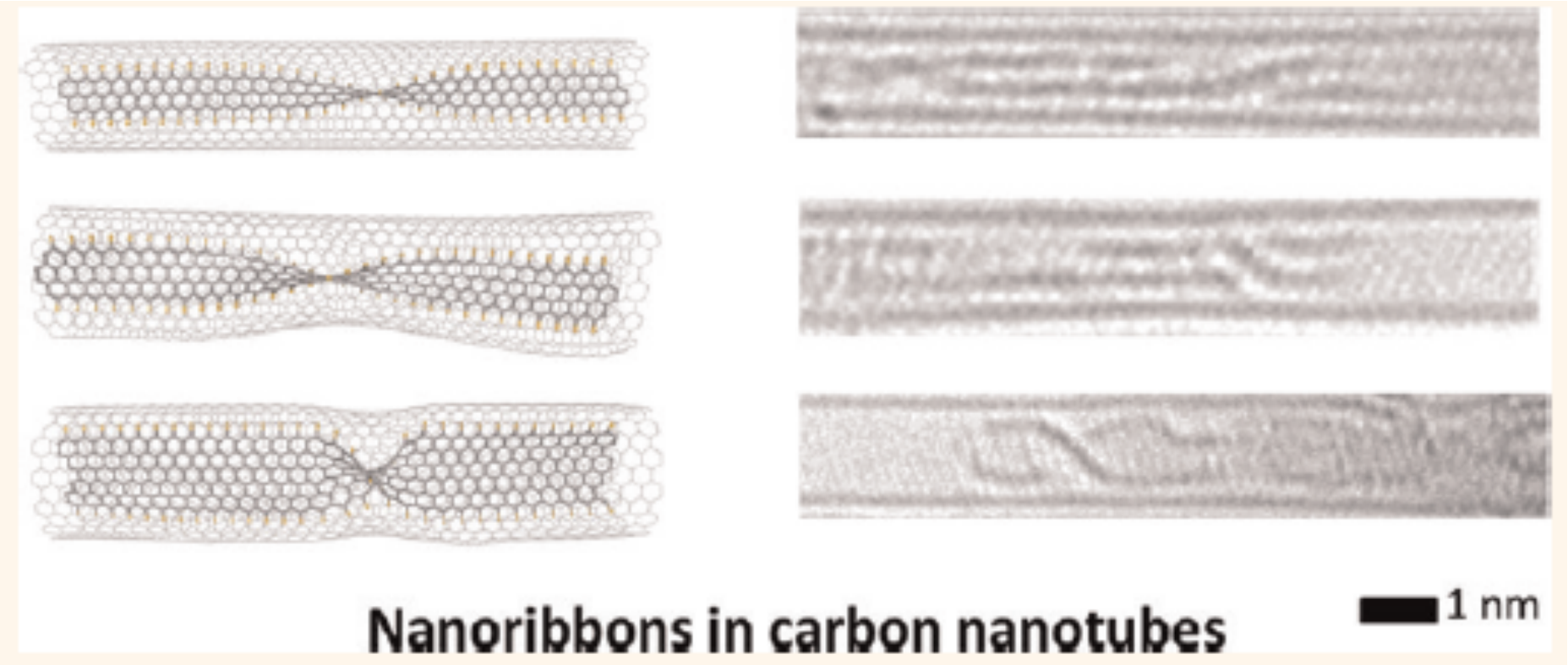


T.W. Chamberlain et al., ACS Nano 6, 3943 (2012)

Calculated & Observed: AC-HRTEM

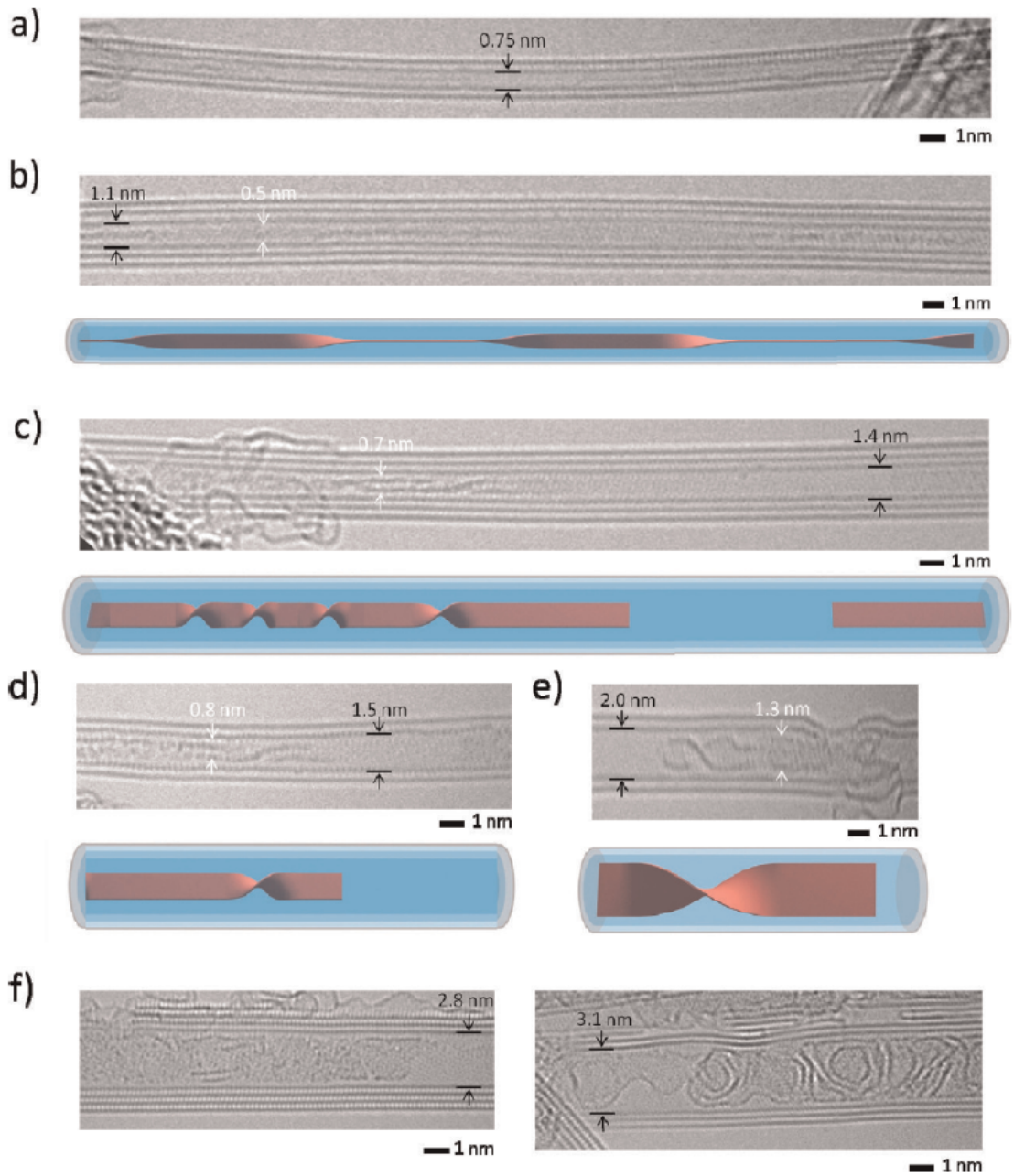


Twisting of GNRs inside nanotubes

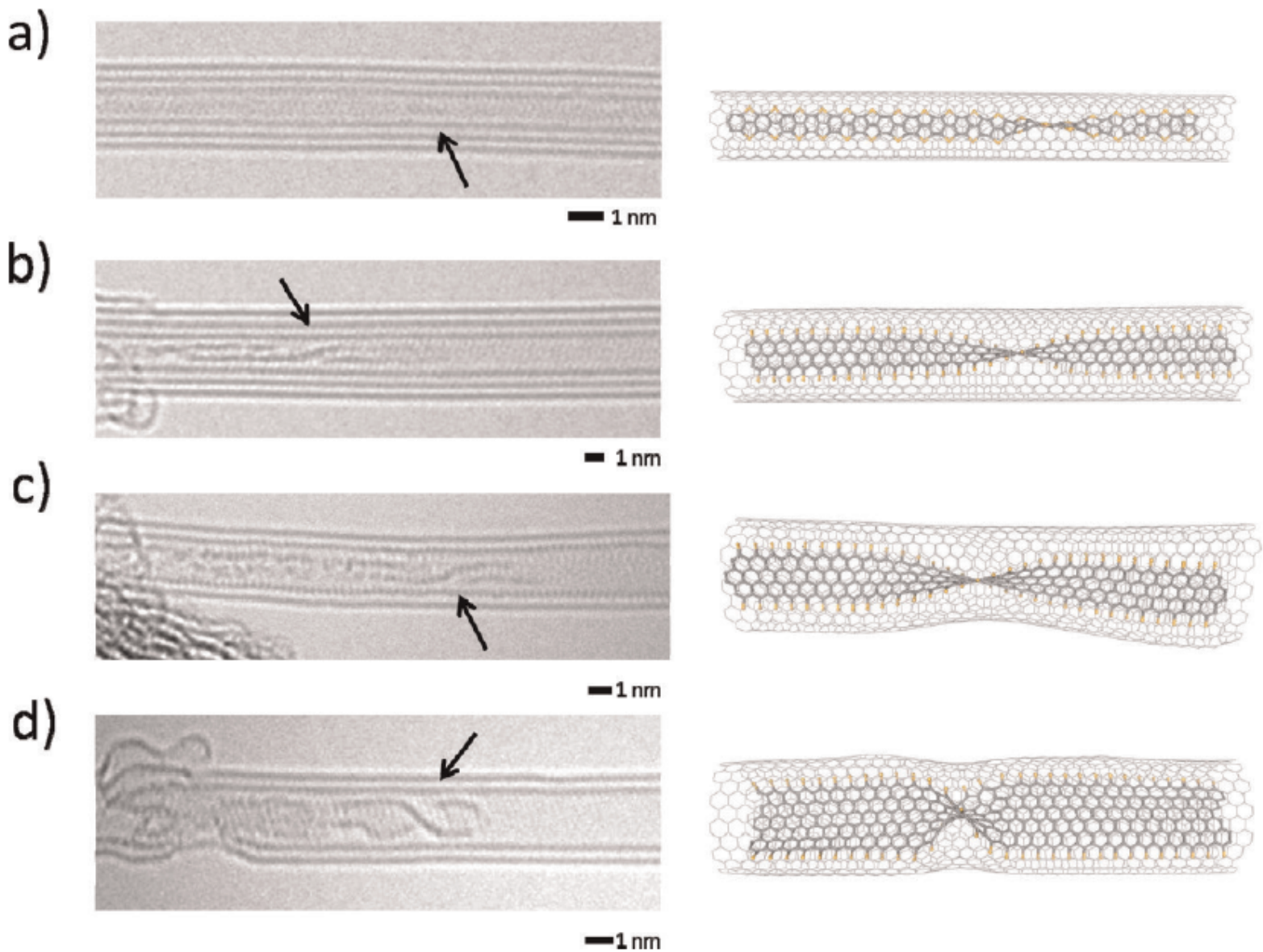


T.W. Chamberlain et al., ACS Nano 6, 3943 (2012)

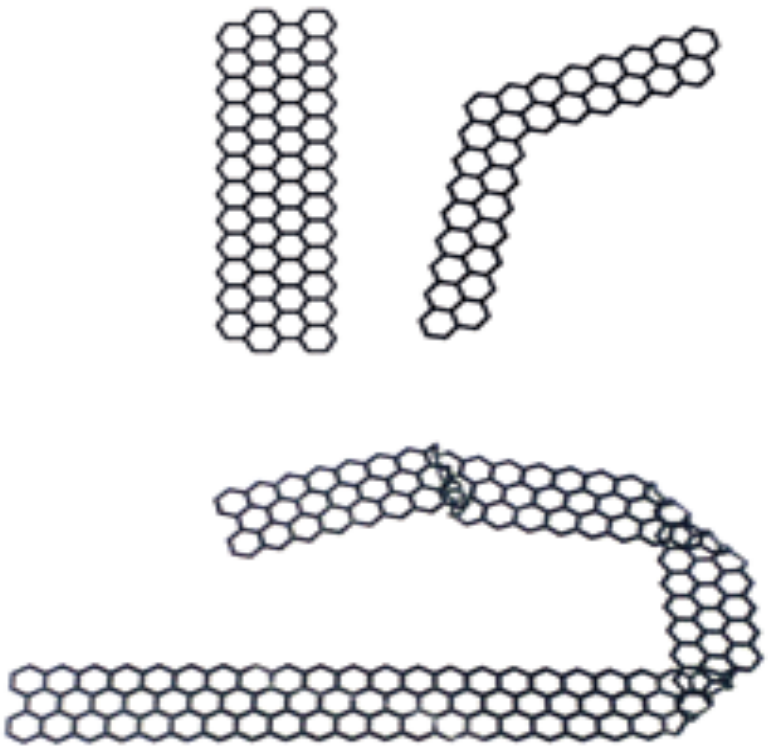
Nanotube diameter vs. GNR width/twists

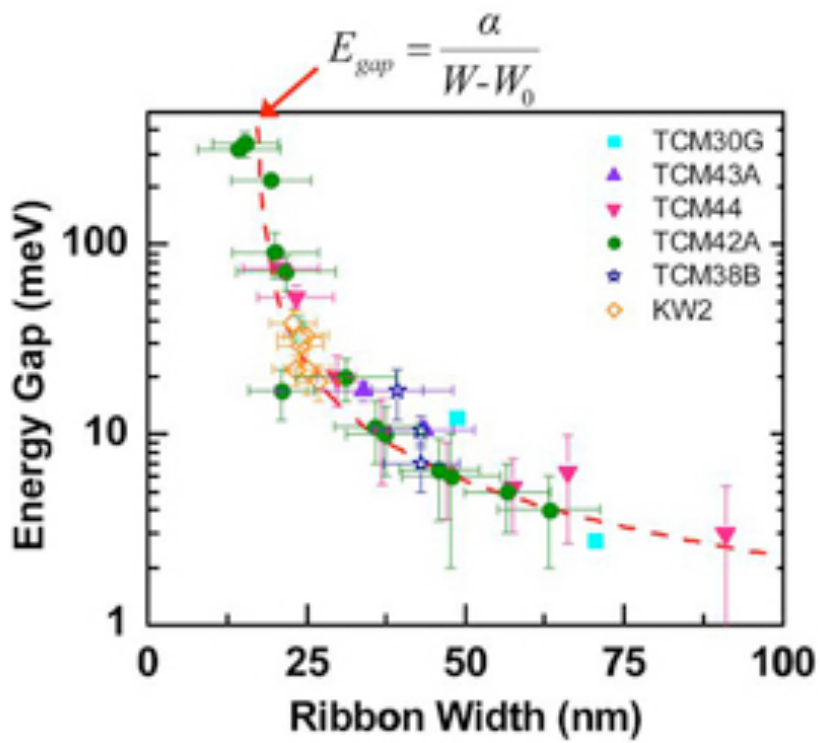


Helical pitch vs. GNR width (MWNT)



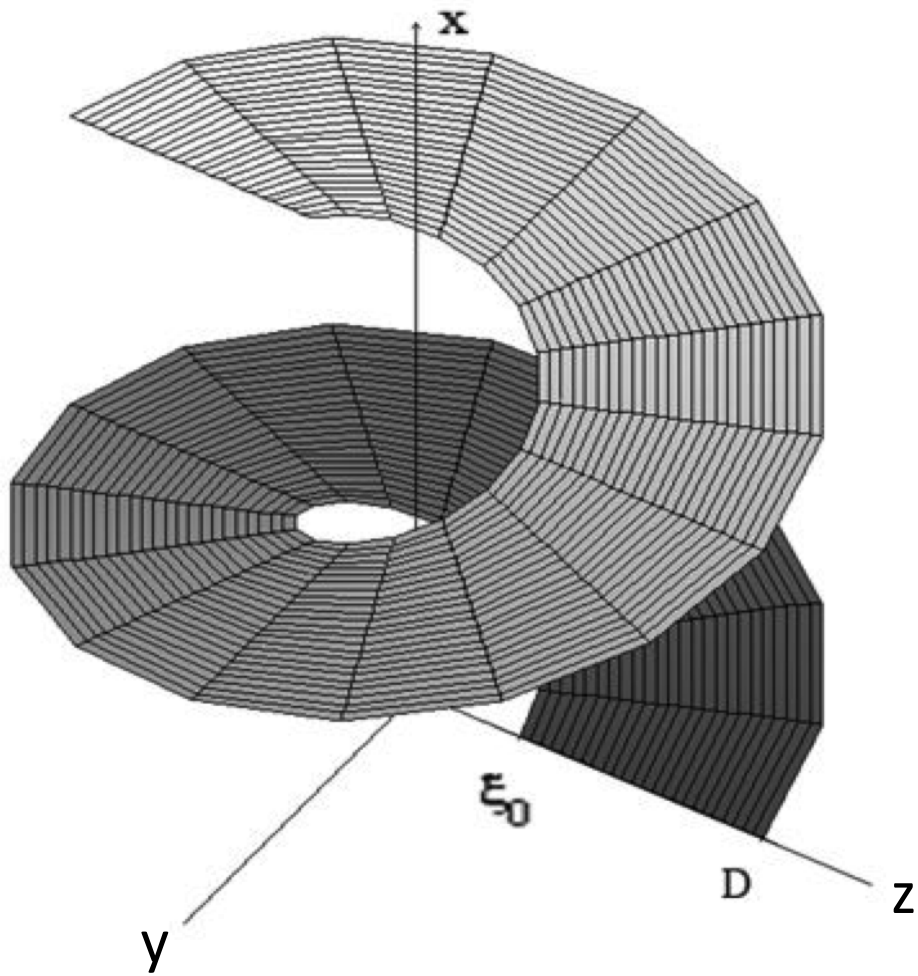
Graphene ribbons





M.Y. Han et al., PRL 98, 206801 (2007)

Helicoidal Ribbon

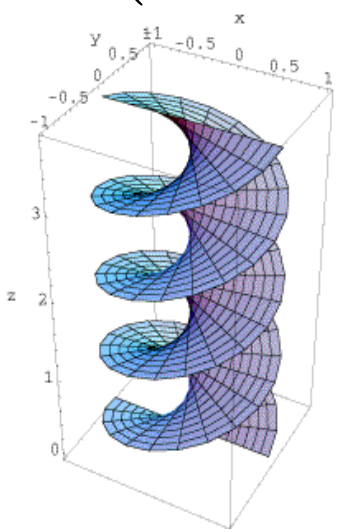


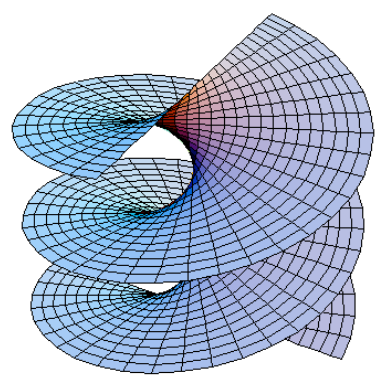
$$\vec{r} = x\,\vec{e}_x + \xi\,[\cos(\omega x)\,\vec{e}_y + \sin(\omega x)\,\vec{e}_z]$$

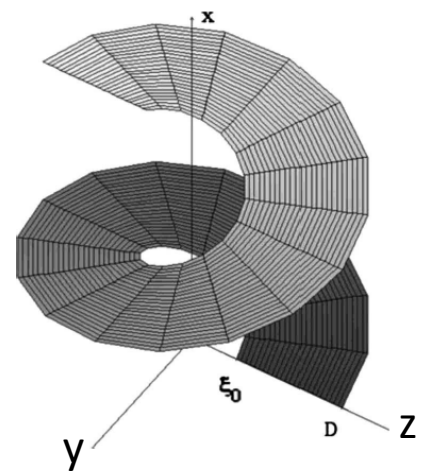
Helicoidal geometry: Dirac Equation

$$\vec{r} = x\,\vec{e}_x + \xi\,[\cos(\omega x)\,\vec{e}_y + \sin(\omega x)\,\vec{e}_z]$$

$$|d\vec{r}|^2 = (1+\omega^2\xi^2)dx^2 + d\xi^2 = h_1^2 dx^2 + h_2^2 d\xi^2$$







$$\begin{pmatrix} -k_+ & \frac{ik_x}{\sqrt{1+\omega^2\xi^2}} - i\partial_\xi \\ \frac{ik_x}{\sqrt{1+\omega^2\xi^2}} + i\partial_\xi & -k_- \end{pmatrix} \begin{pmatrix} \chi_A \\ \chi_B \end{pmatrix} = 0$$

$$k_\pm = \pm E/\hbar v_F$$

Effective Schrodinger Equations

Angular momentum:

$$L_x = -\frac{i\hbar}{\omega}\frac{\partial}{\partial x}\ , \qquad L_x\phi(x) = \hbar \mathrm{m}\phi(x)$$

$$k_x = \mathrm{m}\omega, \qquad \mathrm{m} \in \mathbb{Z}$$

Two spinor components, isospin states:

$$\partial_\xi^2 \chi_A + U_A(x)\chi_A = -k_\xi^2 \chi_A$$

$$-\partial_\xi^2 \chi_B + U_B(x)\chi_B = -k_\xi^2 \chi_B$$

$$k_\xi^2 = k_+ k_- = -E^2/(\hbar v_F)^2$$

Effective potential: chiral electrons

$$U_A = W_{\mathrm{m}}^2 - W'_{\mathrm{m}} = \frac{k_x^2}{1+\omega^2\xi^2} + \frac{k_x\omega^2}{(1+\omega^2\xi^2)^{3/2}}\,\xi$$

$$U_B = W_{\mathrm{m}}^2 + W'_{\mathrm{m}} = \frac{k_x^2}{1+\omega^2\xi^2} - \frac{k_x\omega^2}{(1+\omega^2\xi^2)^{3/2}}\,\xi$$

$$W_{\mathrm{m}} = k_x/\sqrt{1+\omega^2\xi^2}$$

Maxima:

$$\xi_{extr} = \frac{1}{|\omega|}\frac{\sqrt{1+|\mathrm{m}|^2-\sqrt{|\mathrm{m}|^4-3|\mathrm{m}|^2}}}{\sqrt{2(1-|\mathrm{m}|^2)}}$$

$$|\mathrm{m}| = 1, \quad \xi_{extr} = 1/(\omega\sqrt{8})$$

Narrow nanoribbons

$$W < 1/(\omega\sqrt{8}) = L/(4\sqrt{2}\pi \mathrm{n})$$

$$U_A \approx k_x^2 + k_x\omega^2\xi, \quad U_B \approx k_x^2 - k_x\omega^2\xi$$

Effective Schrodinger equations:

$$-\partial_\xi^2\chi_A + \left(k^2 + k_x\omega^2\xi\right)\chi_A = 0$$

$$-\partial_\xi^2\chi_B + \left(k^2 - k_x\omega^2\xi\right)\chi_B = 0$$

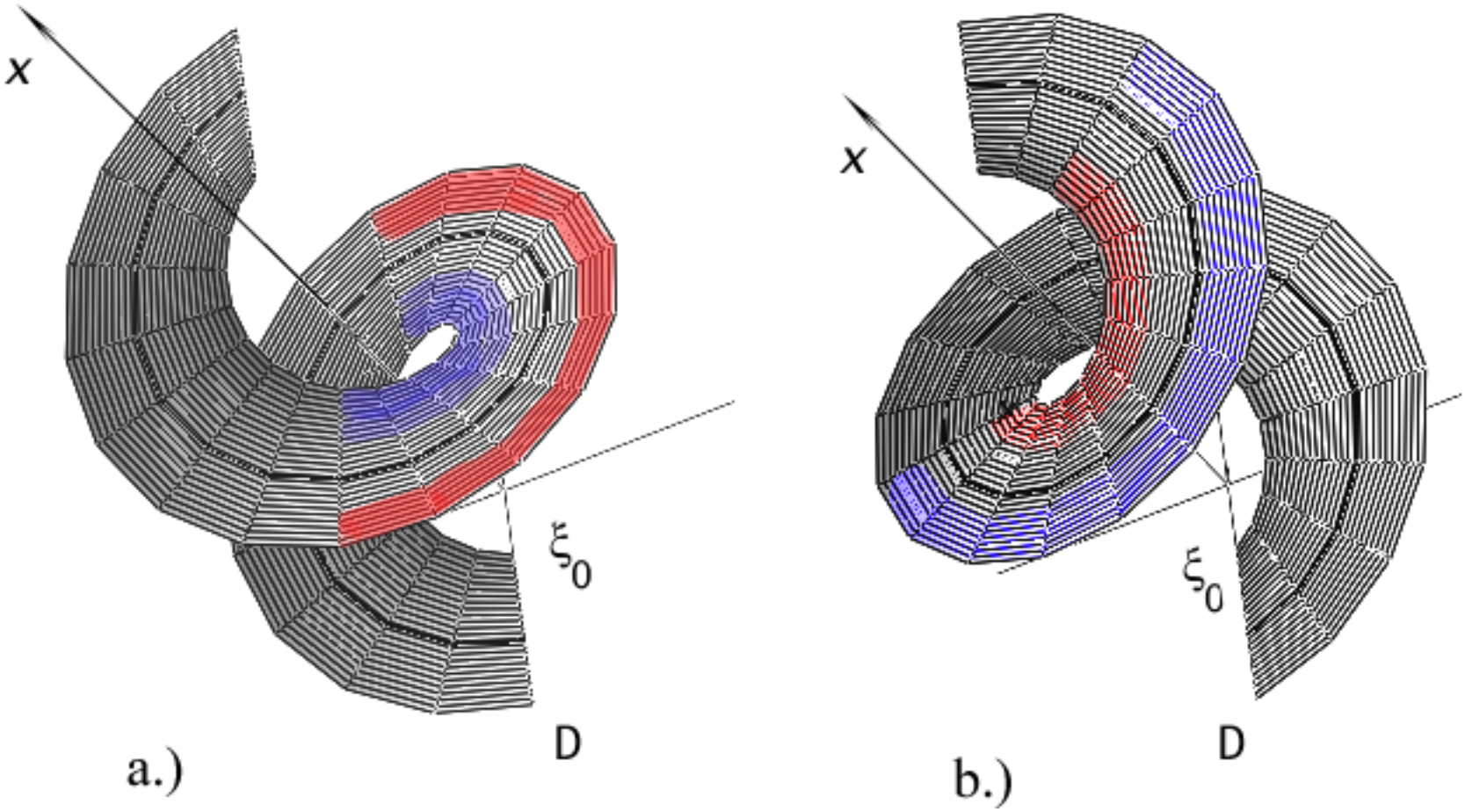
Effective electric field:

$$k_x^2 + k_\xi^2 = k^2$$

$$U_A \propto e\mathcal{E}\xi, \quad U_B \propto -e\mathcal{E}\xi$$

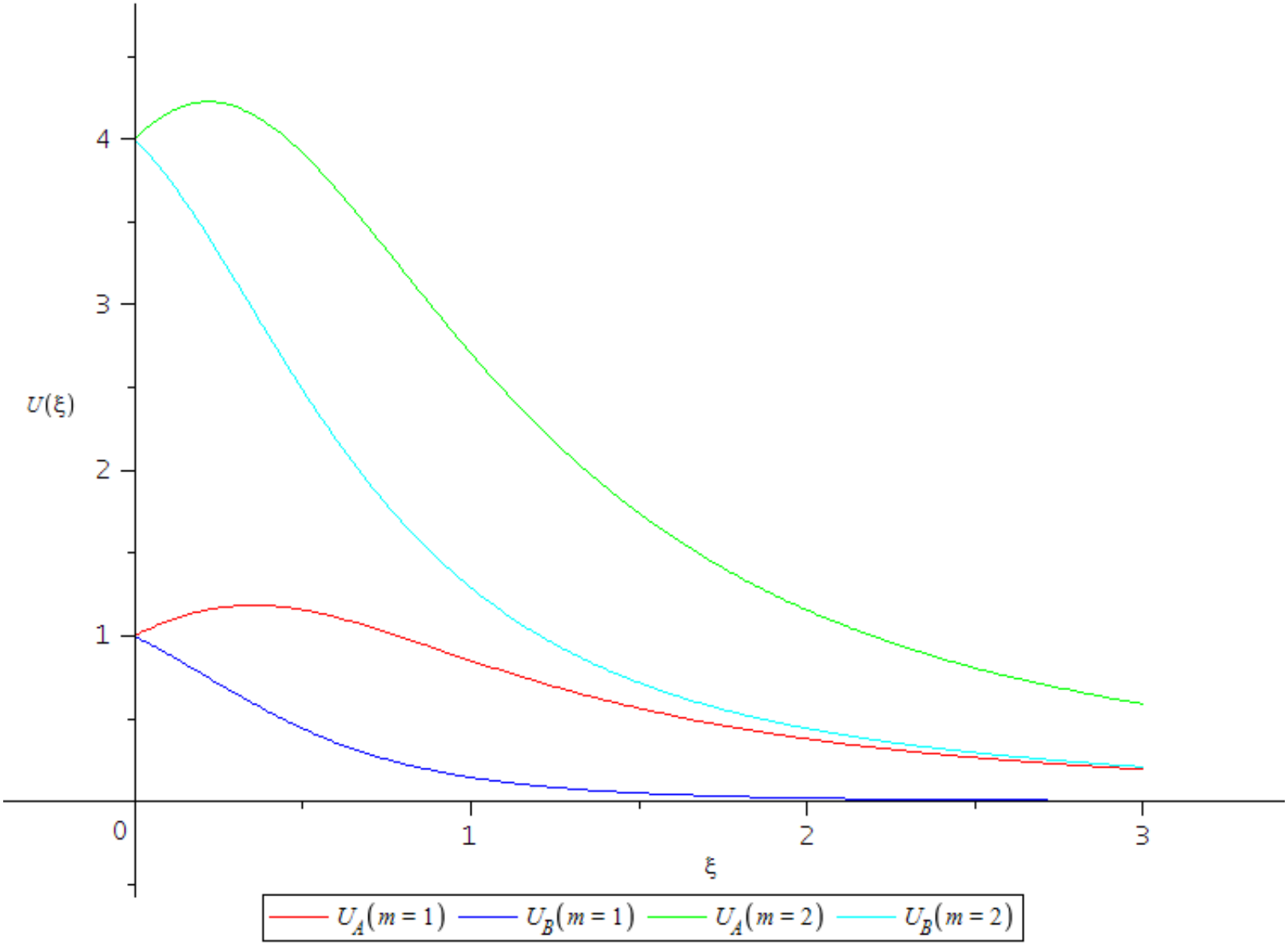
$$\nu \approx v_F\sqrt{|\mathrm{m}||\mathrm{n}|^3 2\pi W/L^3} \qquad \nu \approx |\mathrm{n}|\frac{v_F}{L}\sqrt{\frac{|\mathrm{m}|}{2\sqrt{2}}}$$

Helicoidal Nanoribbons: Two Chiralities

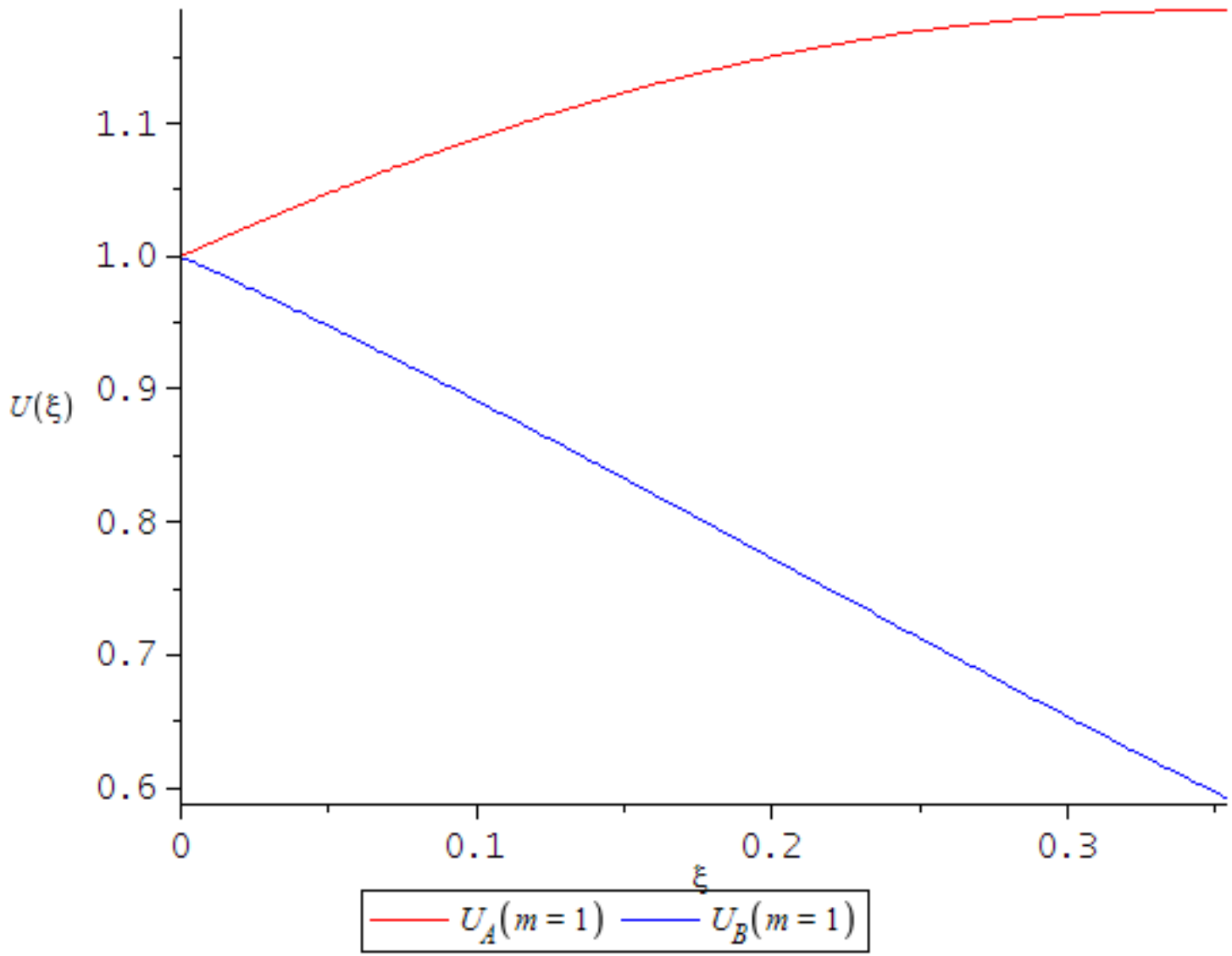


V.A. Atanasov and A. Saxena, Phys. Rev. B 92, 035440 (2015)

Potential on each isospin state vs. width



Potential acting on two iso-spin states



Conclusions

- Chiraltronics: twist separates chiral electrons.
- Relativistic electrons on a confined geometry.
- Geometry induced effective electric field.
- Reminiscent of quantum Hall effect.
- Isospin transitions: THz radiation.
- Possible experimental verification on GNRs.

PART-2

Skyrmions, Merons and Monopoles: Topological Excitations in Chiral Magnets

Avadh Saxena

Los Alamos National Lab

Collaborators: Shizeng Lin, Cristian D. Batista

BES

LA-UR: 15-23587

OUTLINE

1. What is a Skyrmion? Historical perspective.

2. Experimental observation: Chiral magnets.

3. Minimal model: Exchange with DM interaction.

4. Half-skyrmions (merons) and monopoles.

5. Anapoles (toroidal moment).

6. Comparion with vortices and domain walls.

7. Conclusions: Spintronics & other applications.

THEORETICAL DIVISION
MAY 10, 1945

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V.F. Weisskopf, Deputy Division Leader, E-217, Ext 74
J. von Neumann, Consultant, E-205 , Ext 72 R3

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R.F. Christy, Section Leader E-120, Ext 178 R2
K. Fuchs, Section Leader, E-118, Ext 77
Baroody, E.M., E-121, Ext 178 R2
Calkin, J.W., E-117, Ext 77
Inglis, D.R., T-35, Ext 54
Keller, J., E-120, Ext 178 R2
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Wing, Milton, E-222, Ext 468
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Huber, T/4 .C., E-120, Ext 70
Hudson, H., E-214, Ext 73 R1
Inglis, B., E-212, Ext 73 R2
Johnson, M., E-212, Ext 73 R2
Kellogg, T/5 H., E-203, Ext 69 R3
Langer, B., E-201, Ext 79
Page, T/3 W., E-214, Ext 73 R1
Rau, E., T/3, E-120, Ext 70
Staley, T/3 J., E-212, Ext 73 R2
Teller, M., E-214, Ext 73 R1
Vuletic, T/5 V., E-211. Ext 73 R2
Wilson, F., E-212, Ext 73 R2
Wright, T/5 E., E-213, Ext 73 R1
Young, T/Sgt., G., E-211, Ext 73 R2

GROUP T-6
E. Nelson, Leader, E-116
N. Metropolis, Alt. Leader, E-115, Ext 78
R. Feynman, Consultant, E-206, Ext 72
Ewing, F.E., E-112, Ext 75
Goldberg, T/5., E-108, Ext 76
Kemeny, Pvt. J., E-105, Ext 75
Hamming, R.W., E-114, Ext 78
Heermans, Corp. A., E-105, Ext 75
Heller, T/5 A., E-105, Ext 75
Hurwitz, T/5 D., E-105, Ext 75
Johnston, T/3 J., E-103, Ext 75
Kington, T/4 J., E-105, Ext 75
Livesay, N., E-112, Ext 75
Ninger, H., E-105, Ext 75
Noah, F.E., E-105, Ext 75
Vorwald, T/5 A., E-105, Ext 75
Zimmerman, T/3 W., E-105, Ext 75

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J. Hirschfelder, Leader, T-30, Ext 206
J. Magee, Alt. Leader, T-30, Ext 206
Brummer, T/5 E., T-28, Ext 206
Feckete, T/4 P., T-26, Ext 206
Larson, T/4 L., T-28, Ext 206
Ostrow, E., T-30, Ext 206
Schwartz, T/4 P., T-28, Ext 206

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G. Placzek, Leader, E-220, Ext 468
Mark, C., E-221, Ext 468 R2
Carlson, B., E-221, Ext 468 R2
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Day

Computing vs. phenomena

ELECTRONICS

PHOTONICS

SPINTRONICS

VALLEYTRONICS

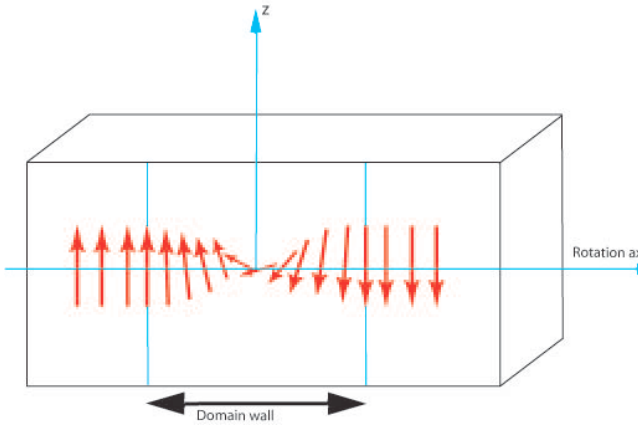
MOTT-TRONICS

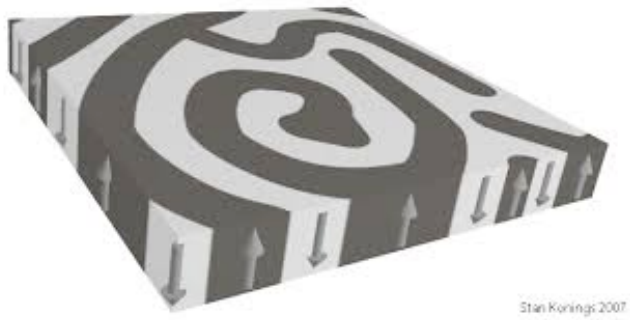
MOLETRONICS

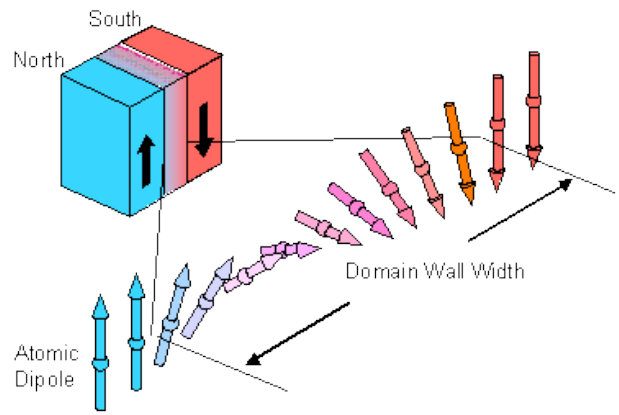
SKYRMIONICS

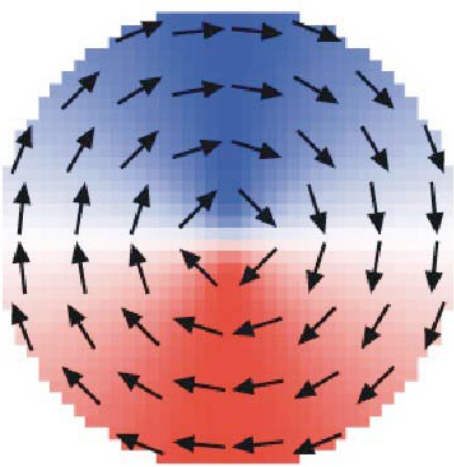
… … …

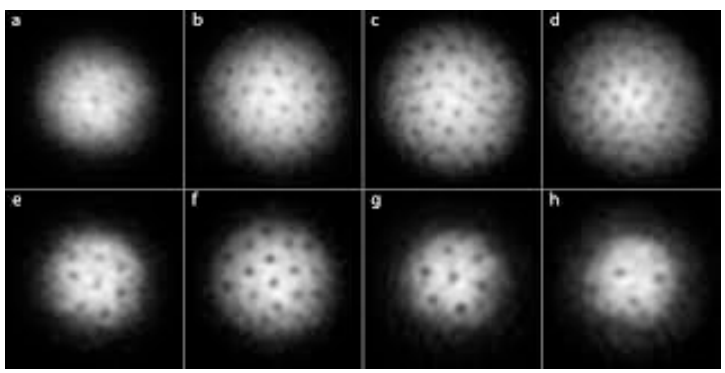
Magnetic domain walls and vortices

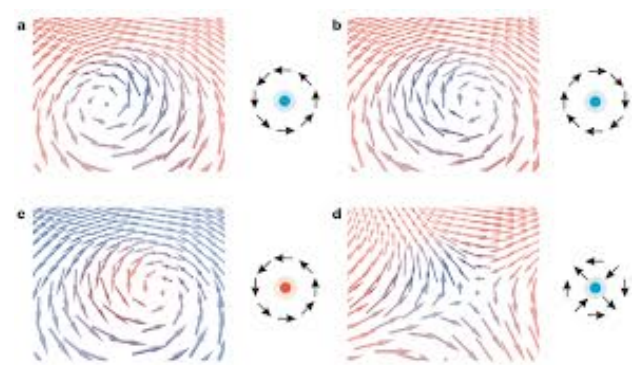




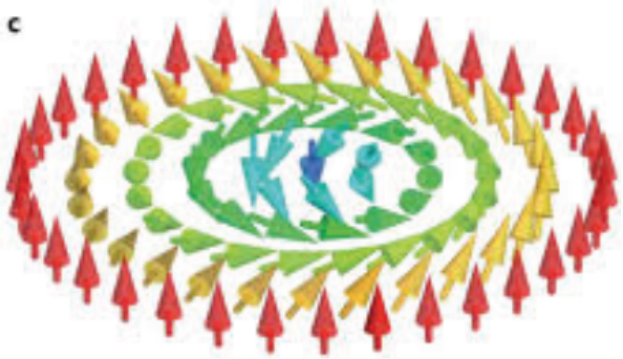








Spin texture:



- Topology:
  - Basic concepts
  - Experiments
- Dynamic phase transition in metallic chiral magnets with skyrmions under current drive
- Equation of motion for rigid skyrmions and its applications; internal modes.
- Skyrmions driven by temperature gradient and magnon current in insulators
- Applications

Dec 5, 1922 – June 25, 1987

Tony Hilton Royle Skyrme, (1922–1987) was a British physicist. He first proposed modeling the effective interaction between nucleons in nuclei by a zero-range potential, an idea still widely used today in nuclear structure and in equation of state for neutron stars. However, he is best known for formulating the first topological soliton to model a particle, the Skyrmion. … Skyrme was awarded the Hughes Medal by the Royal Society in 1985.

from Wikipedia

- **Low energy effective field for QCD**:

$$L = \int \left\{ \frac{F_\pi^2}{16} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32e^2} \mathrm{Tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger][\partial^\mu U U^\dagger, \partial^\nu U U^\dagger]) \right\} d^3x \,,$$

$U(t,r)$: SU(2) field

T. H. R. Skyrme, Proc. R. Soc. London, Ser. A 260, 127 (1961).

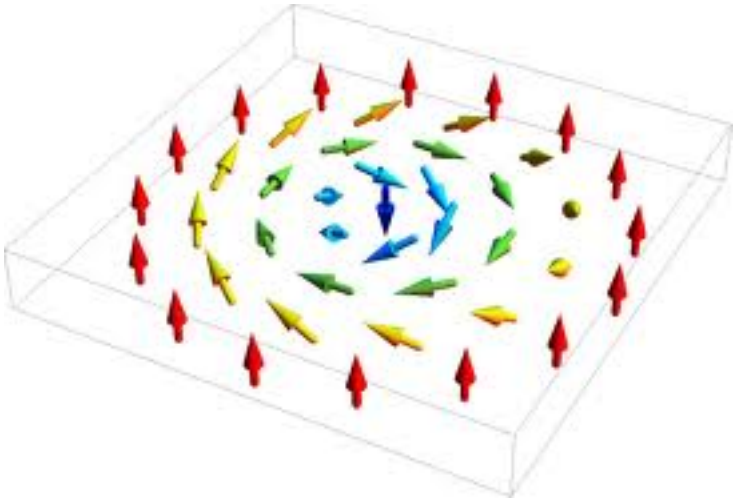
- Supports topological solution, now known as skyrmion.
- Skyrmions are realized in condensed matter systems, such as quantum Hall systems, Bose-Einstein condensates, multiband superconductors, … .

Skyrmion: A topological twist (or kink) in spin space.

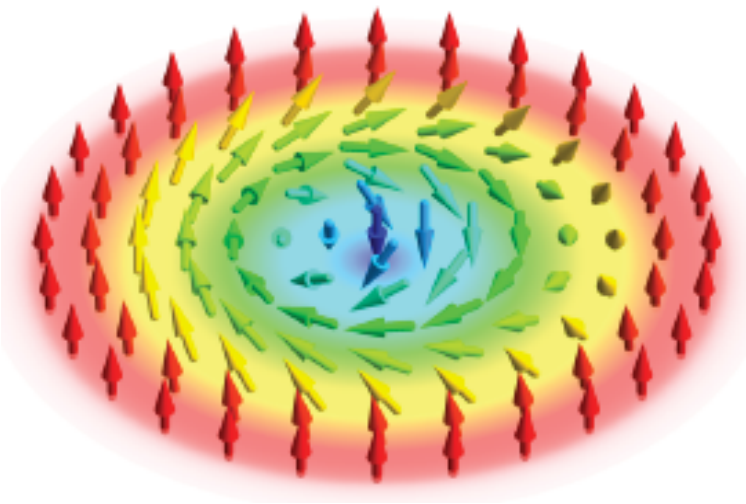
Nonlinear field theory: A soliton with spin and statistics different from those of the underlying fields.

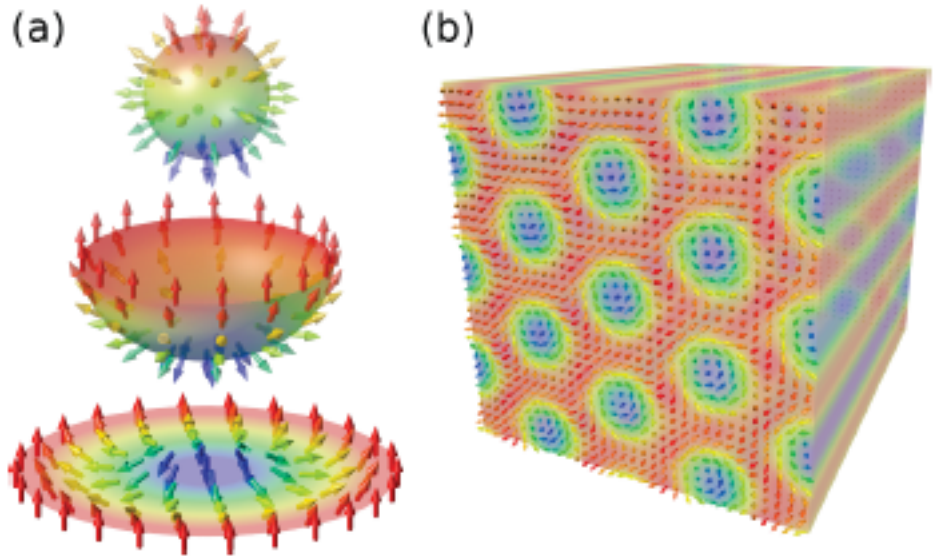
Skyrme (1958): Representation of a nucleon (fermion) as a topological soliton of bosonic pion fields.

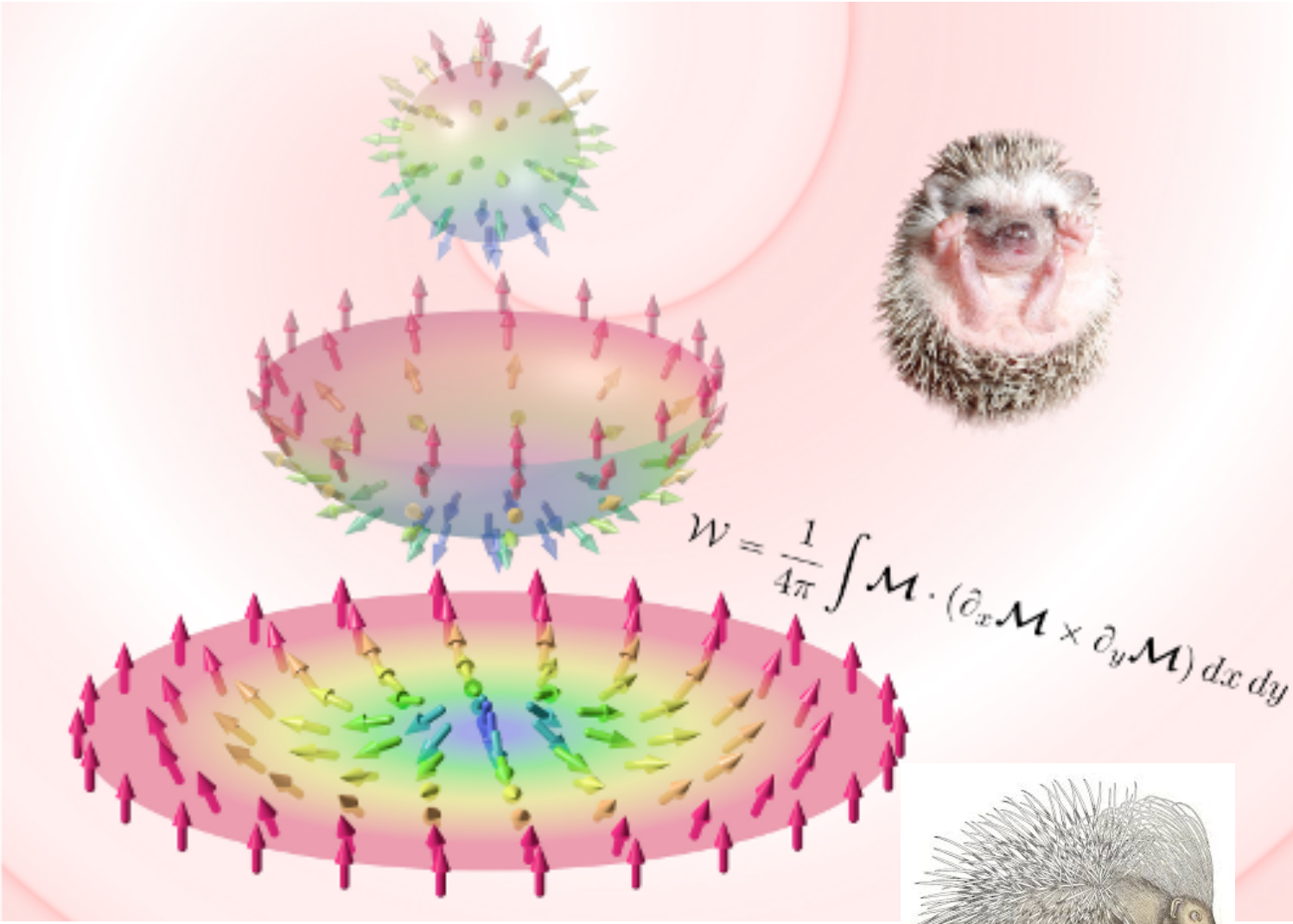
Skyrmions have been observed in 2DEG (quantum Hall systems), chiral magnets, multiferroics, BECs, insulators, superconductors, Chiral nematic liquid crystals, etc.



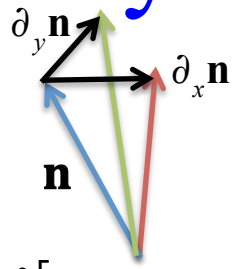
A SKYRMION “COVERS” THE SPHERE ONCE



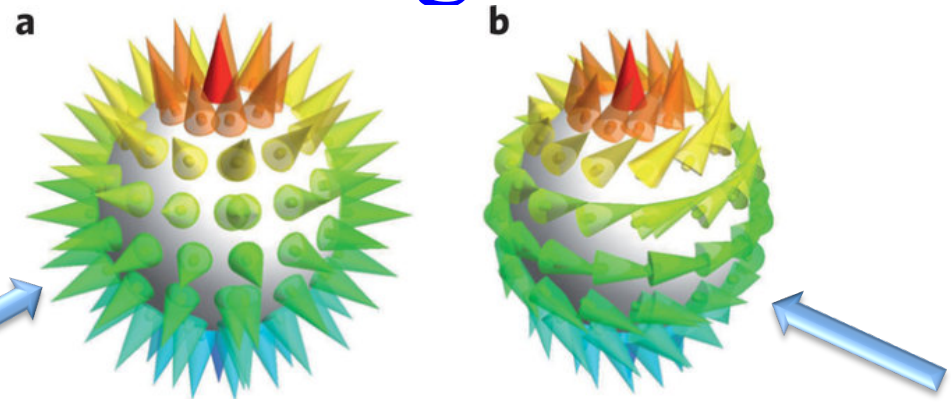


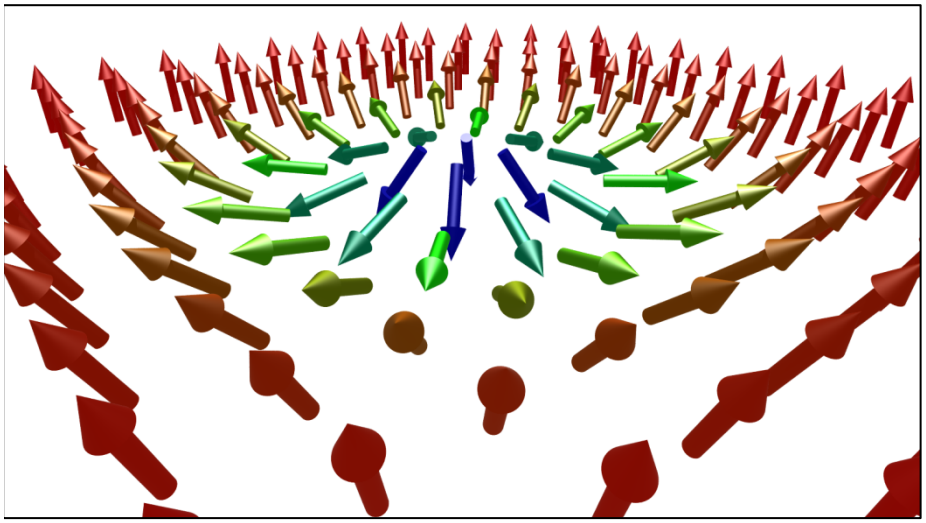

$\mathcal{W} = \frac{1}{4\pi} \int \boldsymbol{\mathcal{M}} \cdot (\partial_x \boldsymbol{\mathcal{M}} \times \partial_y \boldsymbol{\mathcal{M}})\, dx\, dy$

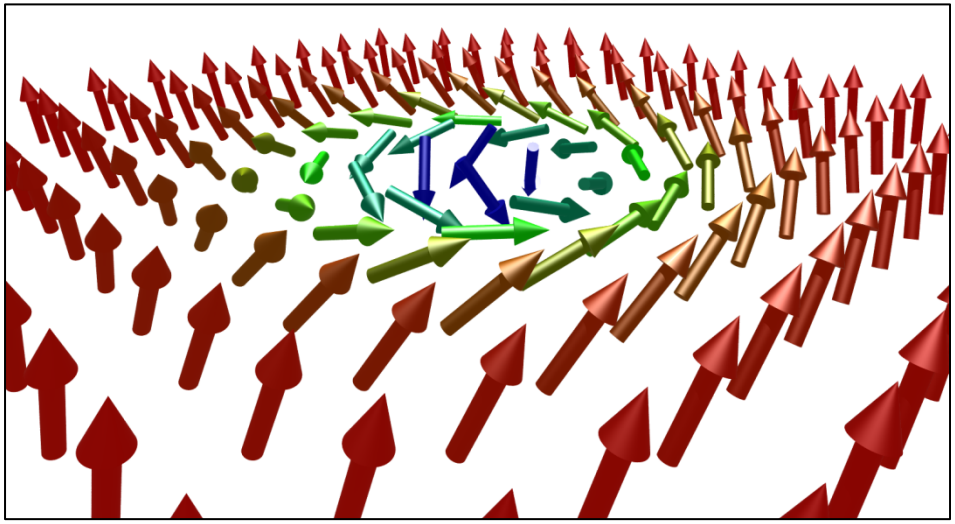
Skyrmions in magnets



$$Q = \frac{1}{4\pi}\int dr^2 \left[\mathbf{n}\cdot(\partial_x \mathbf{n}\times\partial_y \mathbf{n})\right] = \pm 1$$



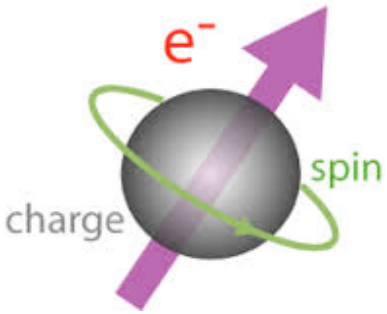




Hedgehog:

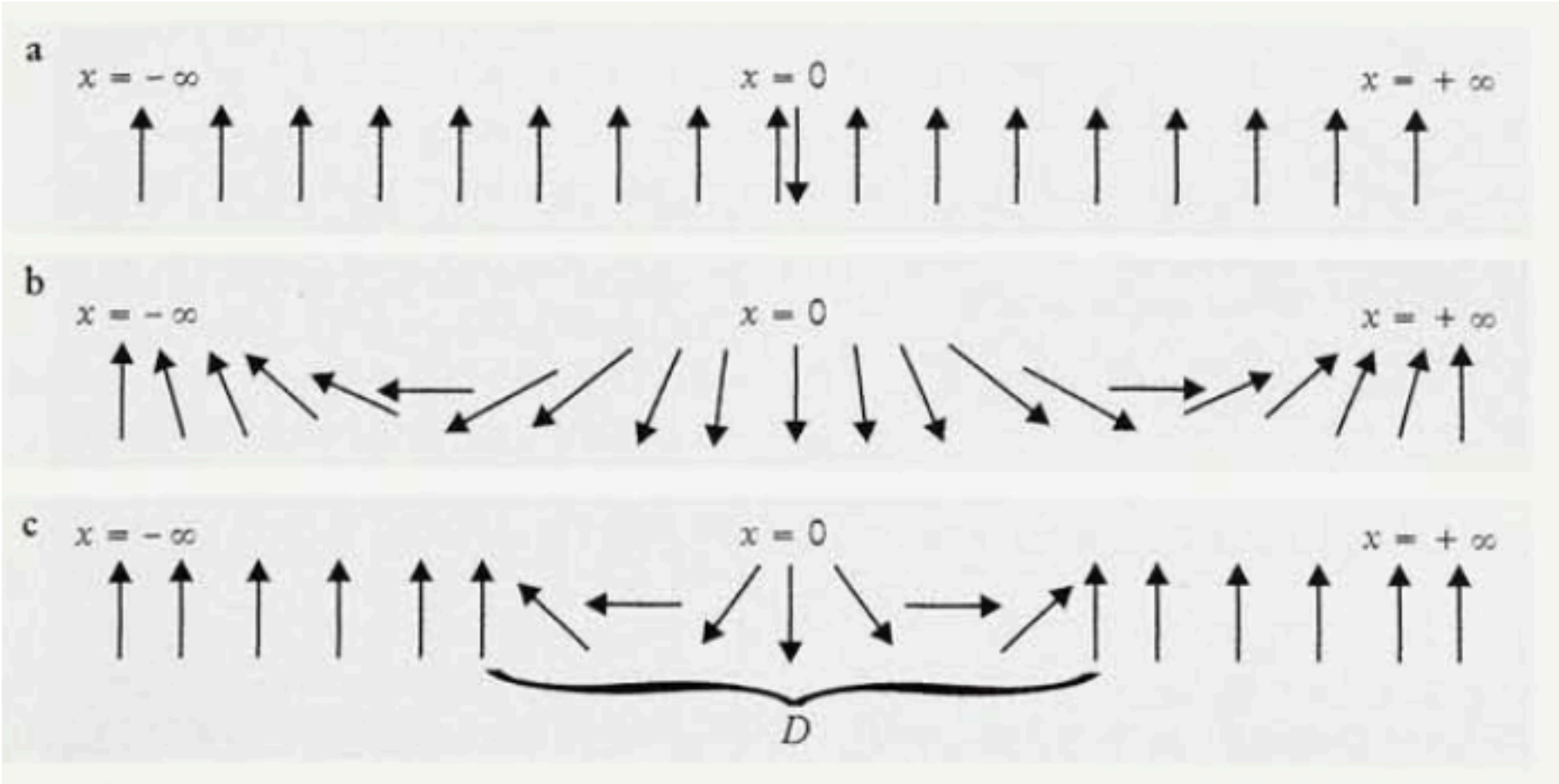
INFORMATION STORAGE and COMPUTATION:

- Electronics (charge, q)
- Spintronics (spin, s)
- Skyrmionics (topological charge, Q)



"TOPOLOGICAL QUANTUM COMPUTING"

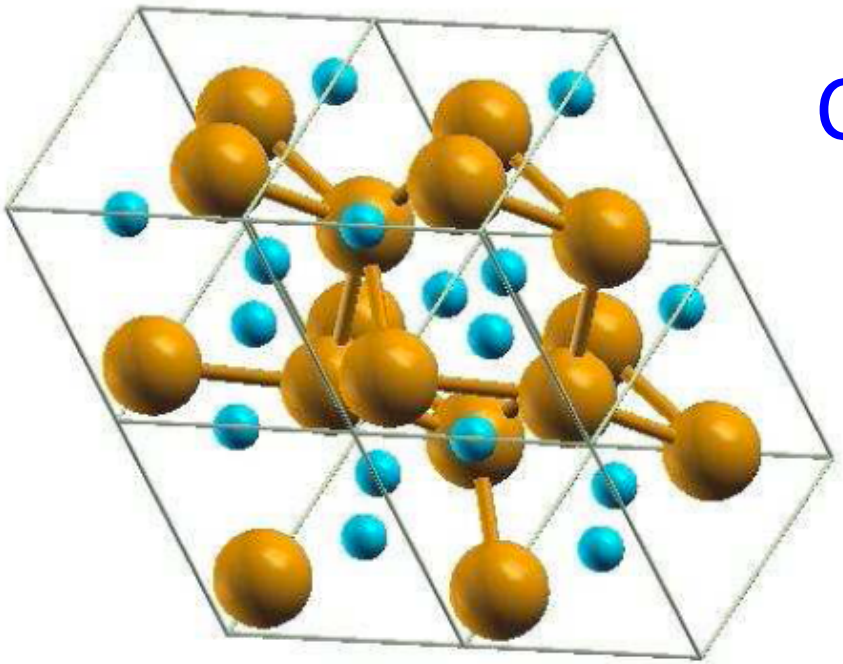
SKYrmion is the Limit



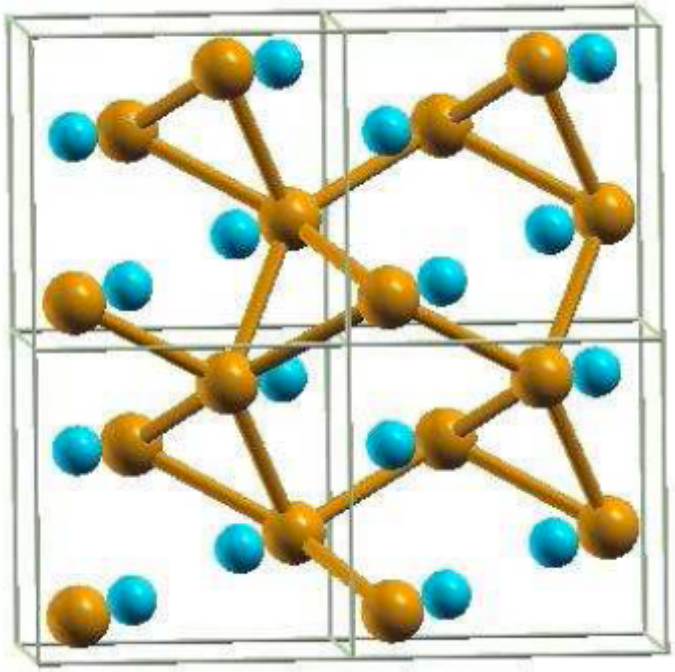
Spins along an axis in a 2D electron system

Ray Ladbury, Phys. Today 47 (July), 19 (1995)

B20 Crystal Structure (no inversion symmetry)



Chiral magnet: MnSi

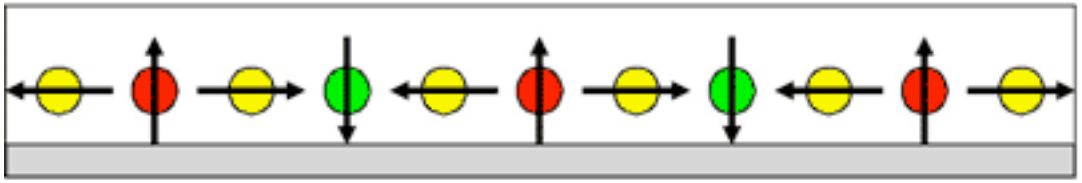


$P2_13$

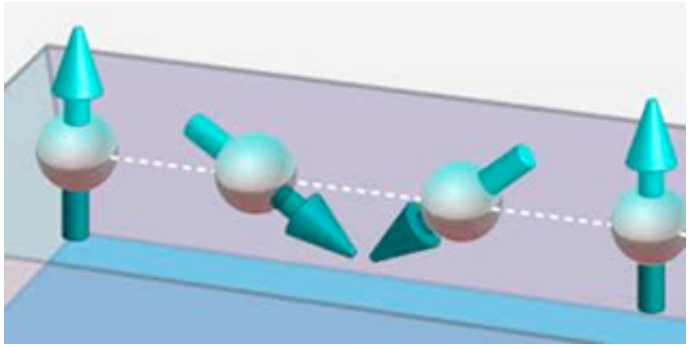
The Dzyaloshinskii-Moriya (DM) Interaction

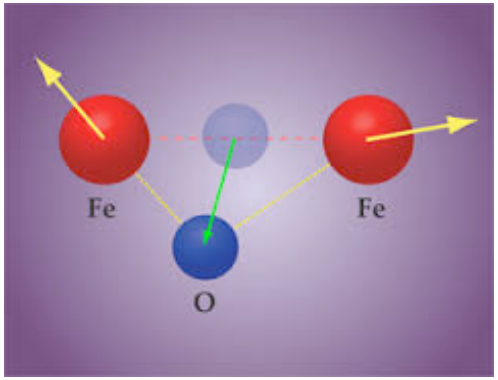
Dzyaloshinskii-Moriya interaction due to spin-orbit coupling

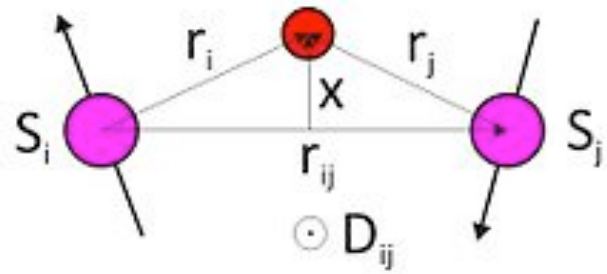
$$E_{\mathrm{DM}} = \sum_{i,j} \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$



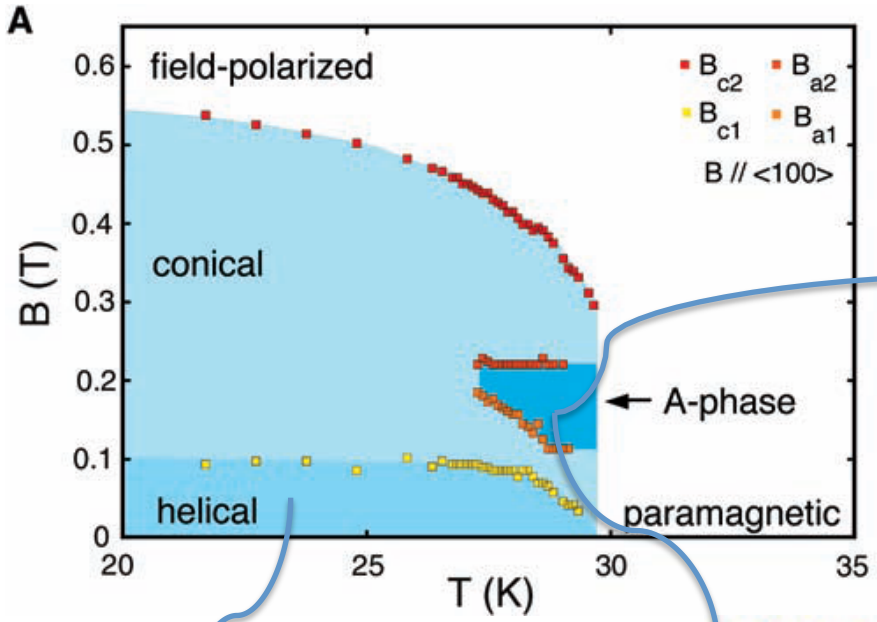
spin spiral with unique rotational sense



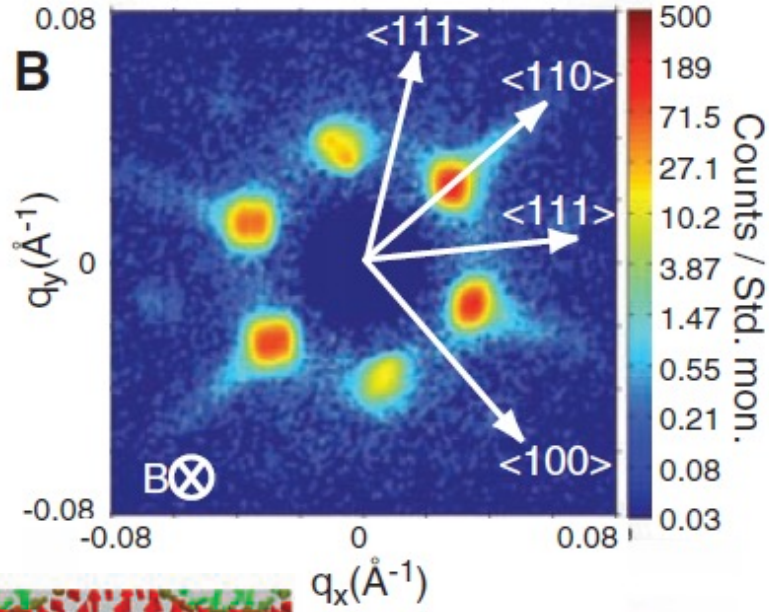


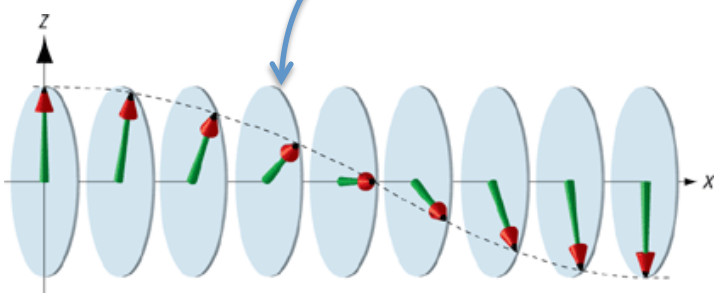


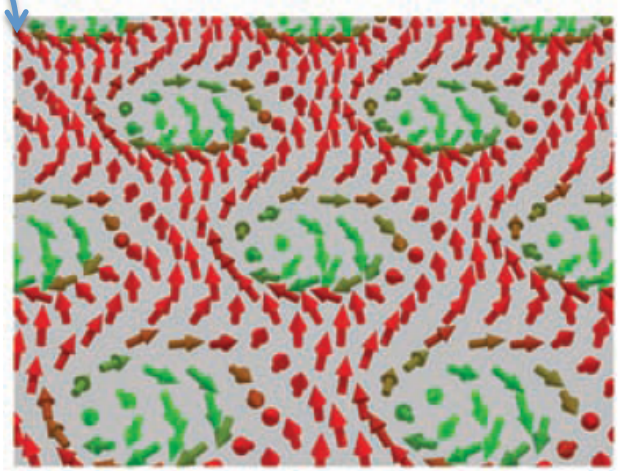
First experimental observation



SANS



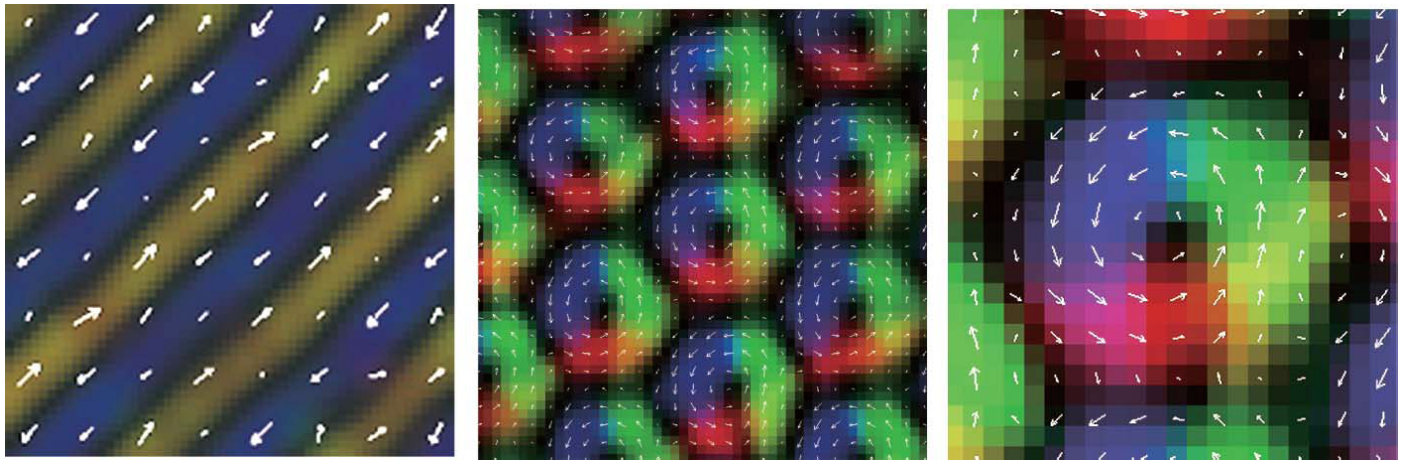




Bulk chiral magnet: MnSi

S. Mühlbauer, et al. Science 323, 915 (2009).

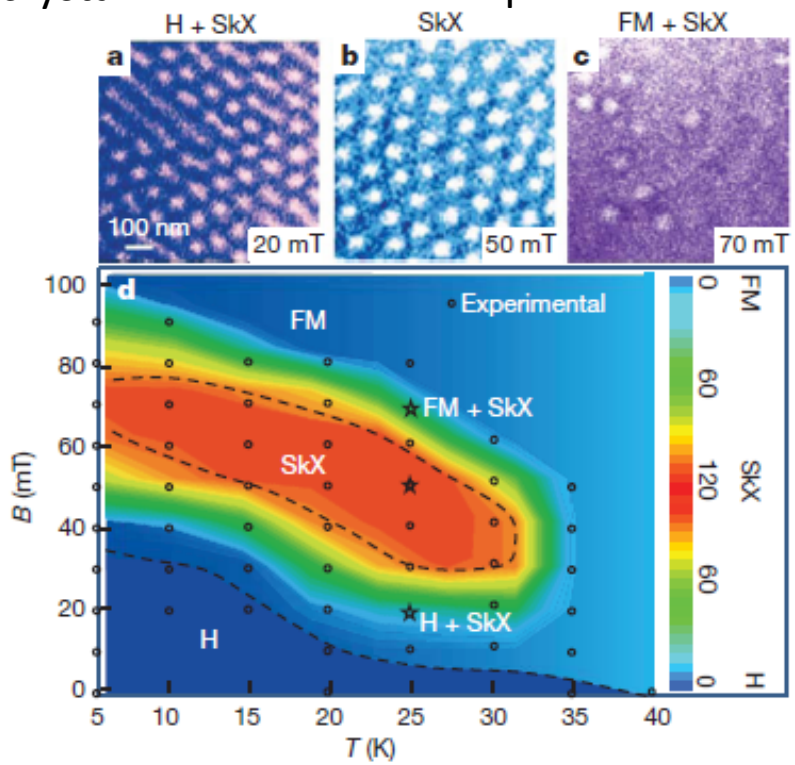
Real space observation of skyrmions in thin films



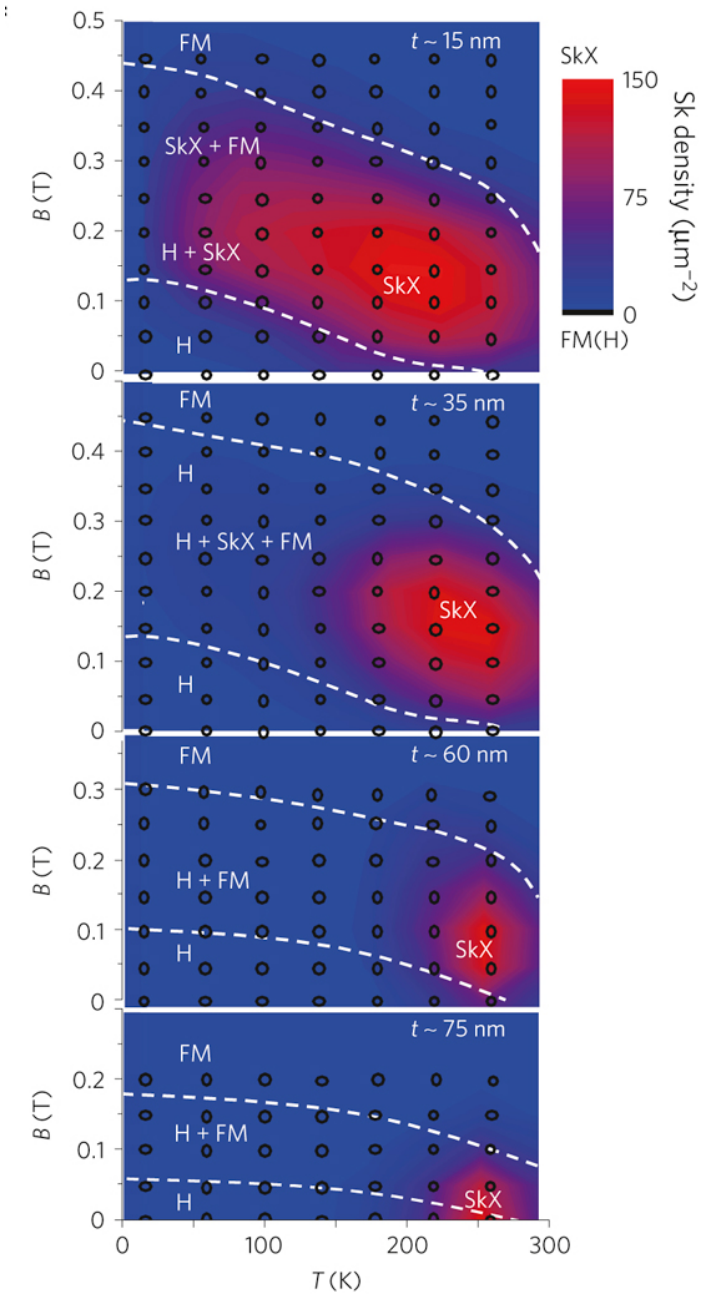
Helical Skyrmion crystal Close-up

- Lorentz transmission electron microscopy; measure the in-plane component of spin.
- Material: $Fe_{0.5}Co_{0.5}Si$

X. Z. Yu et al. Nature 465, 901 (2010).



Skyrmions are more stable in thin films



Experiments: FeGe

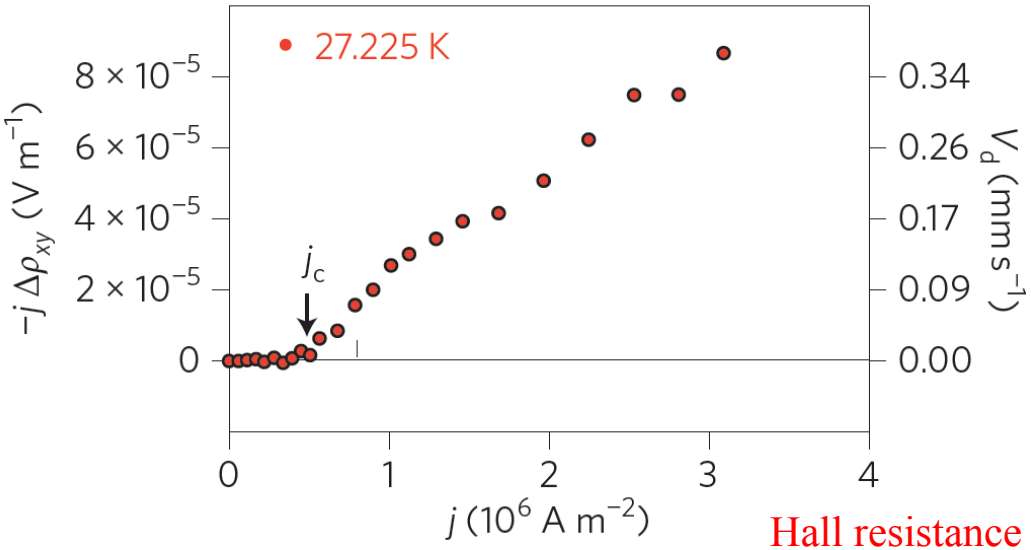
Z. Yu, et al., Nature Materials 10, 106 (2011).

Theory: stabilizing by out-of-plane anisotropy and dipole-dipole interaction

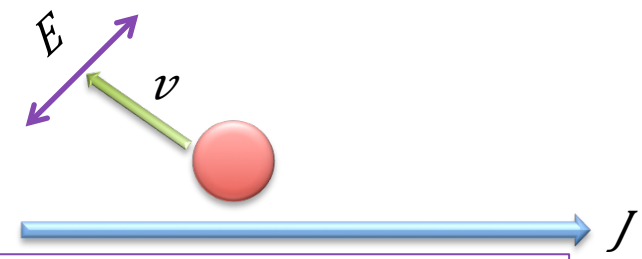
S. D. Yi et al., Phys. Rev. B 80, 054416 (2009).
A. B. Butenko et al., Phys. Rev. B82, 052403 (2010).
M. N. Wilson et al., Phys. Rev. B86, 144420 (2012).

Driven skyrmions by current:

Material: MnSi, Schulz, et al., Nature Physics 8, 301 (2012).



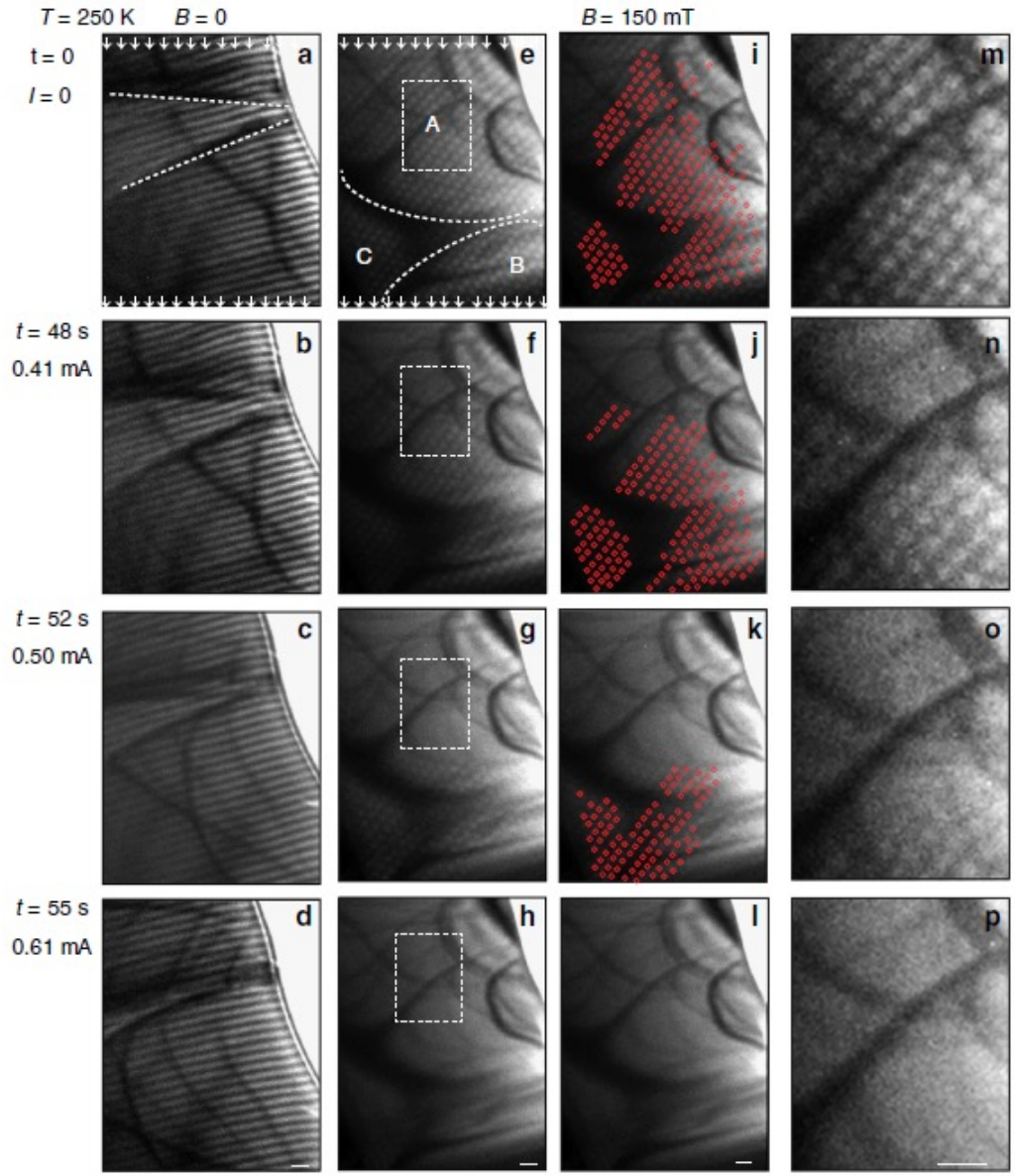
Hall resistance measurement and skyrmion velocity



- Hall resistance is due to the emergent electric field induced by the motion of skyrmion.
- The depinning current is extremely low $J_c \sim 10^6$ A/m^2. For magnetic domain walls $J_c \sim 10^{11}$ A/m^2.

Jonietz et al., Science 330, 1648 (2010).
Yu et al, Nature Communications 3, 988 (2012).

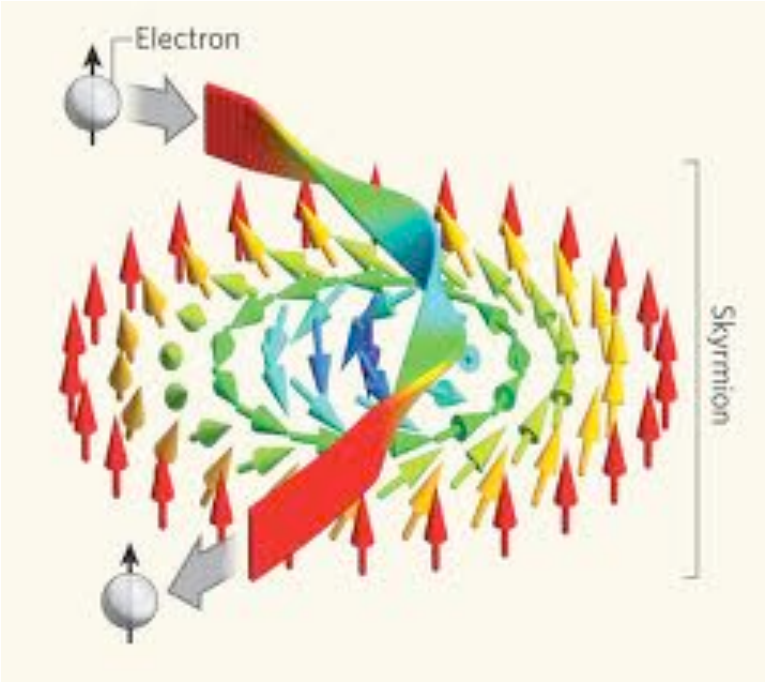
Real time observation of skyrmion motion

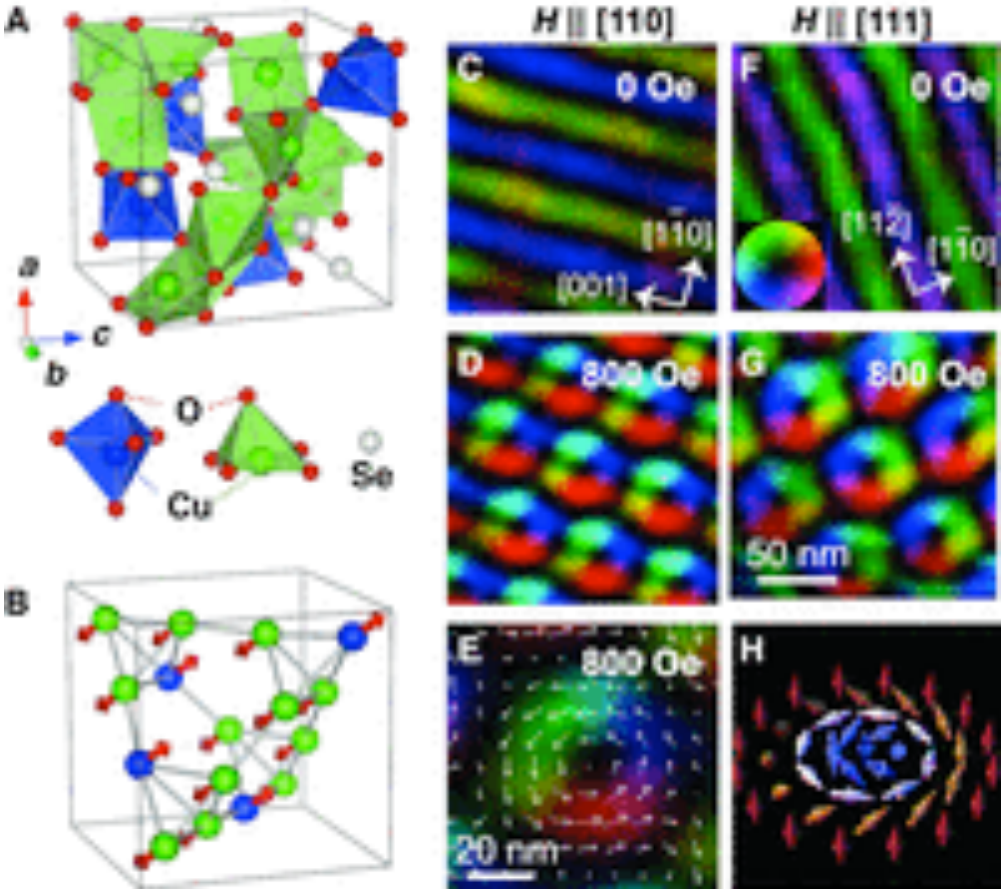


Material: FeGe

Yu et al, Nature Communications 3, 988 (2012).

Multiferroic skyrmions: Cu_2OSeO_3





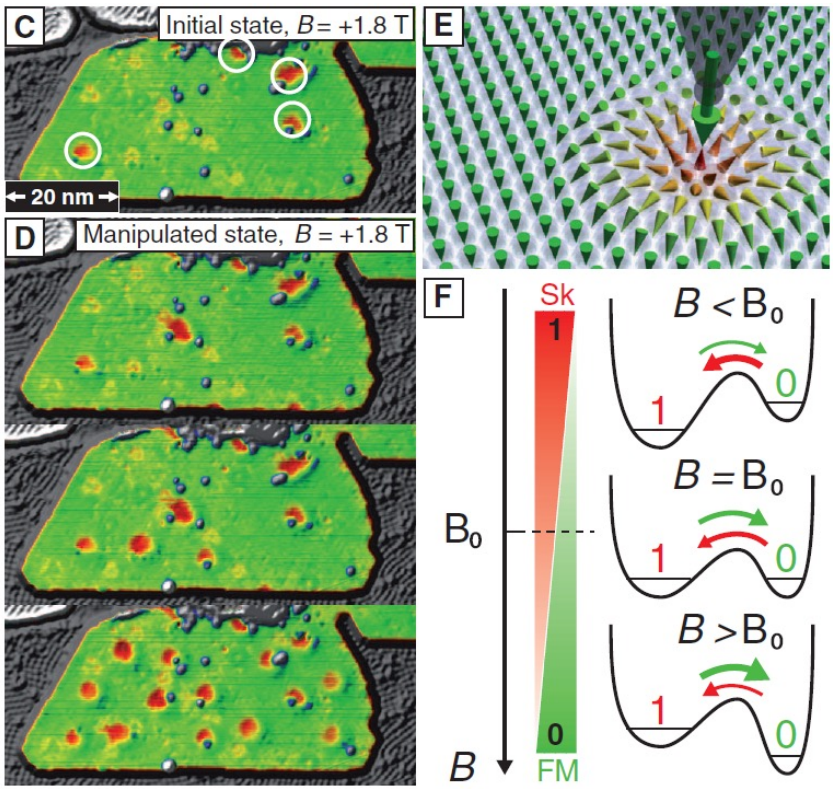
S. Seki et al., Science **336**, 198 (2012)

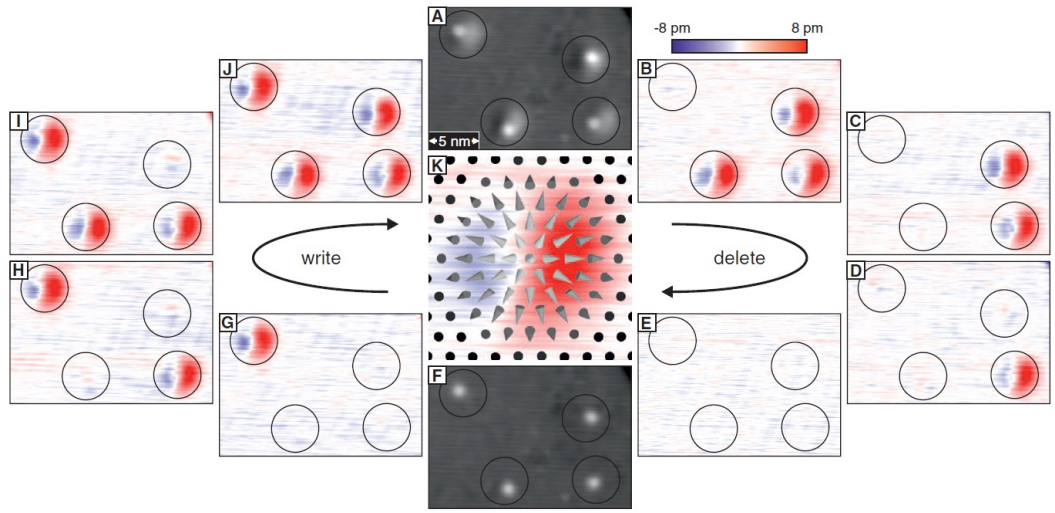
Magnetoelectric skyrmions: LTEM

Skyrmions also seen in $BiFeO_3$ films:

Possible toroidal moment!

Writing and deleting skyrmions

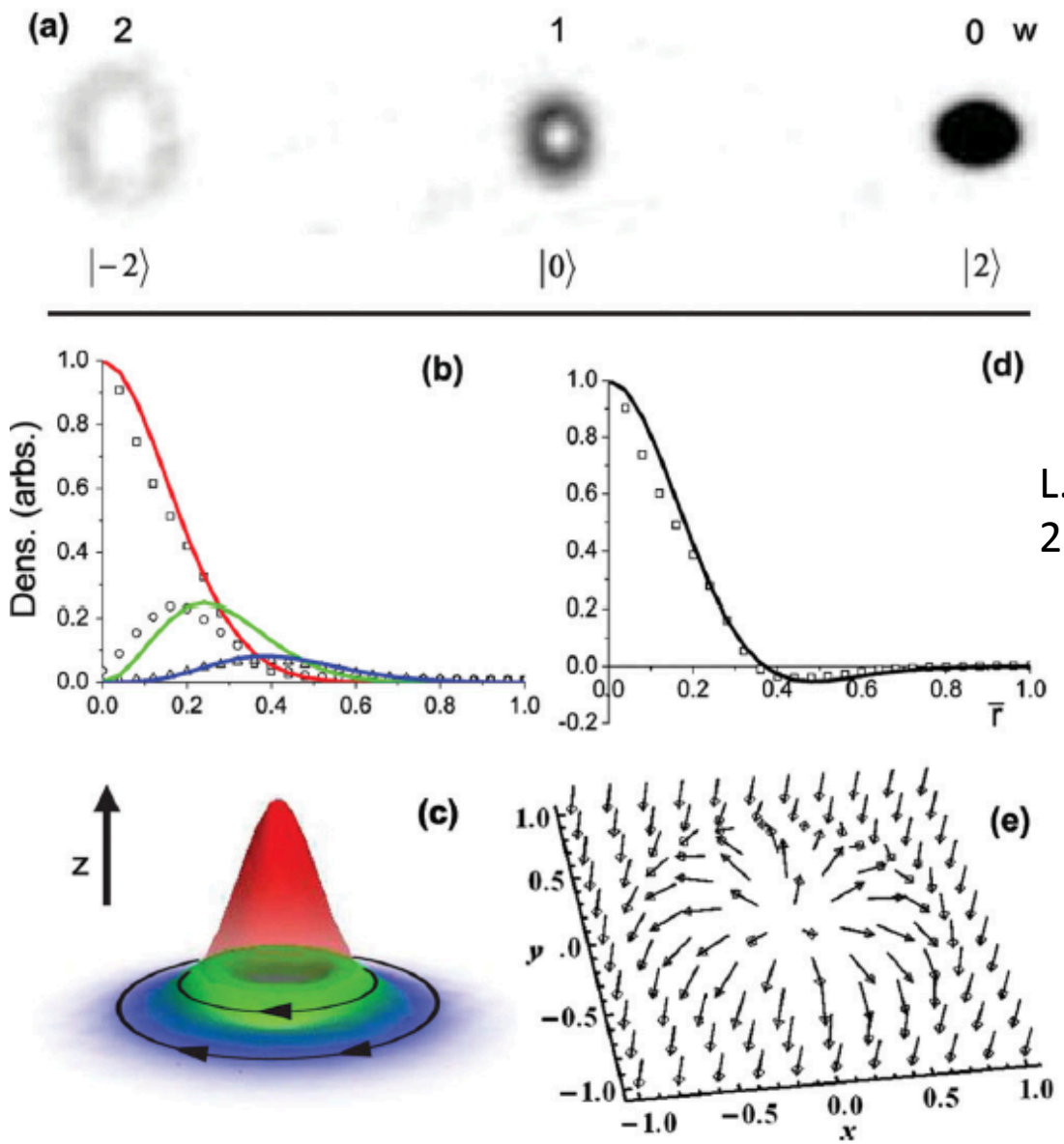




- Controlled creation and removal of a skyrmion by a spin polarized STM tip.
- Demonstration of application of skyrmions in information storage.

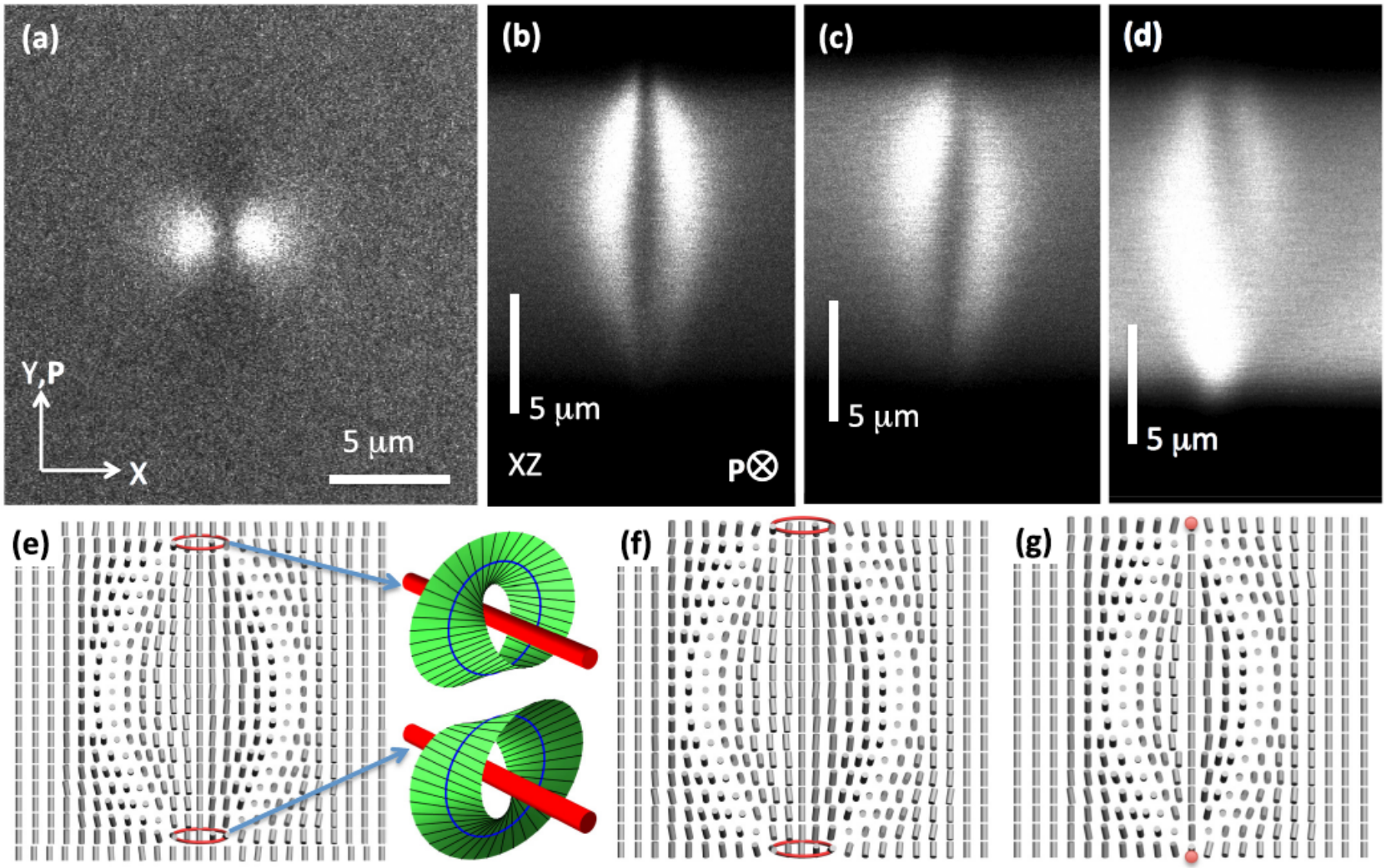
N. Romming *et al*. Science 341, 636 (2013).

Skyrmions in spin-2 BEC (^{87}Rb)



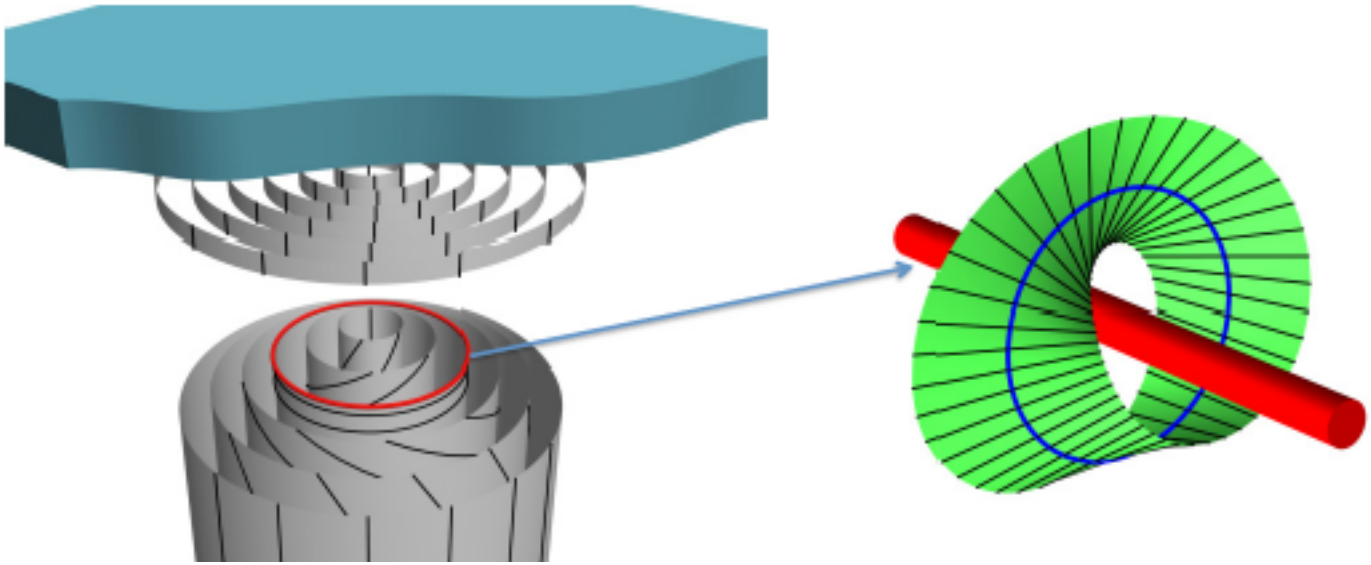
L.S. Leslie et al., PRL **103**, 250401 (2009)

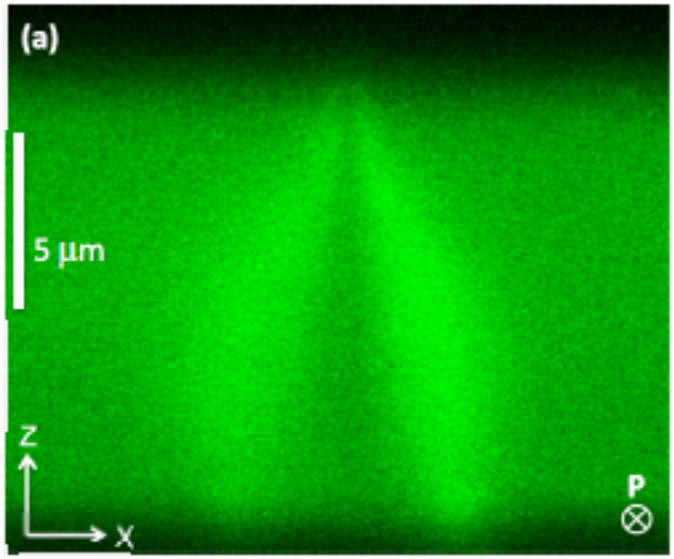
Skyrmion observation in liquid crystals

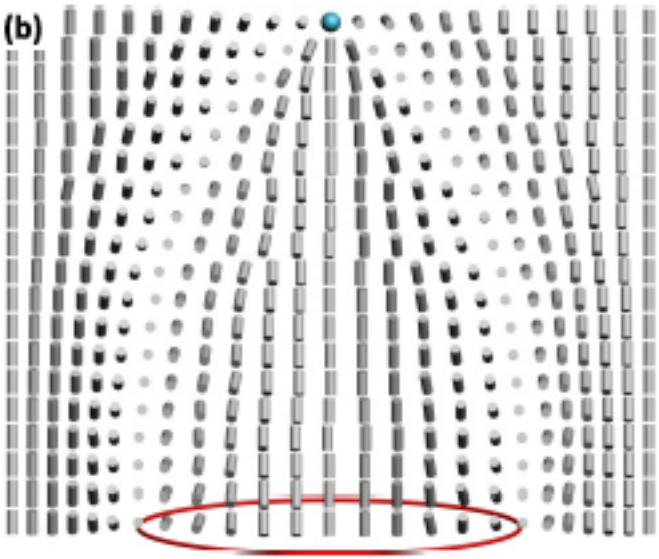


P.J. Ackerman et al., PRE 90, 012505 (2014)

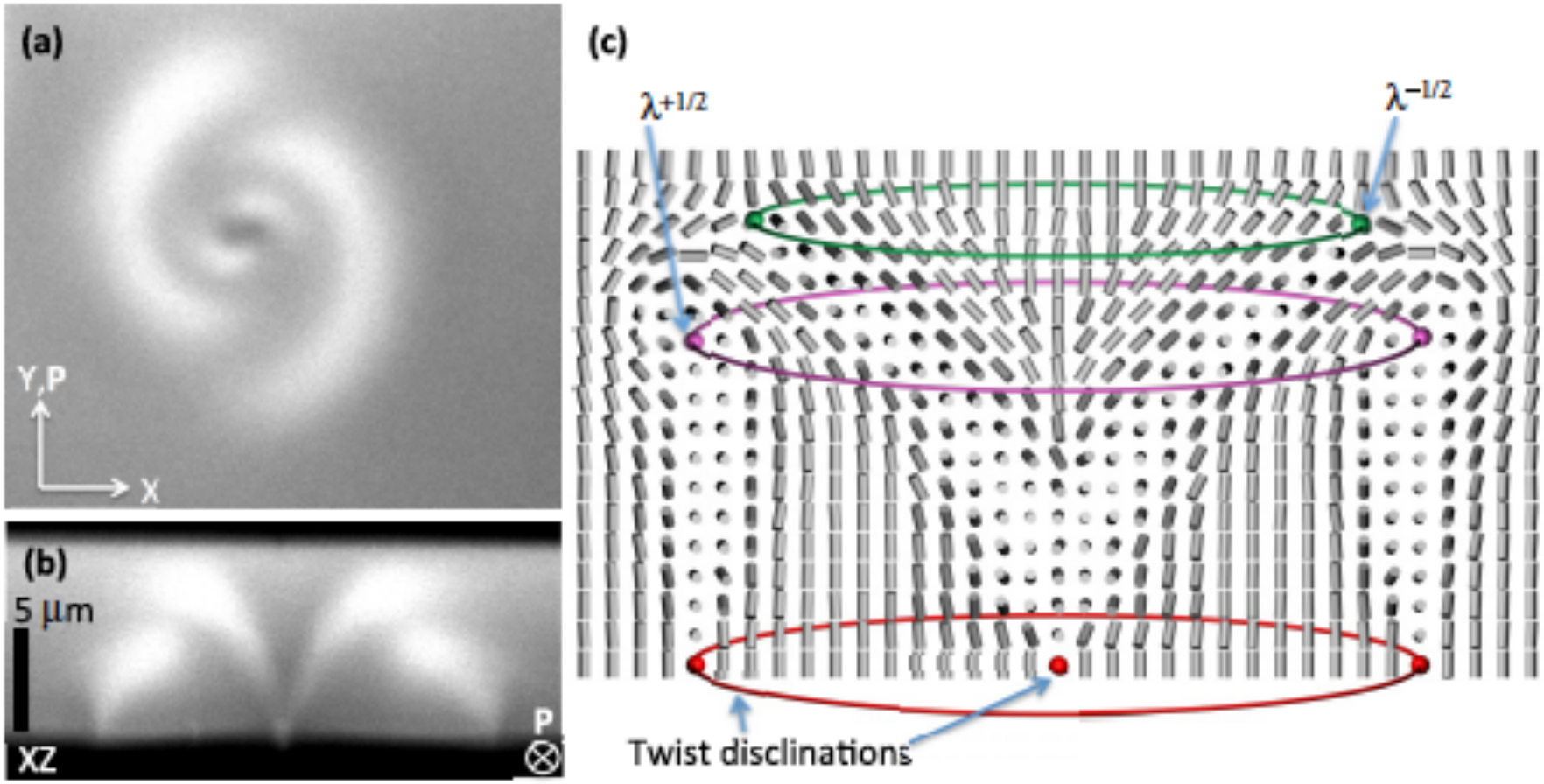
Twist dislocation loop



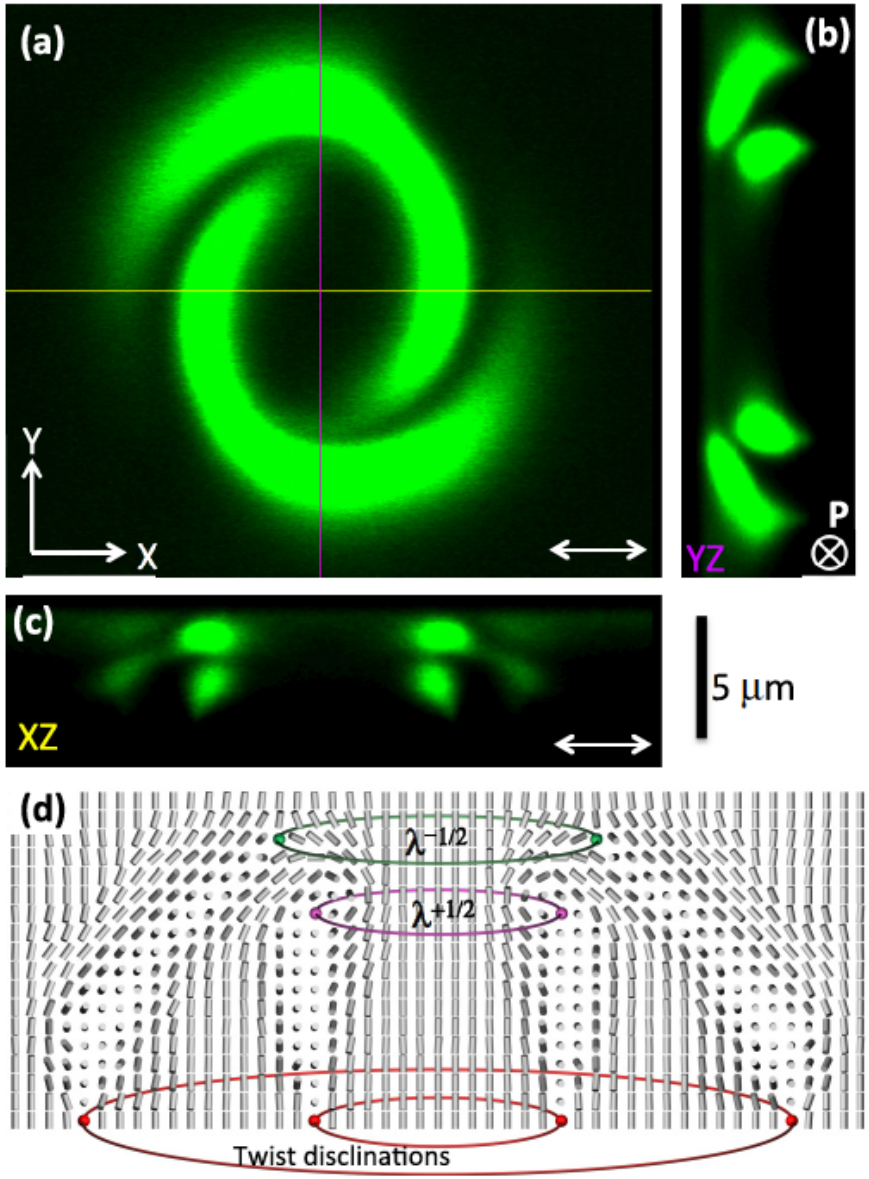




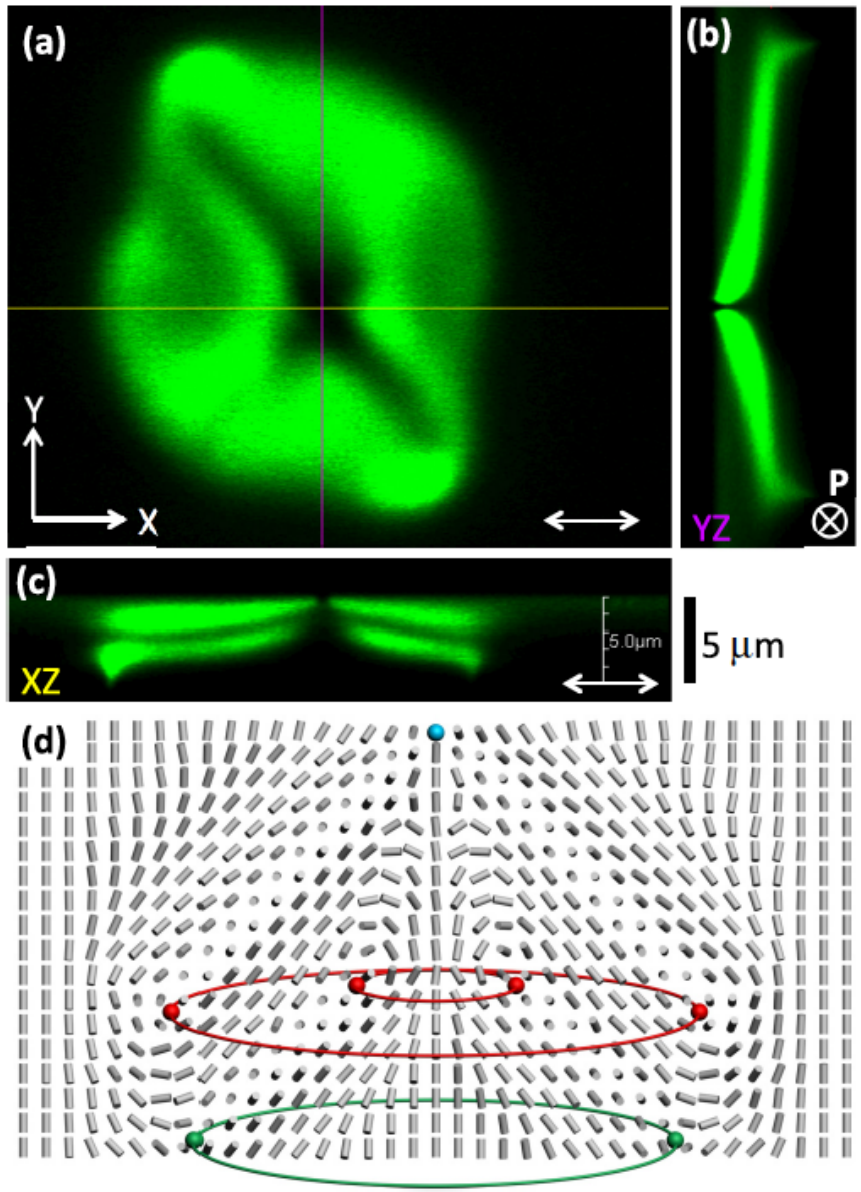
Axially symmetric structure



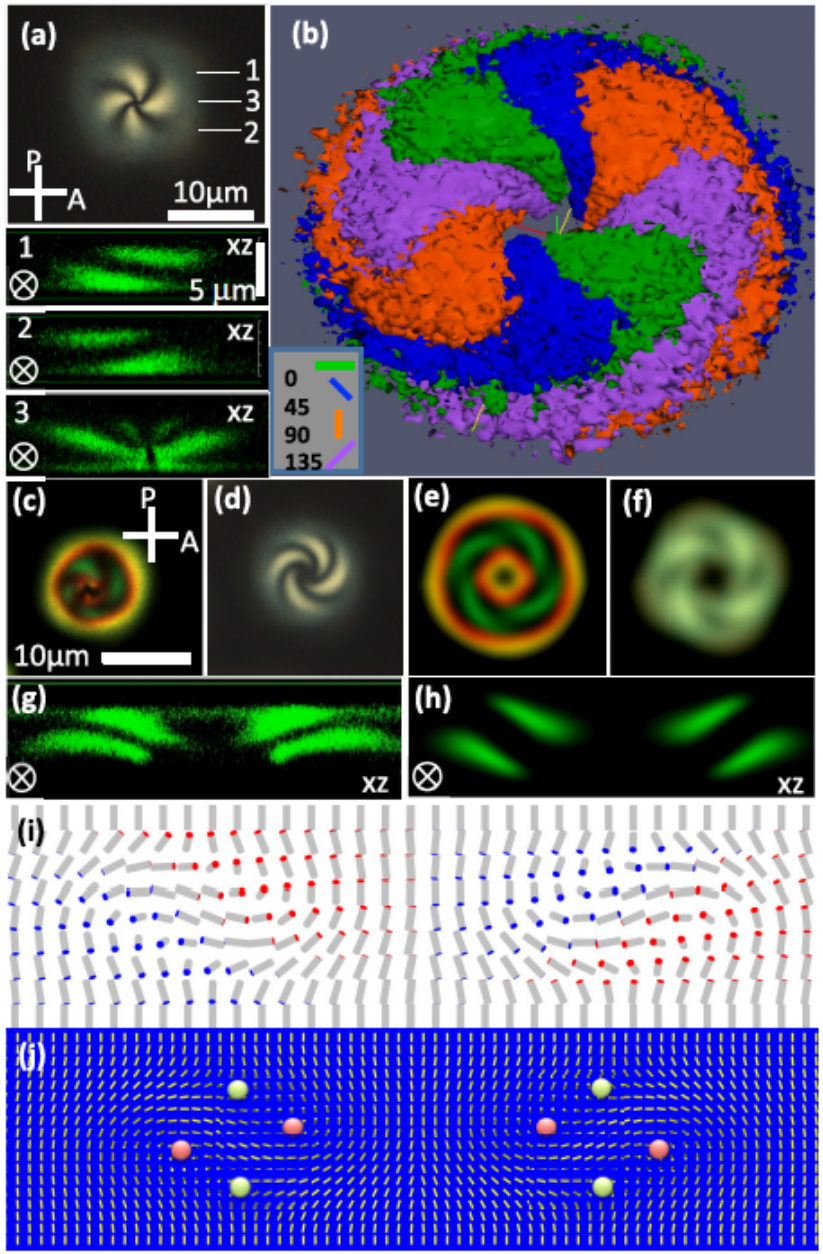
Localized twisted configuration



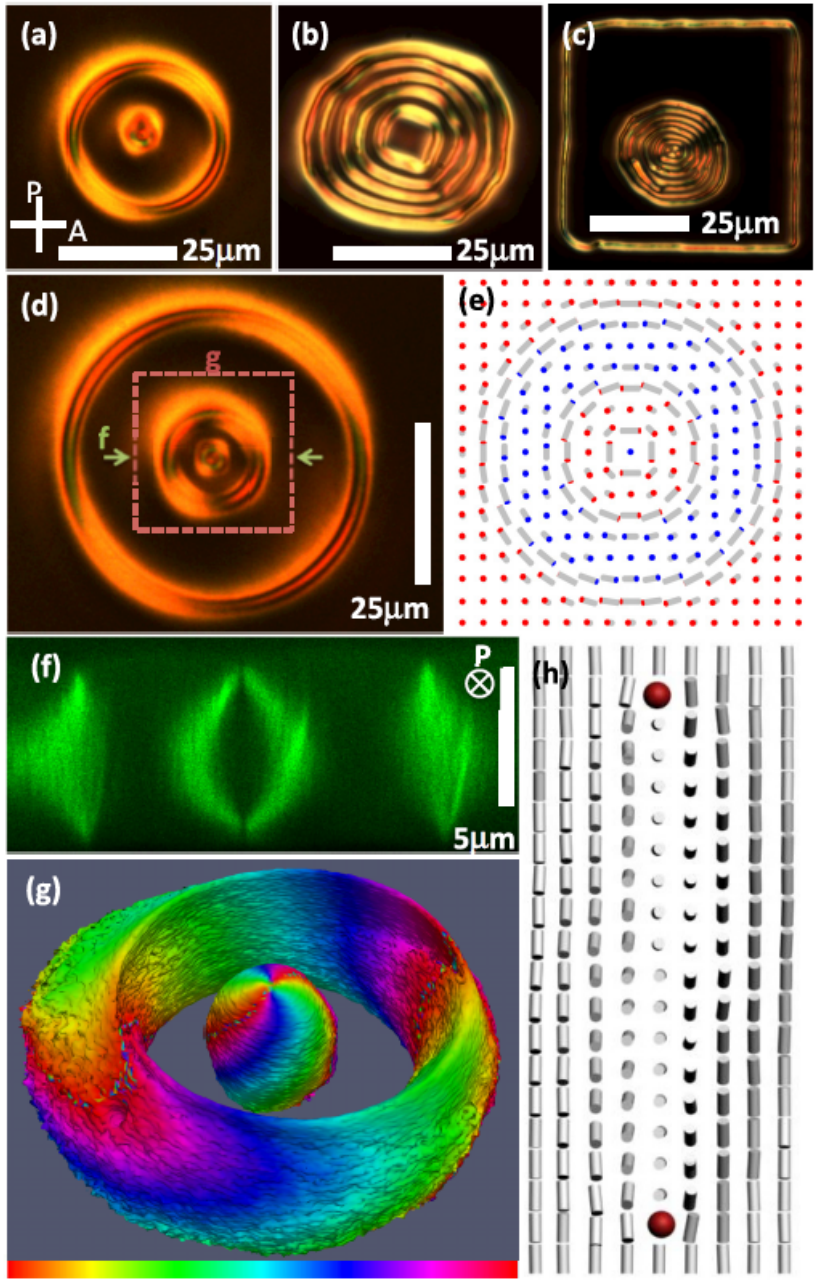
Localized axisymmetric twisted configuration



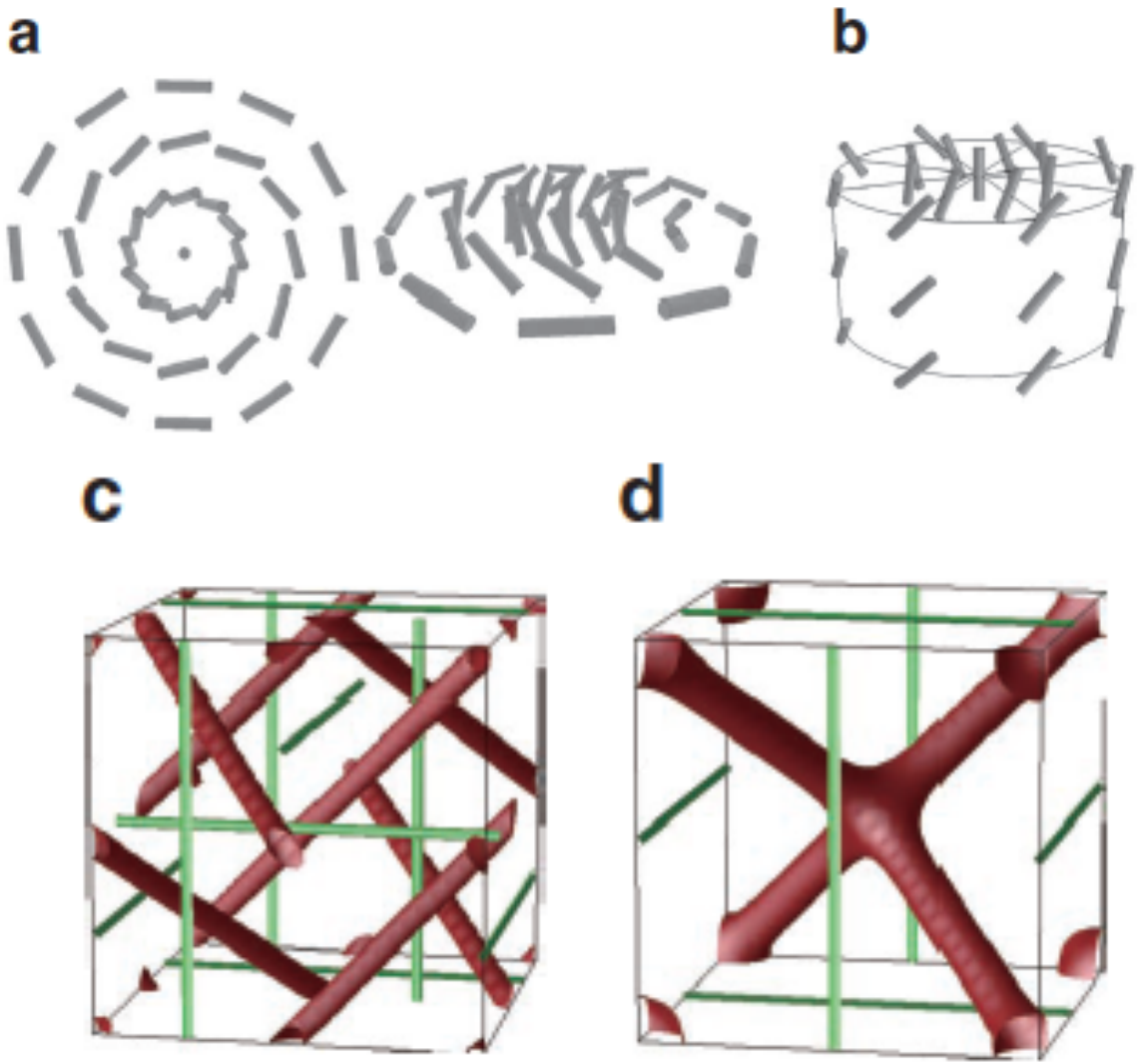
Cholesteric localized structures



Localized director structures

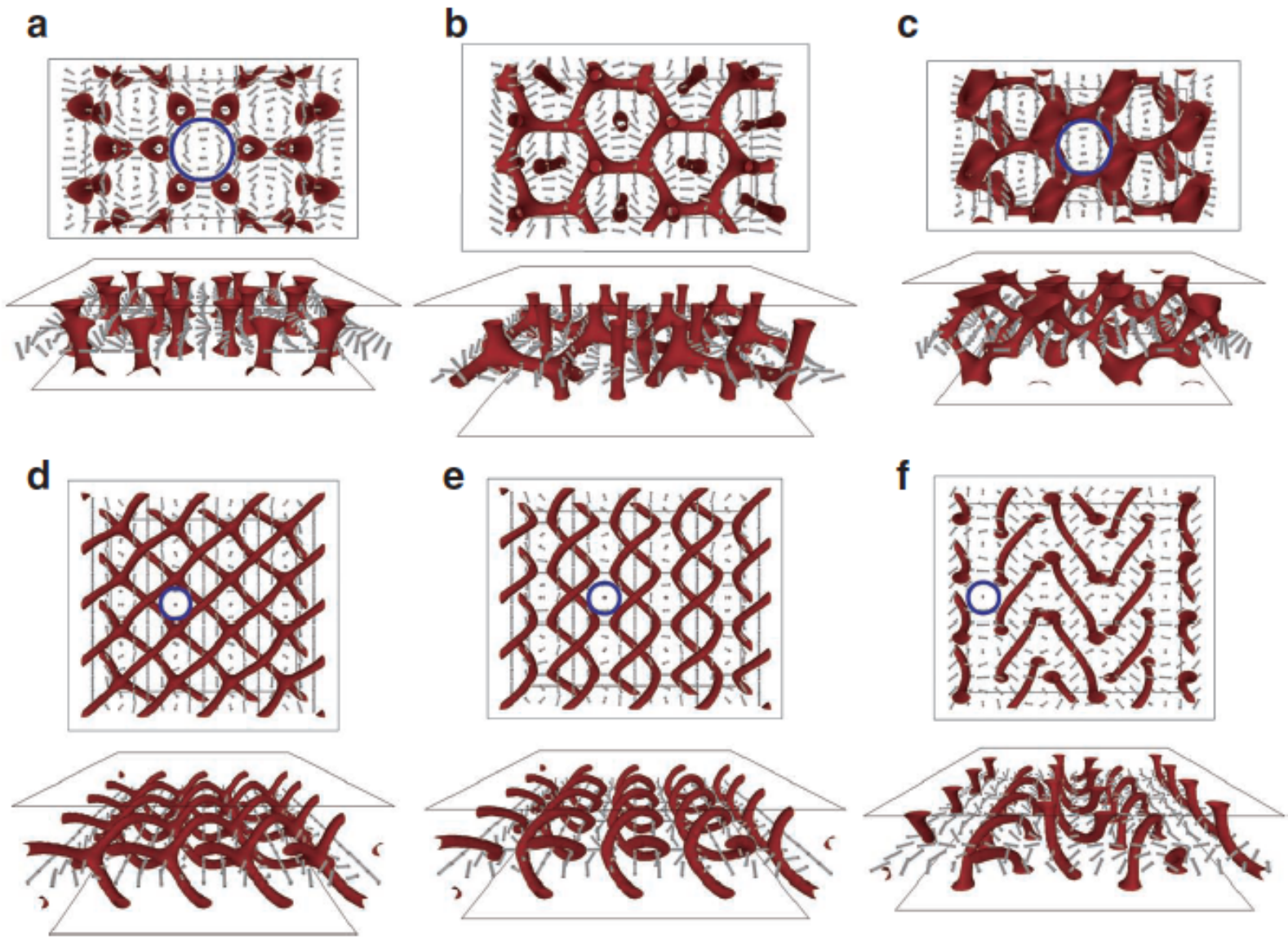


Skyrmions in Chiral Nematic Liquid Crystals

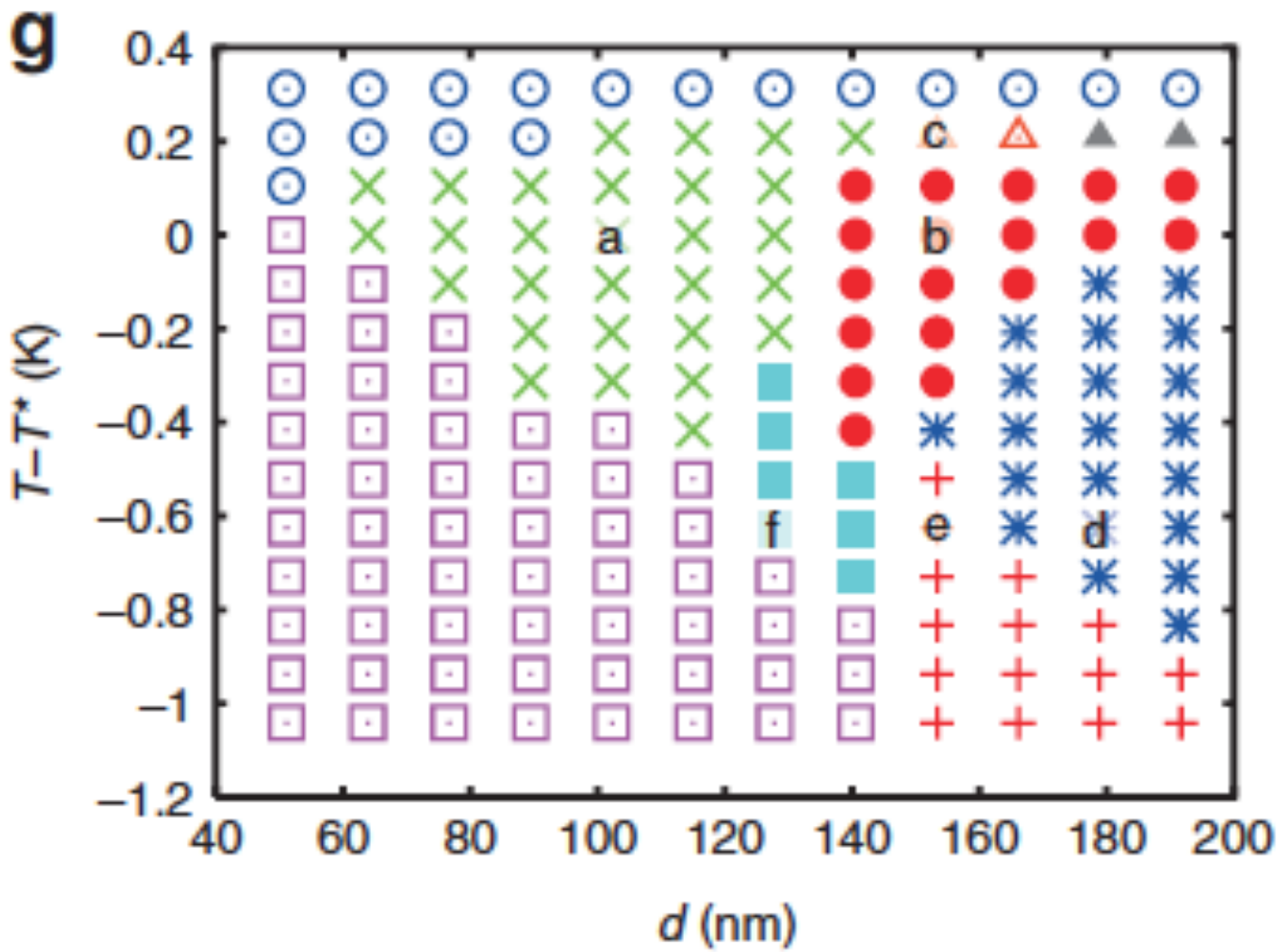


J. Fukuda and S. Zumer, Nature Commun. (2011)

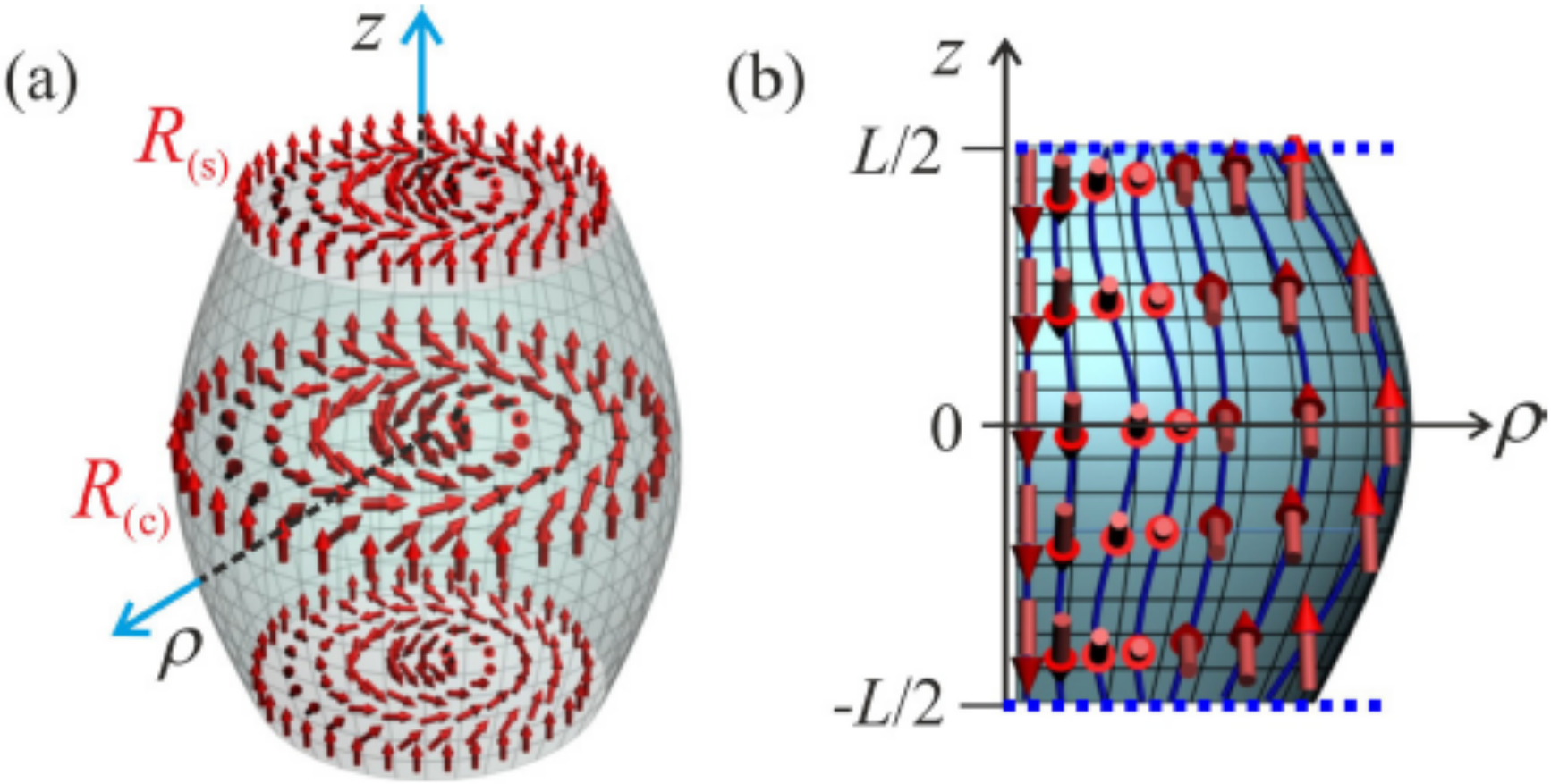
Orientational Order: Confined Liquid Crystal



Phase Diagram: T vs. Film Thickness



Axisymmetric skyrmion: Chiral Liquid Crystal



A.O. Leonov et al., PRE 90, 042502 (2014)

Frank Free-Energy Density Functional

$$f(\mathbf{n}) = \frac{K_1}{2}(\mathrm{div}\,\mathbf{n})^2 + \frac{K_2}{2}(\mathbf{n}\cdot\mathrm{rot}\,\mathbf{n} - q_0)^2 + \frac{K_3}{2}(\mathbf{n}\cdot\mathrm{rot}\,\mathbf{n})^2 - \frac{\varepsilon_a}{2}(\mathbf{n}\cdot\mathbf{E})^2 - \frac{\chi_a}{2}(\mathbf{n}\cdot\mathbf{H})^2.$$

Material parameters:

$$p = \frac{2\pi}{q_0}, \quad E_0 = \frac{\pi K q_0}{2}\sqrt{\frac{K}{\varepsilon_a}},$$

Cylindrical coordinates: $\mathbf{r} = (\rho\cos\varphi, \rho\sin\varphi, z)$, $\mathbf{n} = (\sin\theta\cos\psi, \sin\theta\sin\psi, \cos\theta)$.

$$F = \frac{K}{2}\int_0^{2\pi} d\varphi \int_{-L/2}^{L/2} dz \int_0^{\infty} \rho d\rho\,[w(\theta,\psi) + w_s(\theta)]\,,$$

$$w = \left(\frac{\partial\theta}{\partial z}\right)^2 + \left(\frac{\partial\theta}{\partial\rho}\right)^2 + \frac{\sin^2\theta}{\rho^2}\left(\frac{\partial\psi}{\partial\varphi}\right)^2 + \frac{\varepsilon_a E^2}{K}\sin^2\theta + 2q_0\left[\left(\frac{\partial\theta}{\partial\rho}\right) + \frac{\sin\theta\cos\theta}{\rho}\left(\frac{\partial\psi}{\partial\varphi}\right)\right]\sin(\psi - \phi),$$

$$w_s(\theta) = (K_s/K)\sin^2\theta\,\delta(z \pm L/2),$$

Minimization and Euler equation

$$\psi = \varphi + \pi/2,$$

Euler equation:

$$\frac{\partial^2\theta}{\partial z^2} + \frac{1}{\rho}\frac{\partial\theta}{\partial\rho} + \frac{\partial^2\theta}{\partial\rho^2} - \frac{1}{\rho^2}\sin\theta\cos\theta$$
$$- \frac{2q_0}{\rho}\sin^2\theta - \frac{\varepsilon_a E^2}{K}\sin\theta\cos\theta = 0,$$

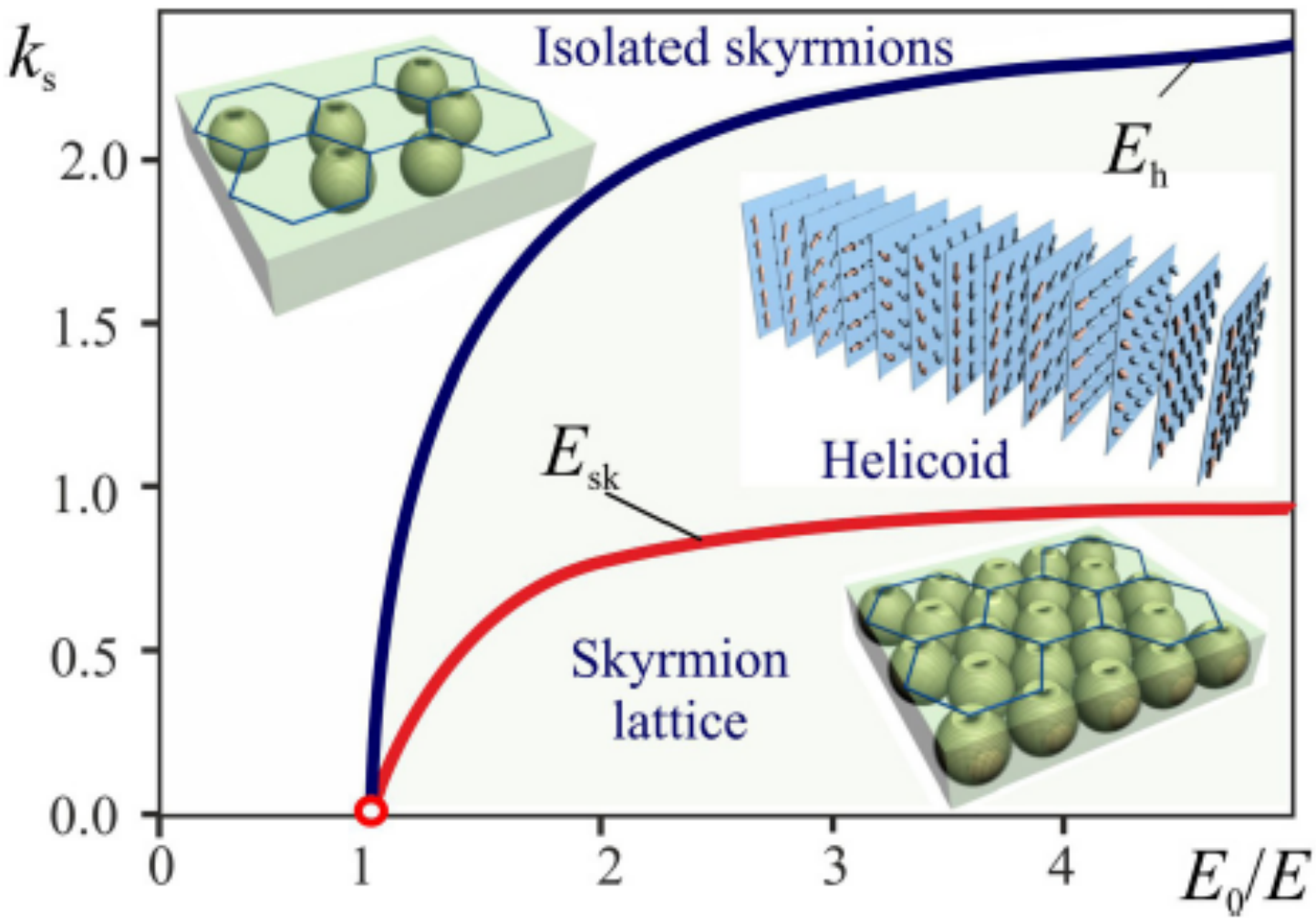
Boundary condition: $\theta(0,z) = \pi, \theta(\infty,z) = 0,$

$$\left(\frac{\partial\theta}{\partial z} + \frac{K_s}{K}\sin\theta\cos\theta\right)\Bigg|_{z=\pm L/2} = 0.$$

Three control parameters:

$$E/E_0, \quad k_s = K_s/(Kq_0), \quad \nu = L/p,$$

Phase Diagram: Helicoid & Skyrmion Lattice



Minimal model:

- For magnets without inversion symmetry, Dzyaloshinskii-Moriya interaction

$$H = E_1 + E_2 + E_3 = \int \mathrm{d}r^d [\frac{J_{ex}}{2} \sum_{\mu} (\partial_\mu \mathbf{n})^2 + D\mathbf{n} \cdot \nabla \times \mathbf{n} - \mathbf{B} \cdot \mathbf{n}]$$

n is a unit vector representing the spin direction

- Derrick's theorem: necessary but not sufficient condition for the existence of topological excitations

  If the energy of the system does not have stationary point upon rescaling the length $r \to \lambda r$, then there is no stable topological excitation.

- after rescaling $r \to r/\lambda$

$$H = \lambda^{2-d} E_1 + \lambda^{1-d} E_2 + \lambda^{-d} E_3$$

For skyrmions $E_1>0$, $E_3>0$ and $E_2<0$

- DM interaction stabilizes skyrmion in 2D and 3D.

Theoretical prediction: Bogdanov and Yablonskii, Sov. Phys. JETP 68, 101 (1989).
Rößler, et al., Nature 442, 797 (2006).

- Other mechanisms to stabilize skyrmions
  - Change the sample geometry (base manifold) A. Saxena, R. Dandoloff, PRB 66, 104414 (2002)
  - Dipole-dipole interaction X. Yu et al. PNAS 109, 8856 (2012).
  - Mutliple-Q spiral condensation Okubo, et al. PRL108, 17206 (2012).
    Kamiya and Batista, PRX 4, 11023 (2014).

Model and equation of motion:

- We consider a magnetic film (2D) with the DM interaction, with the Hamiltonian

$$E = \int dr^2 \left[\frac{J_{ex}}{2} \sum_{\mu} (\partial_\mu \mathbf{n})^2 + D\mathbf{n} \cdot \nabla \times \mathbf{n} - \mathbf{H} \cdot \mathbf{n} \right]$$

- Equation of motion for spins is the Landau-Lifshitz-Gilbert equation

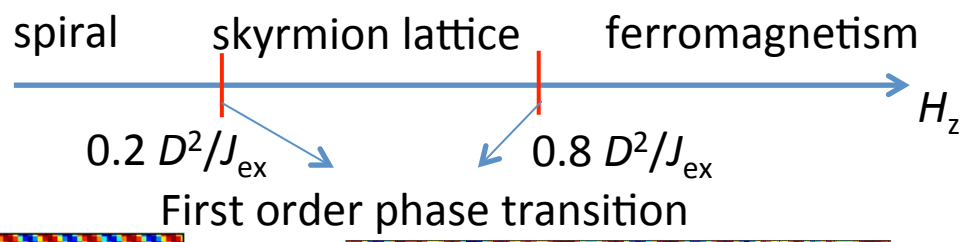
Wess-Zumino action (Berry phase of spin)

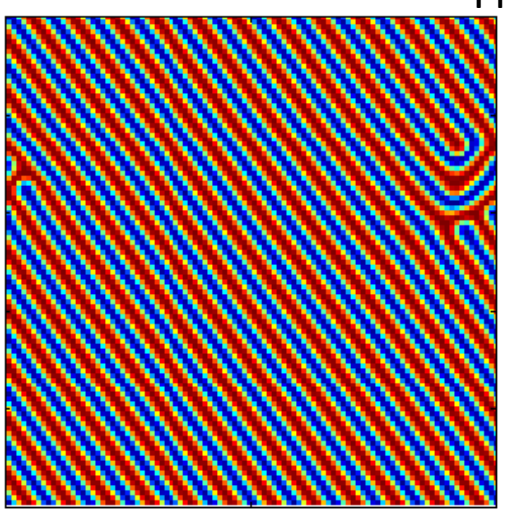
$$\partial_t \mathbf{n} = -\gamma \mathbf{n} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{n} \times \partial_t \mathbf{n} + (\mathbf{J} \cdot \nabla)\mathbf{n}$$

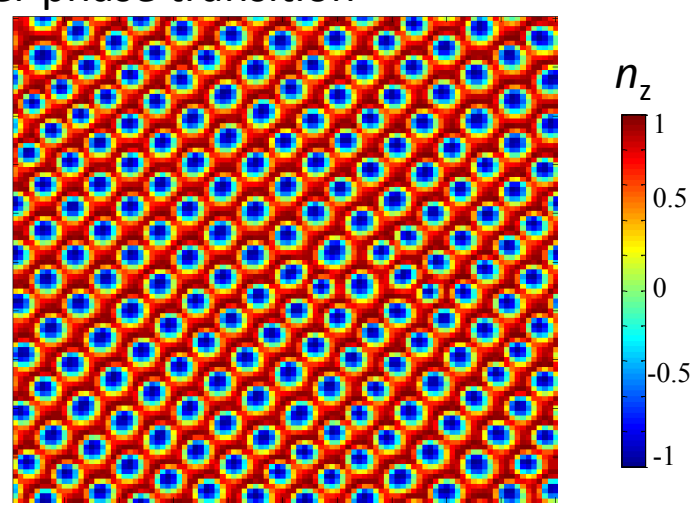
Spin transfer torque

with $\mathbf{H}_{\text{eff}} = -\dfrac{\delta \mathrm{H}}{\delta \mathbf{n}} = J_{ex} \nabla^2 \mathbf{n} - 2D \nabla \times \mathbf{n} + \mathbf{H}$

- Phase diagram at $T=0$







Model and equation of motion

- We consider a magnetic film (2D) with the DM interaction, with the Hamiltonian

$$E = \int dr^2 \left[\frac{J_{ex}}{2} \sum_{\mu} (\partial_\mu \mathbf{n})^2 + D\mathbf{n} \cdot \nabla \times \mathbf{n} - \mathbf{H} \cdot \mathbf{n} \right] + A n_z^2$$

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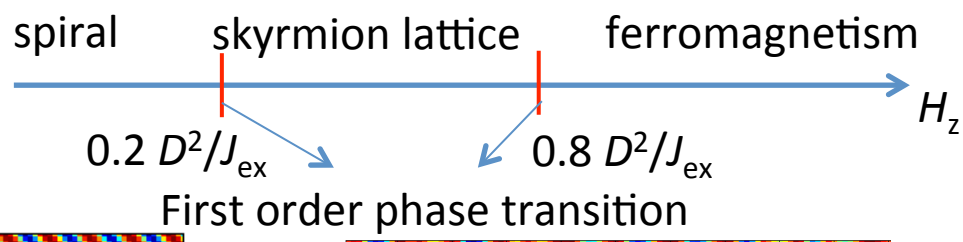
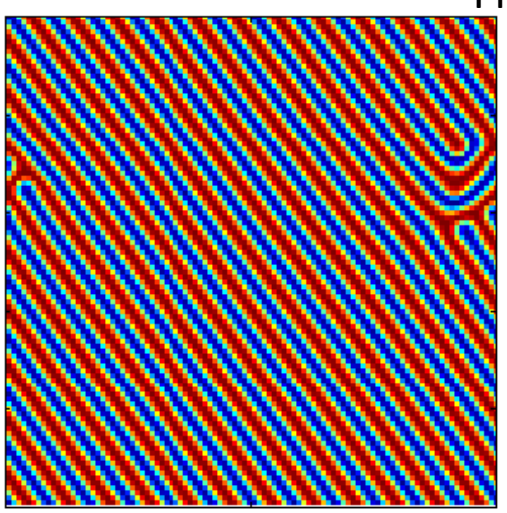
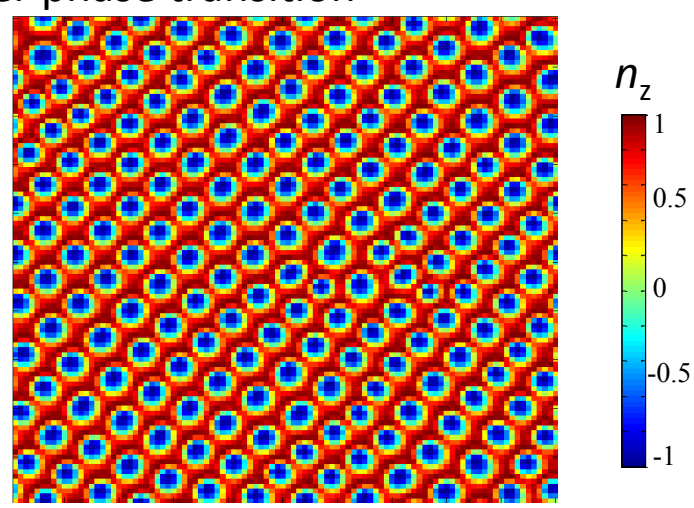
Spin transfer torque

$$\text{with } \mathbf{H}_{\text{eff}} = -\frac{\delta \mathrm{H}}{\delta \mathbf{n}} = J_{ex} \nabla^2 \mathbf{n} - 2D \nabla \times \mathbf{n} + \mathbf{H} - A n_z$$

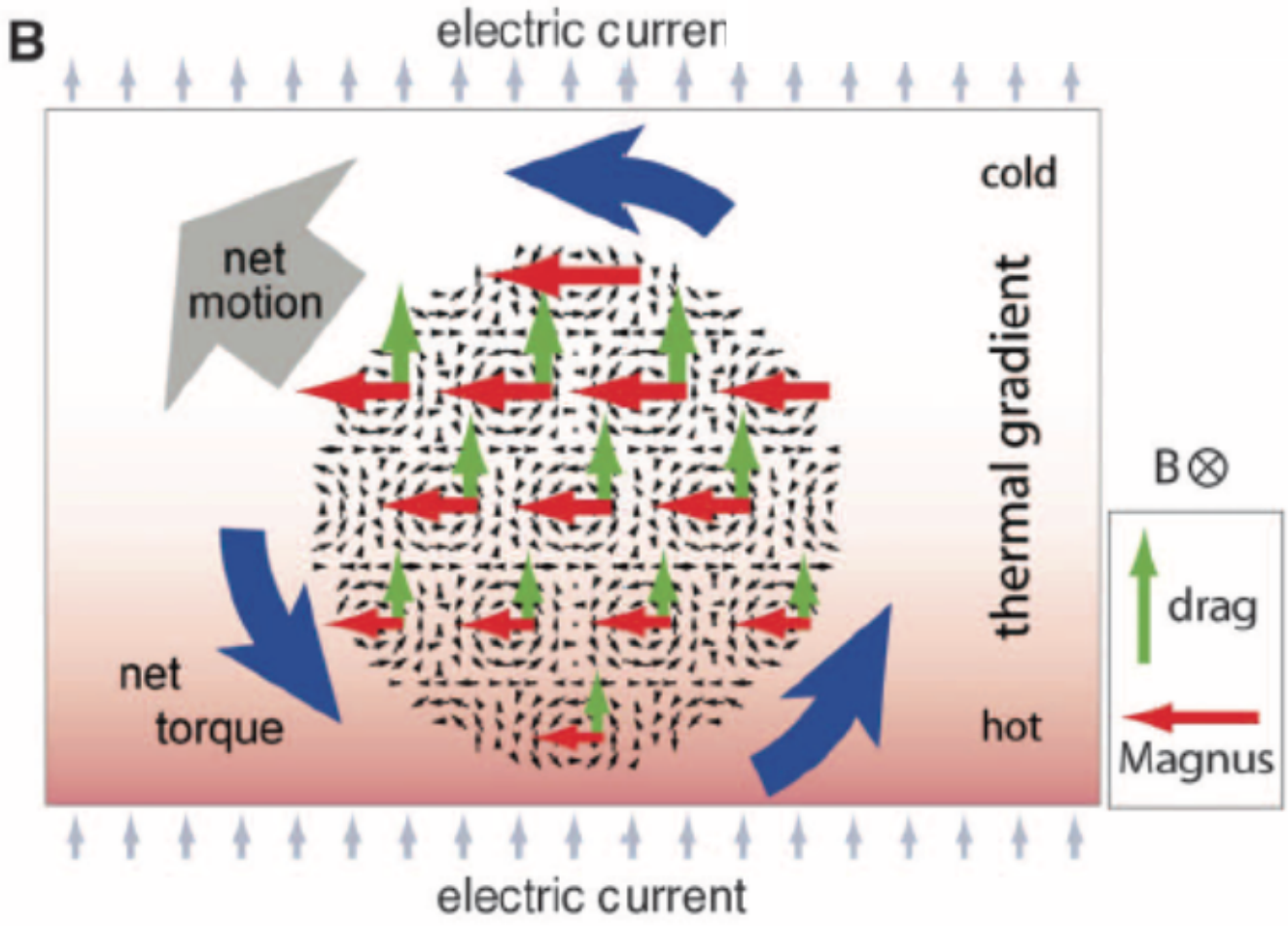
- Phase diagram at $T=0$





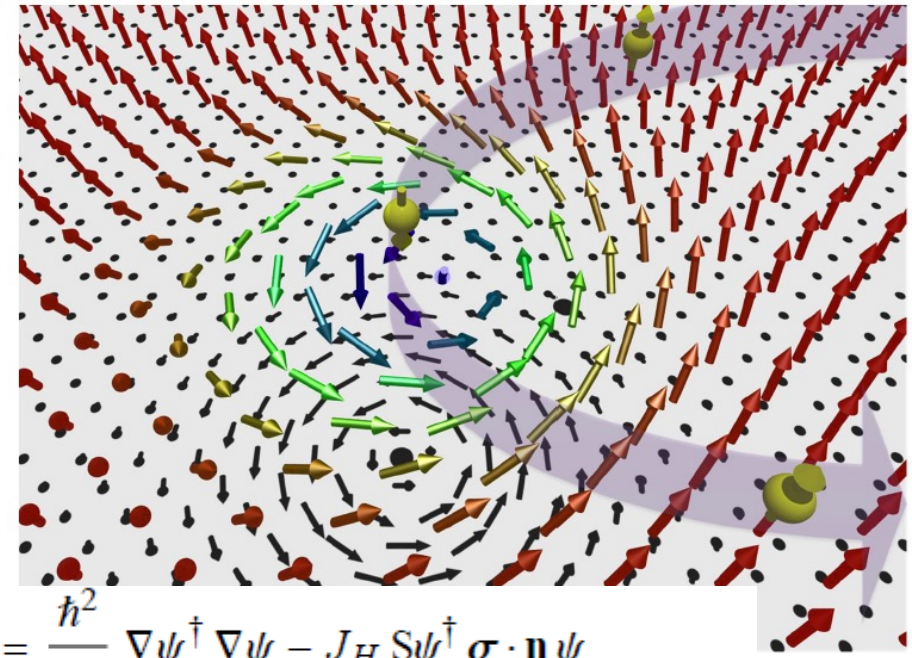


SPIN TRANSFER TORQUE



Jonietz et al., Science (2010)

Emergent electromagnetism in conducting magnets



- Conduction electrons Hamiltonian

$$S_{\rm el} = \int dt\, dx^3 \left[i\hbar \psi^\dagger \dot{\psi} - \mathcal{H} \right] \text{ with } \mathcal{H} = \frac{\hbar^2}{2m} \nabla\psi^\dagger \nabla\psi - J_H S \psi^\dagger \boldsymbol{\sigma}\cdot\mathbf{n}\,\psi$$

- Large Hund's coupling (adiabatic limit) $J_{\rm H} >> E_{\rm F}$

$$S_{el} = \int dr^3 dt [i\hbar\chi^\dagger \dot{\chi} + eA_0 - \frac{1}{2m}[(i\hbar\nabla - \frac{e}{c}\mathbf{A}^*)\chi^\dagger][(-i\hbar\nabla - \frac{e}{c}\mathbf{A})\chi] - \frac{\hbar^2}{8m}(\nabla\mathbf{n})^2 + J_H S$$

- Emergent electromagnetic fields

$$\nabla\times\mathbf{A} = \mathbf{B} = \frac{hc}{2e}[\mathbf{n}\cdot(\partial_x\mathbf{n}\times\partial_y\mathbf{n})],\ \nabla A_0 = \mathbf{E} = \frac{h}{2e}[\mathbf{n}\cdot(\nabla\mathbf{n}\times\partial_t\mathbf{n})]$$

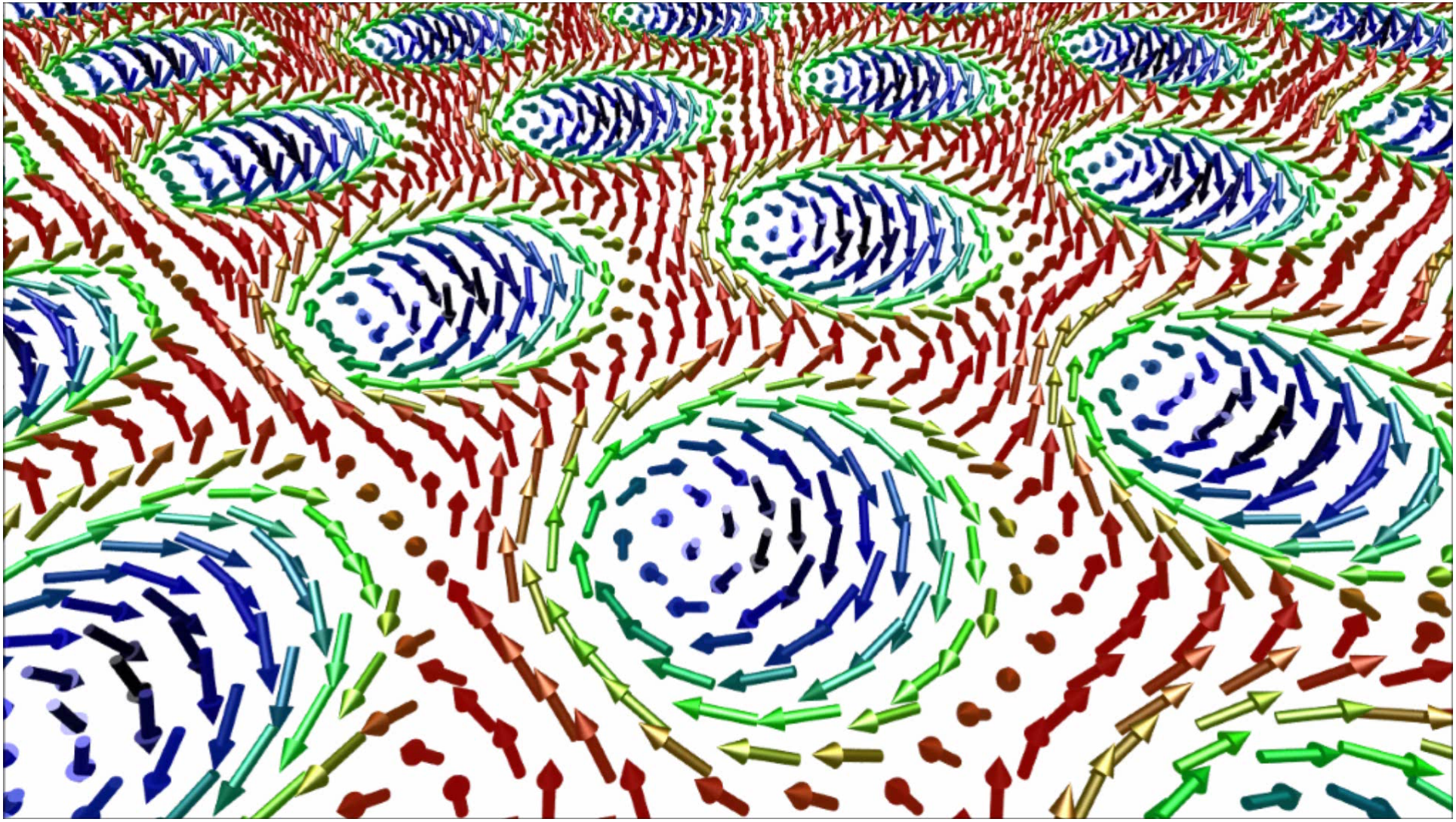
- For skyrmion size 10 nm, B~100 T !!!

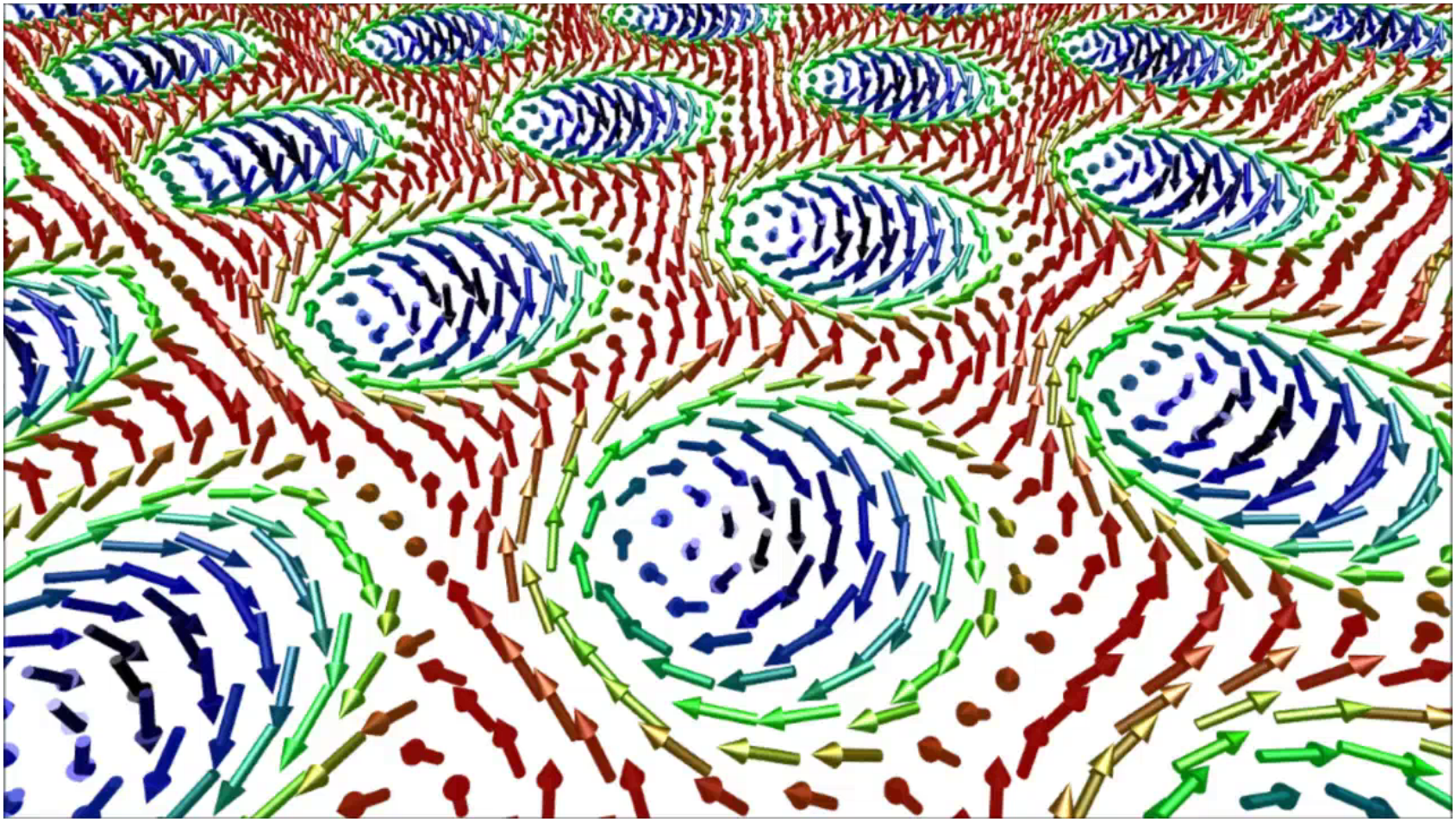
- Topological charge $Q = \frac{1}{4\pi}\int dr^2 \left[\mathbf{n}\cdot(\partial_x\mathbf{n}\times\partial_y\mathbf{n})\right] = \pm 1$

A skyrmion is an emergent mesoscopic particle.

- Radius: 10 ~ 100 nm, containing ~ 100s of spins.
- Energy: ~1 meV.
- Can be driven by current, electric field or temperature gradient.
- Promising applications in spintronics.

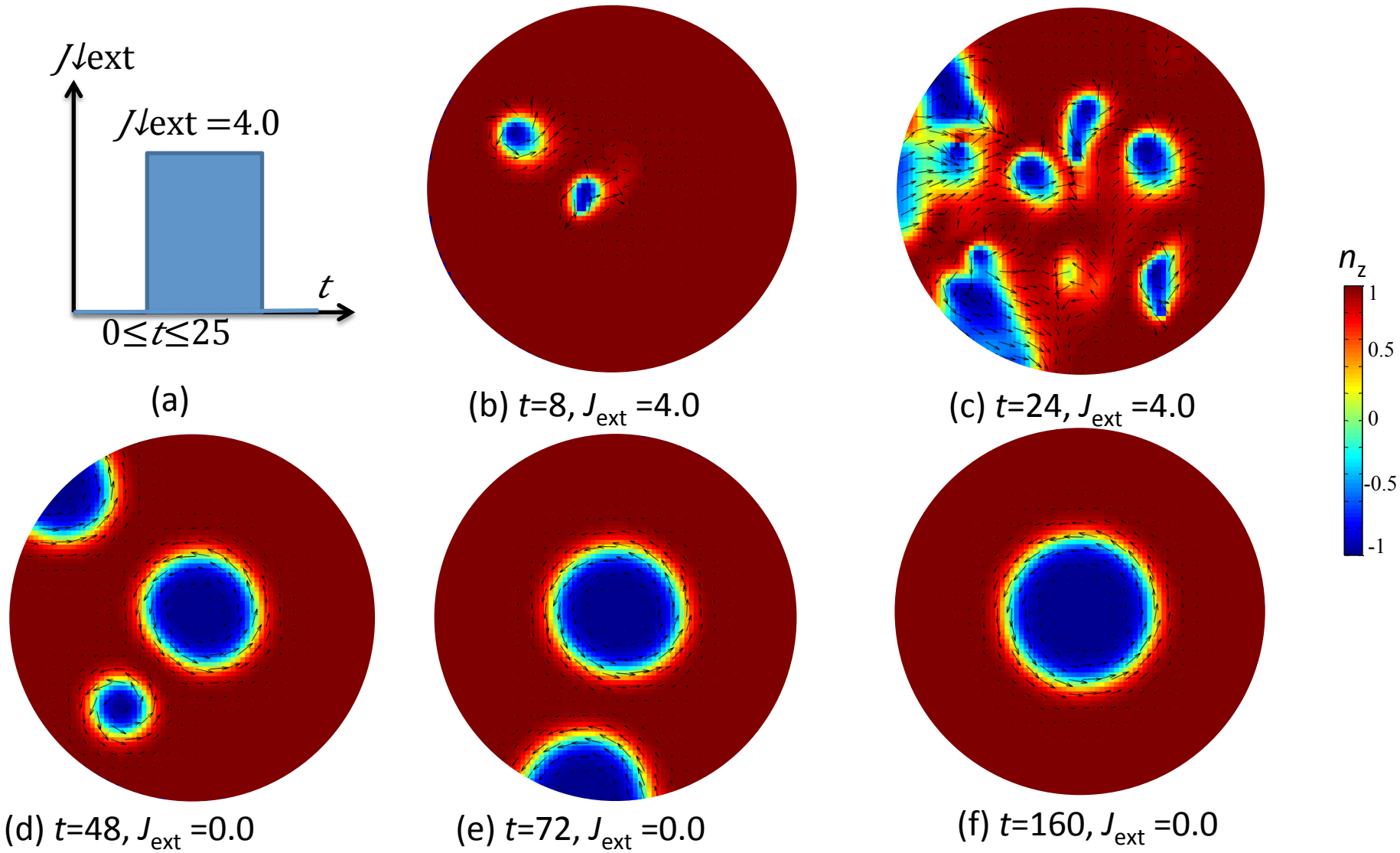
Motion of skyrmion lattice driven by current





Creation of a skyrmion by current pulse:

$J \sim 10^{12}$ A/m^{2}and duration τ>1ns.



SZ Lin, Reichhardt and Saxena, Appl. Phys. Lett. 102, 222405 (2013).

Particle-level description for skyrmions:

- For rigid skyrmions, equation of motion (PRB 87, 2013)

$$\alpha \mathbf{v}_i = \mathbf{F}_M + \mathbf{F}_L + \sum_j \mathbf{F}_d(\mathbf{r}_j - \mathbf{r}_i) + \sum_j \mathbf{F}_{ss}(\mathbf{r}_j - \mathbf{r}_i),$$

Gilbert damping (+electronic conductivity)

Berry phase

Spin transfer torque

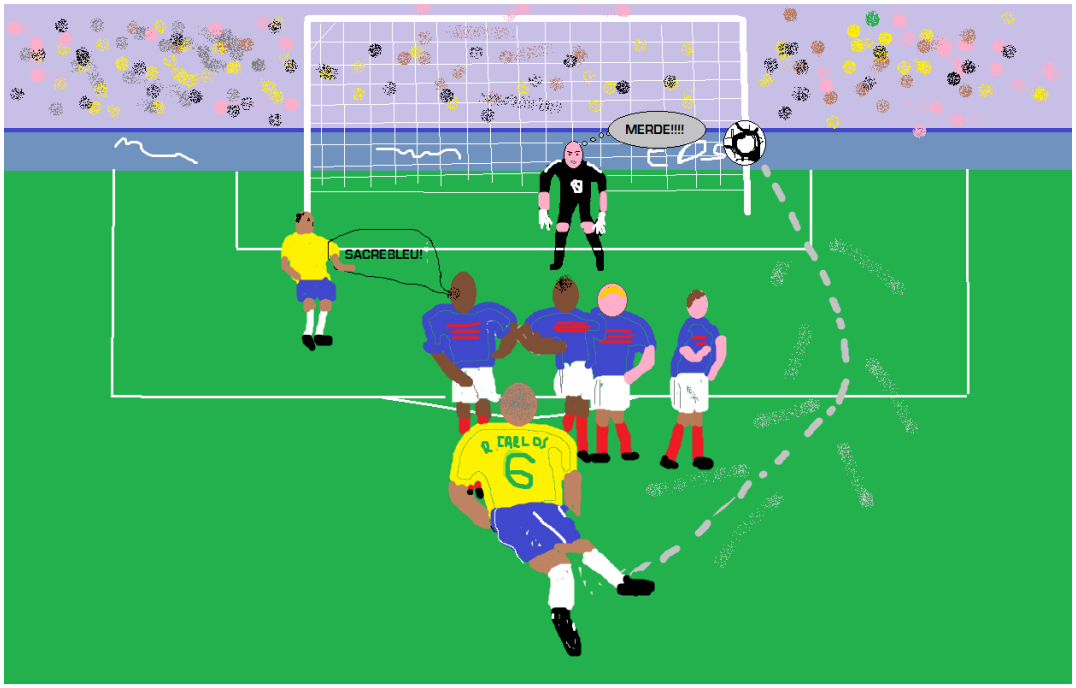
Magnus force $\mathbf{F}_M = 4\pi\gamma^{-1}\hat{z} \times \mathbf{v}_i$, Lorentz force: $\mathbf{F}_L = 2\pi \mathrm{h} e^{-1}\hat{z} \times \mathbf{J}$

- For α=0, Hamiltonian equation of motion: $H = -(J_y - F_x)x + (J_x + F_y)y$

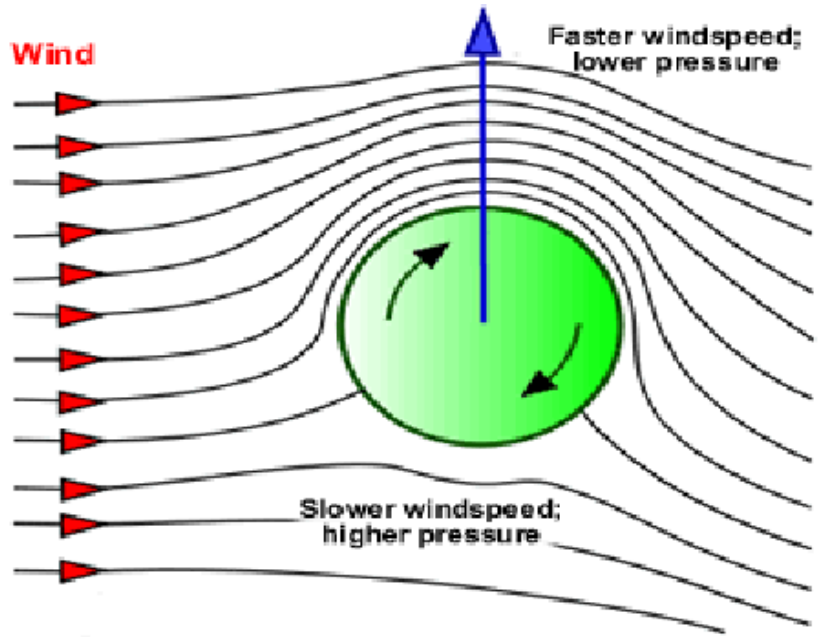
  $\dot{y} = \partial H / \partial x$ and $\dot{x} = -\partial H / \partial y$, x is in conjugate with y!!!
- The topology determines the equation of motion!
- For skyrmions F_M stronger than for vortices
- No intrinsic mass for a rigid skyrmion
- Equivalent to motion of electrons in a strong transverse magnetic field.

PHYSICS OF SOCCER: Magnus force

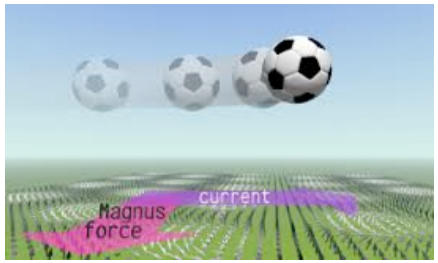
Physics of soccer

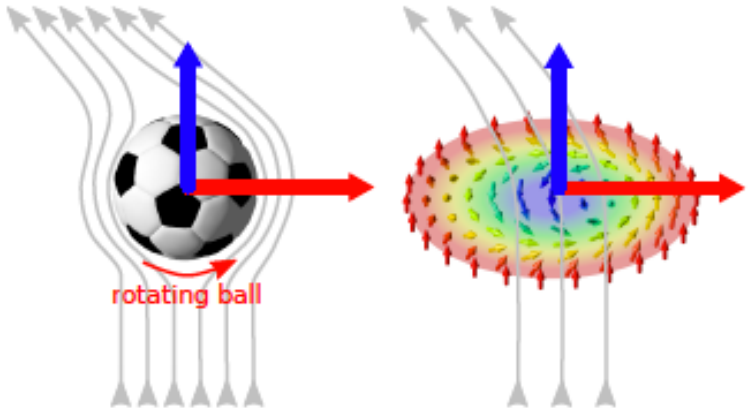


http://valderramarama.com/being-b-art-tards/



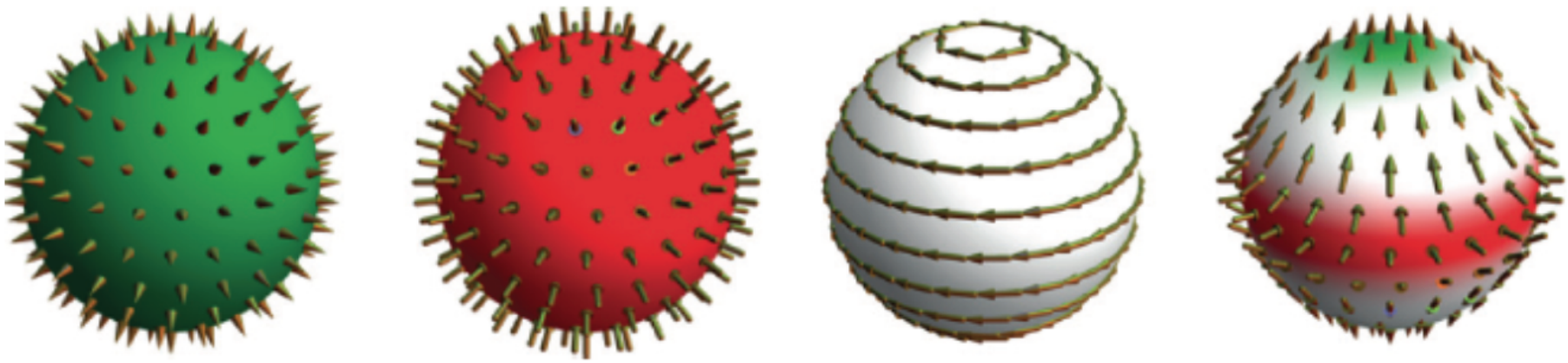
$$\mathbf{F}_M \sim \mathbf{\Omega} \times \mathbf{v}$$





Multipole expansion of magnetic field in a solid where
m is the magnetic dipole moment; a the magnetic monopole;
t the polar toroidal moment (anapole); q the magnetic quadrupole

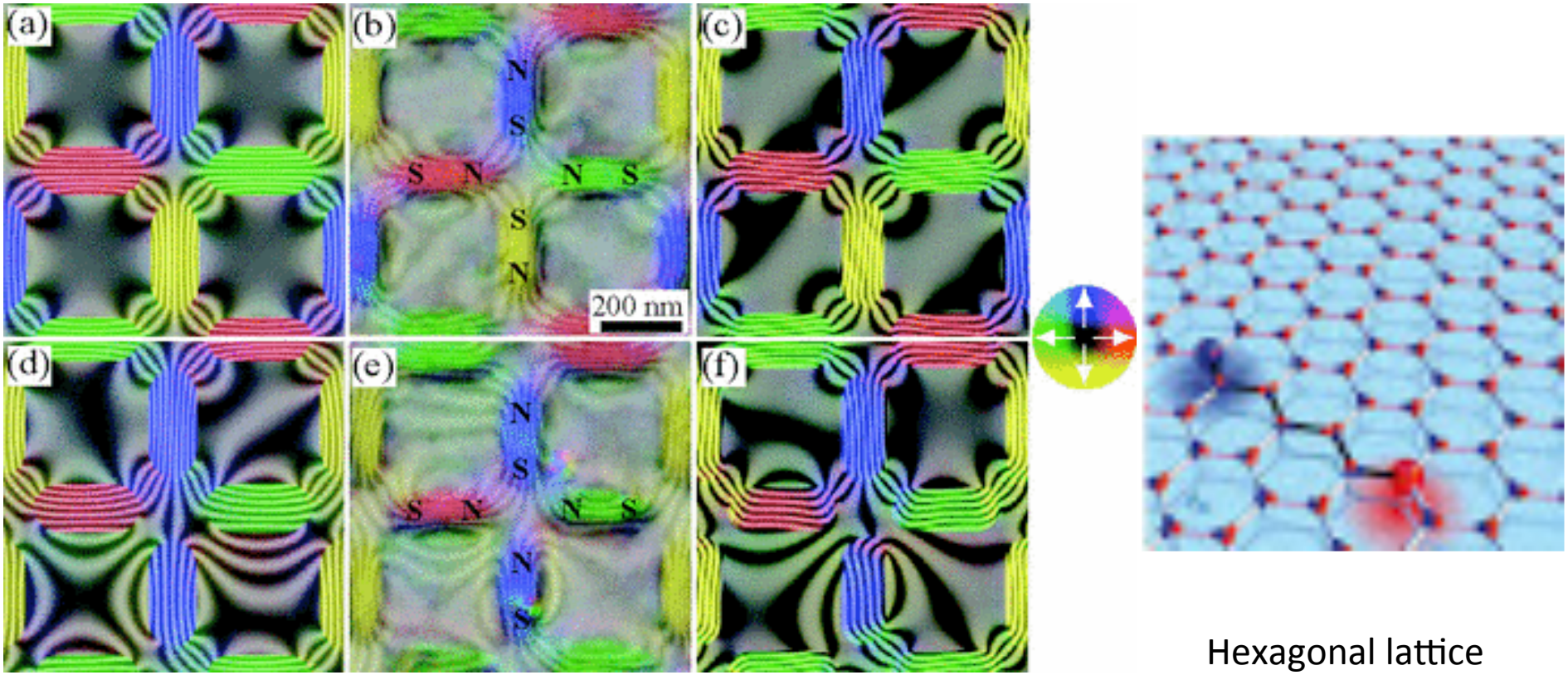
$$H_{\text{int}} = -\boldsymbol{m} \cdot \boldsymbol{H}(0) - a(\nabla \cdot \boldsymbol{H})_{\boldsymbol{r}=0} - \boldsymbol{t} \cdot [\nabla \times \boldsymbol{H}]_{\boldsymbol{r}=0}$$
$$-q_{ij}(\partial_i H_j + \partial_j H_i)_{\boldsymbol{r}=0} - \ldots .$$



+ monopole - monopole z anapole z^2 quadrupole

Dubrovik and Tugushev, Phys. Rev. (2000); Di Matteo, J. Phys. D (2012)
Spaldin et al., Phys. Rev. B (2013)

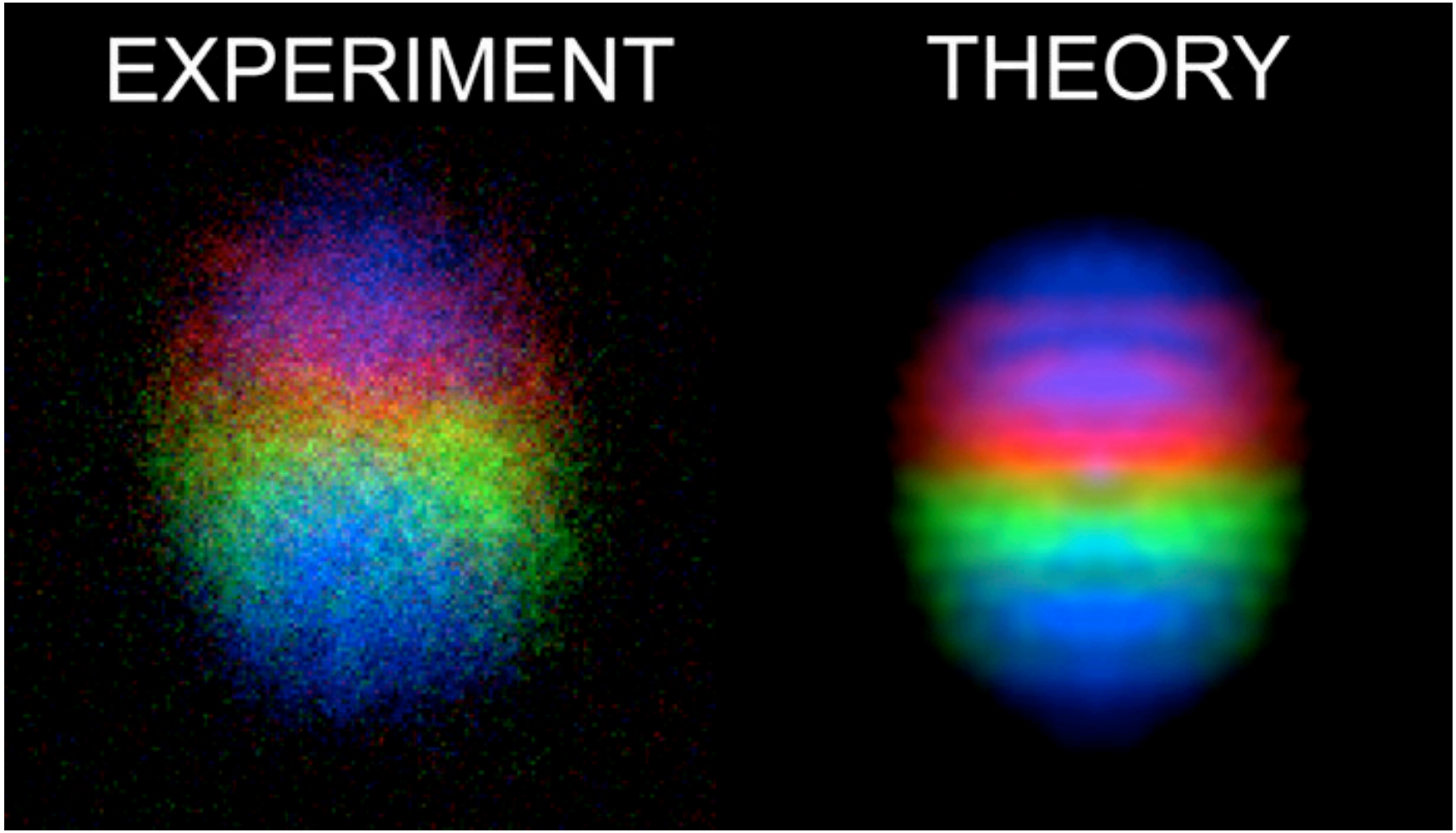
MAGNETIC MONOPOLES IN ARTIFICIAL SPIN ICE



Hexagonal lattice

C. Phatak et al., Phys. Rev. B 83, 174431 (2011)

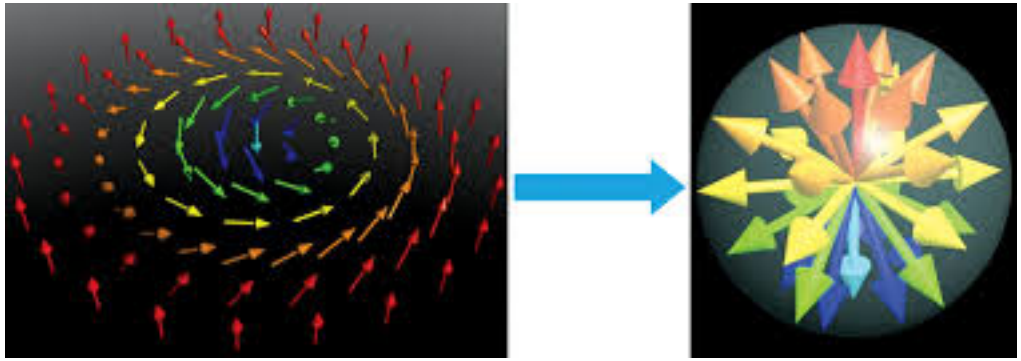
Observation of Monopoles: BEC

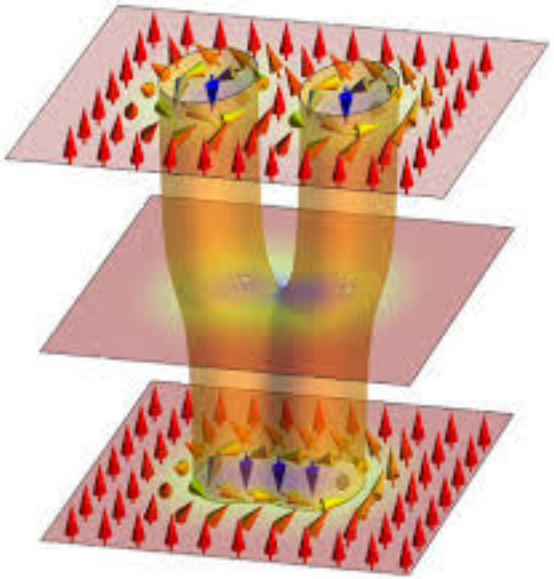


M.W. Ray et al., Science 1258289 (2015)

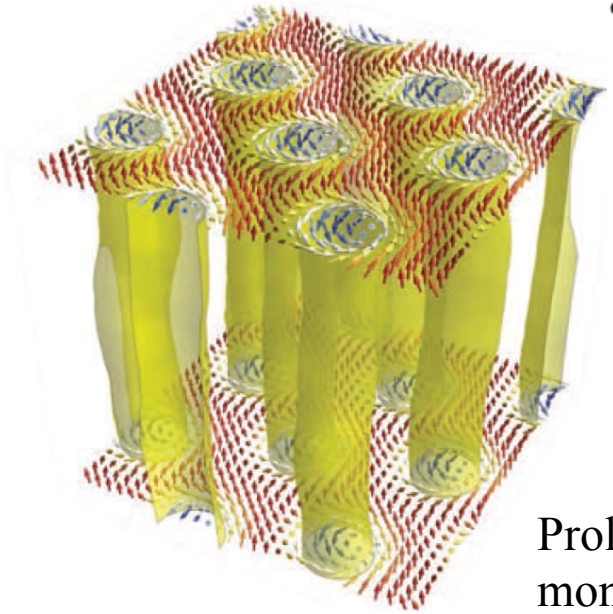
Rb atoms in a synthetic magnetic field

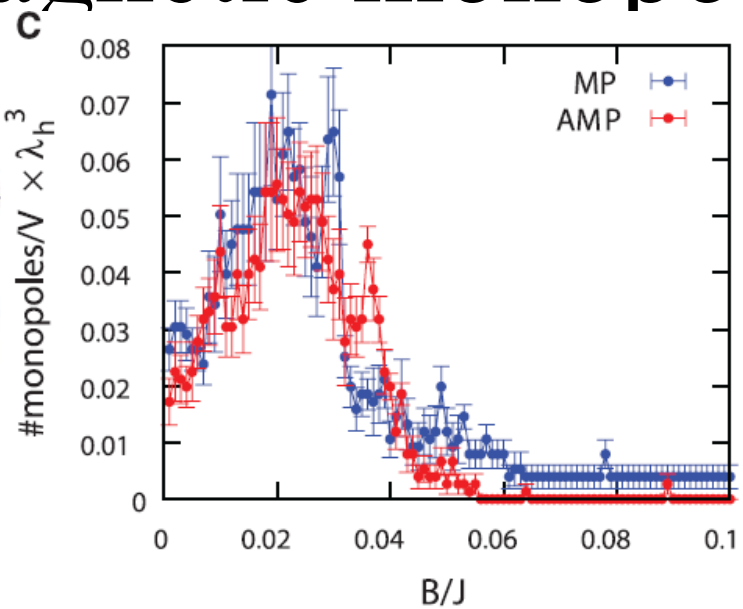
FIELD OF A MAGNETIC MONOPOLE

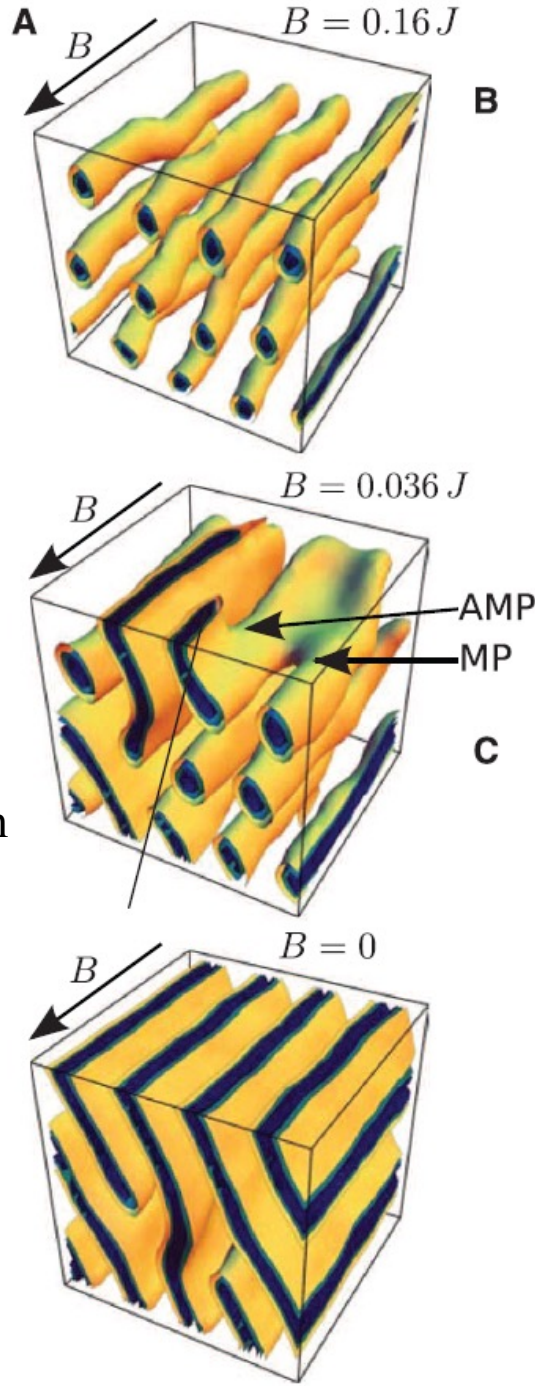




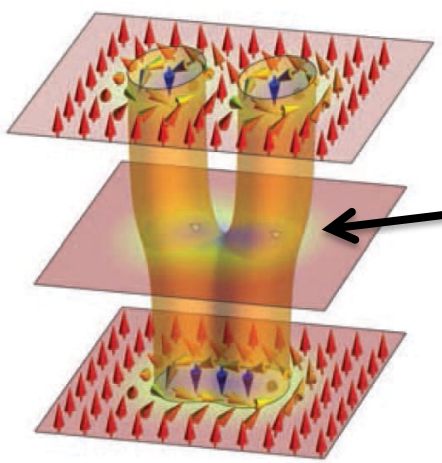
Emergent magnetic monopo

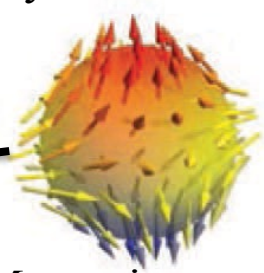






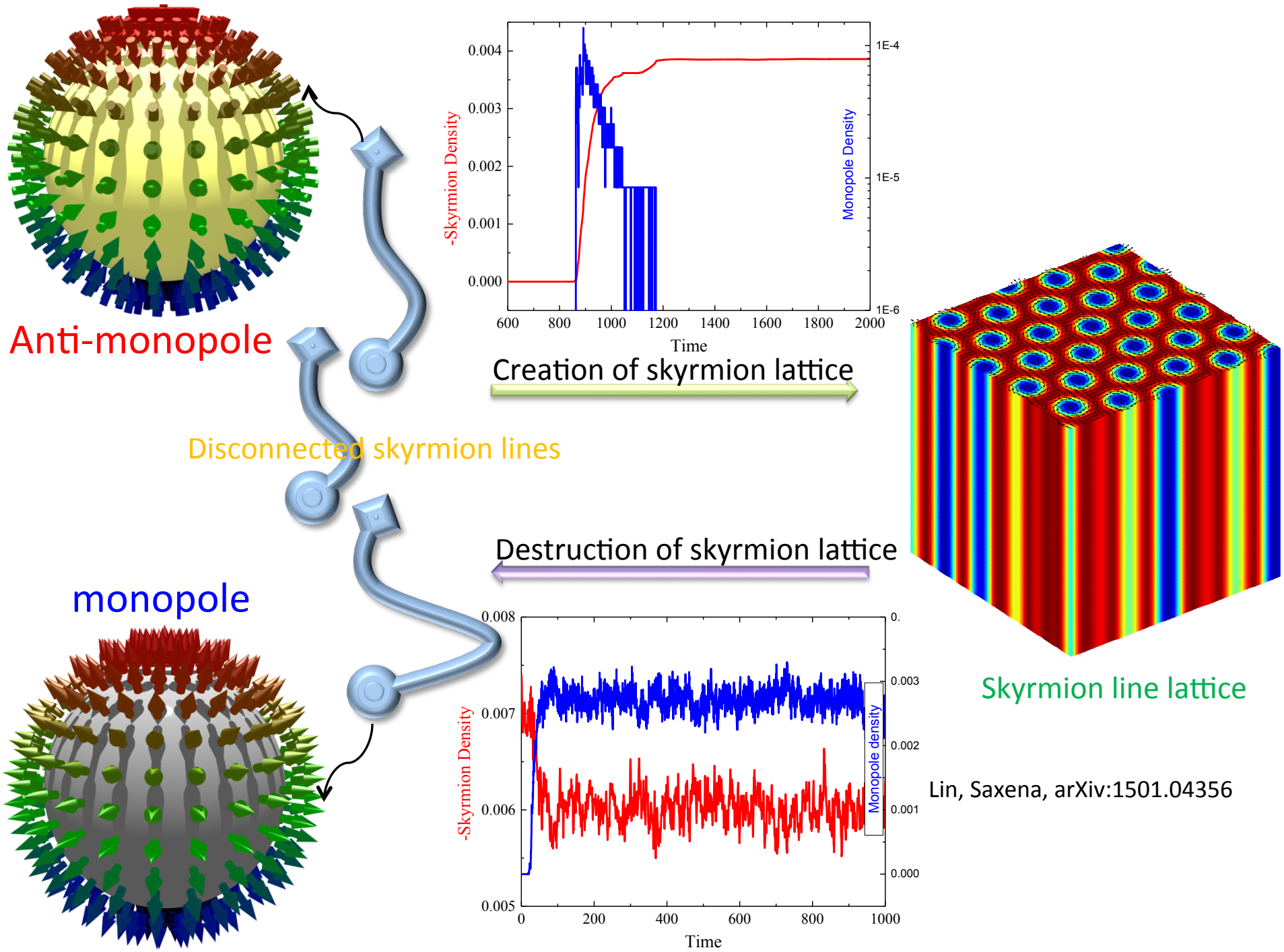
Proliferation of monopole (MP) and anti-monopole (AMP) changes the system from skyrmion lattice phase to the spiral phase.





Magnetic monopole

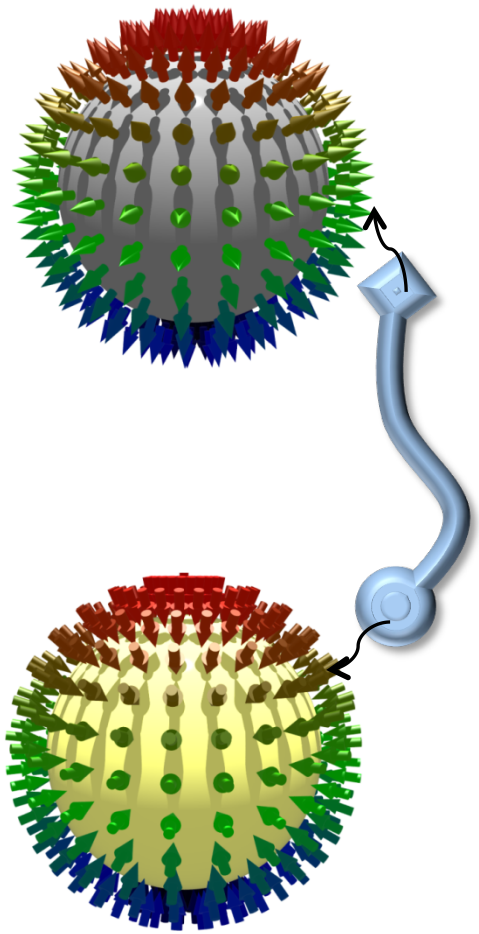
P. Milde *et al*, Science **340**, 1076 (2013).



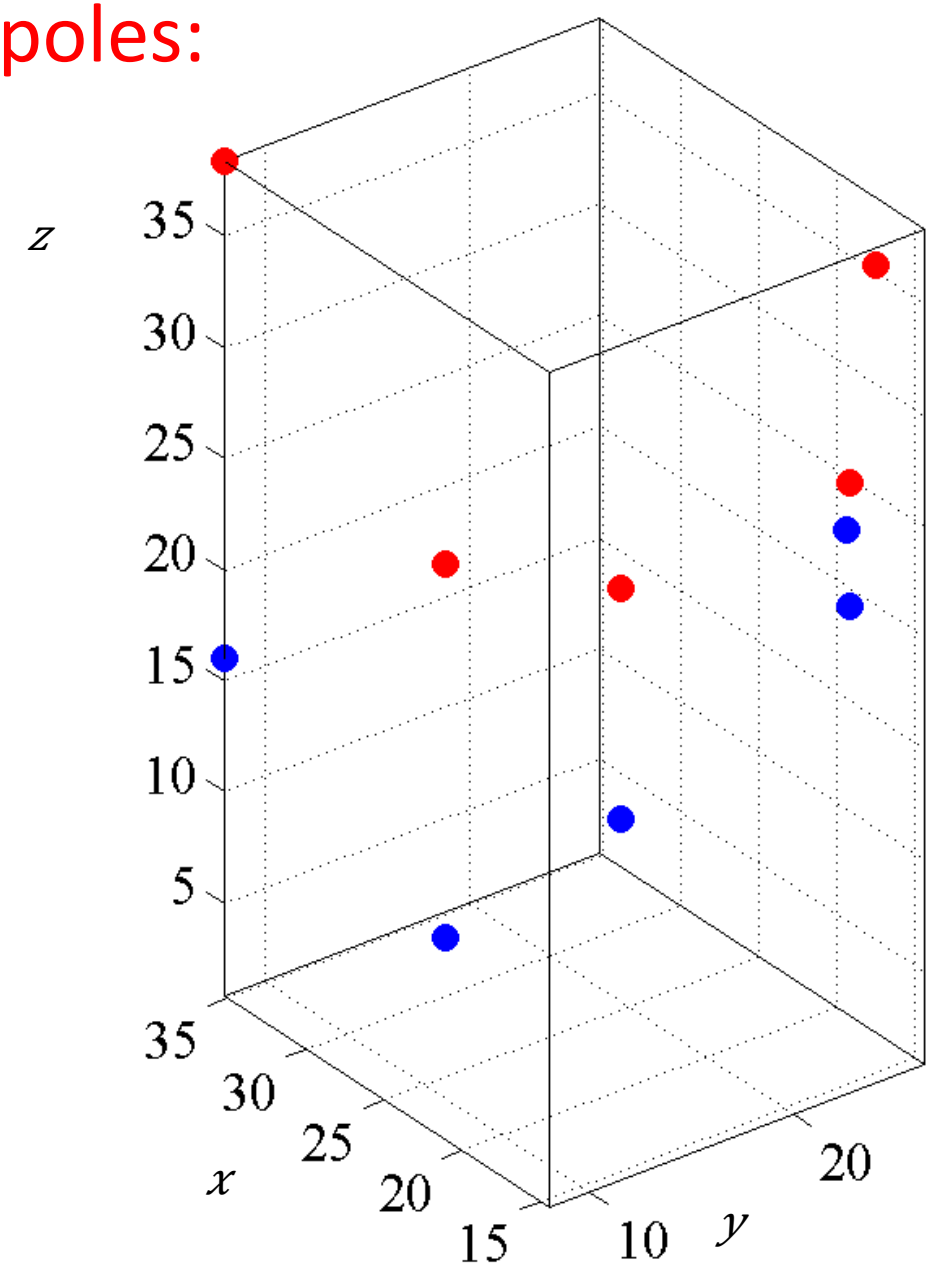
Lin, Saxena, arXiv:1501.04356

Emergent monopoles:

- Skyrmion line in metal can be regarded as emergent magnetic flux line.
- It allows for the existence of monopole, i.e. $\nabla \cdot \mathbf{B}^E = \rho_m$ with $\rho_m \neq 0$
- A skyrmion line segment can is a realization of emergent Dirac string, where there are monopole and antimonopole at the two ends.

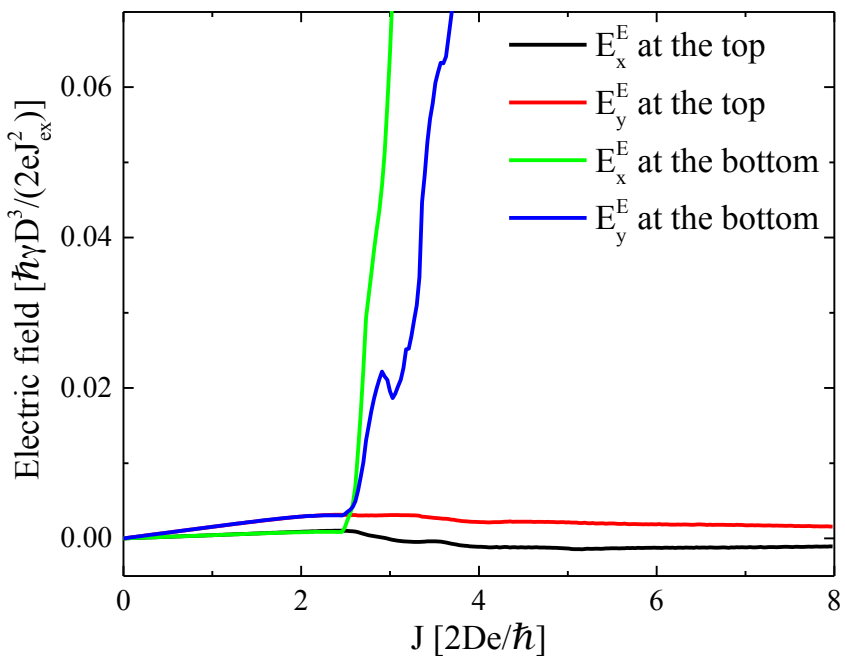


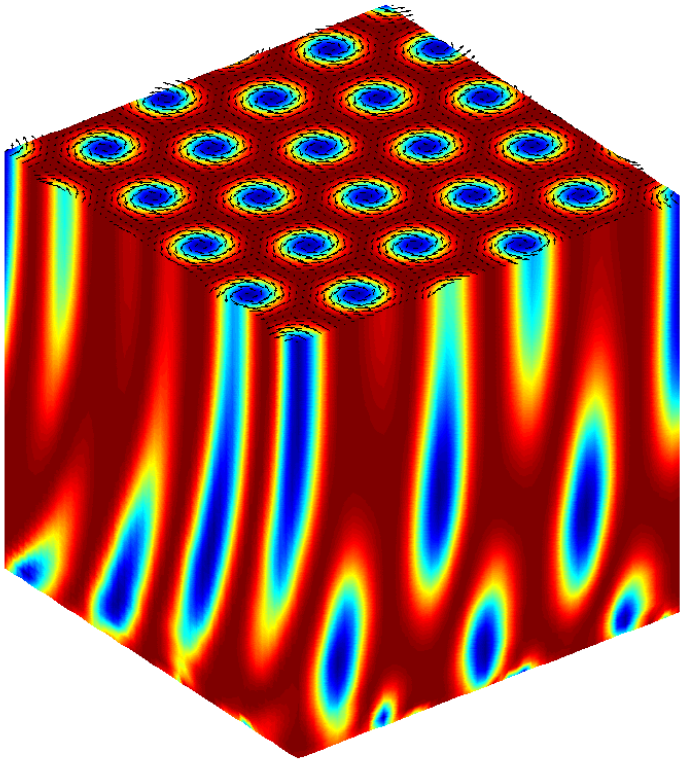
Typical distribution of monopoles and anti-monopoles:

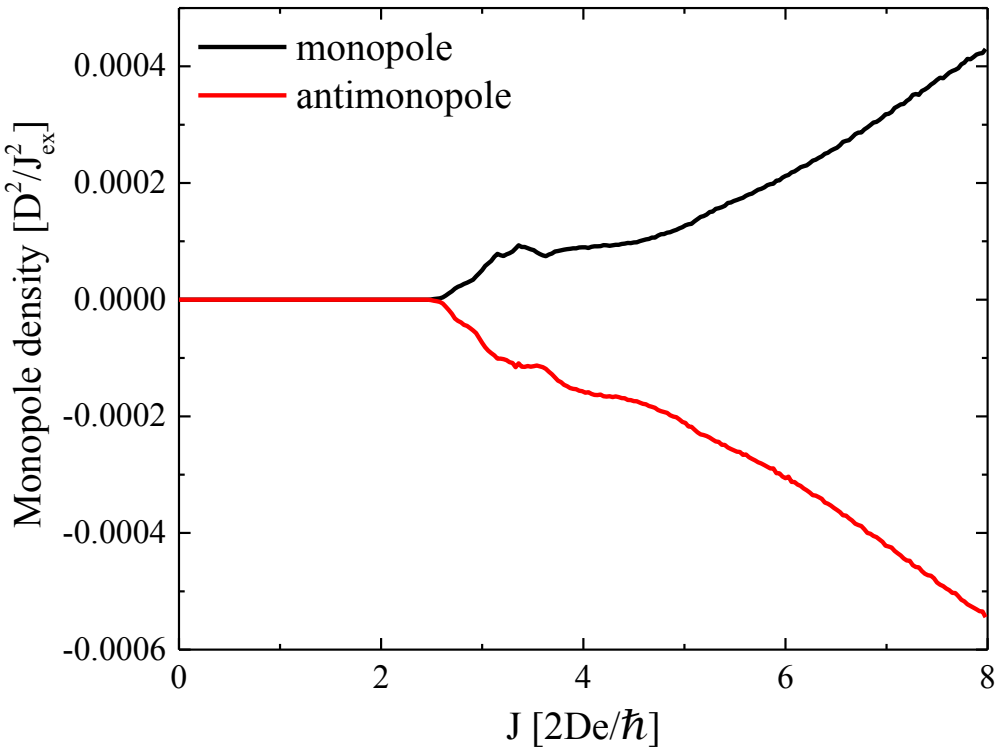


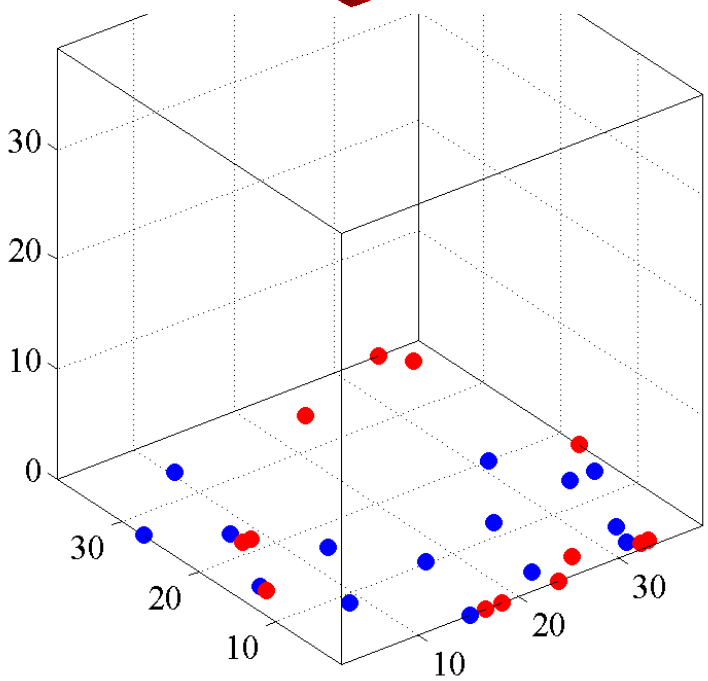
Controlled creation of monopole

- The skyrmion line at the bottom experiences a bigger driving force.
- For a small current, the force differences is balanced by the elastic force of the skyrmion line.
- For a large current, skyrmion line breaks accompanying the creation of monopoles and antimonopoles.









Spin configuration and monopole distribution when skyrmion lines break.

Comparison

	skyrmion	Vortex in SC	domain walls (kink)
Homotopy class	$\pi_2(S^2)$	$\pi_1(S^1)$	$\pi_0(S^0)$
Emergent EM fields	Yes	No	No
Thermodynamic phase	Triangular lattice	Triangular lattice in most cases	1D array
Pair interaction	Short-range	Short-range in bulk Long-range in films	Short-range
Current drive	In metals yes	yes	In metal yes
Equation of motion	Mangus force dominates	Dissipative force dominates	Dissipative force dominates
Pinning	Weak	Strong	Strong
Unstable at high current drive	Yes,	Yes,	Yes,
Magnetoelectric coupling	In insulators yes	No	In insulator yes
Existence at room temperature	Yes	Not realized yet	Yes