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SOURCE MODEL

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THEORY OF HARMONIC RADIATION USING A SINGLE-ELECTRON SOURCE MODEL *

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ABSTRACT

Significant progress has recently been made toward the understanding of the various mechanisms that generate harmonic radiation in plane-polarized free electron lasers. Within the context of a single-frequency coherent-spontaneous emission model, a distributed transverse source function for a single electron has been derived. This source is multiply peaked, with the number of peaks being equal to the harmonic number. The peaks and nulls in the radiation source are analogous to the radiation peaks seen in the spontaneous radiation pattern of a single electron. When the distributed source function is averaged over transverse space, the simplified one-dimensional results are recovered. The distributed source function model predicts the generation of even harmonic radiation with odd-symmetry in the electron wiggle plane (for electrons traveling along the wiggler axis) and odd harmonic radiation patterns with even transverse symmetry.

A method for modeling the multi-pole nature of the harmonic radiation on a discrete grid is described. When the transverse electron beam distribution is slowly varying, all the harmonics can be adequately modeled with multi-poles having only a few peaks. This model has been incorporated into the 3-D FEL simulation code FELEX. Simulations of the Los Alamos and Stanford FEL oscillators have been performed. How the harmonic transverse spatial electric field profiles change for different operating conditions is examined.

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I. Introduction

Radiation at the harmonic frequencies is affected not only by the magnitude of the wiggler magnetic vector potential[1] but also by several other factors including transverse gradients of the electron beam density and the wiggler magnetic field. Additional coupling to the harmonic frequencies can also occur when the electron, optical and wiggler axes are misaligned. Such misalignment can result from gross electron beam offsets, wandering of the electron beam inside the wiggler due to magnetic field errors and betatron motion of the single-electron trajectories.

The dynamics of harmonic radiation in free electron lasers (FELs) differs fundamentally from other laser configurations. The primary difference lies in the macroscopic transverse excursion an electron makes as it radiates. This transverse excursion in conjunction with fluctuations of the electrons' radiation phase gives rise to radiation patterns that can be multiply peaked[2-4]. Specific source patterns arise for each harmonic due to the correlation of the electron's radiation phase with specific transverse locations relative to an electron's guiding center location. Previous models[5,6] ignore these fluctuations by assuming the electron radiates at only its guiding center position. This assumption is valid in only certain limiting cases. The primary consequence of ignoring these fluctuations is the omission of a source term that gives rise to radiation at the even harmonics with a radiation pattern that has odd-symmetry. In this analysis the wiggle motion of the electrons is tied to its radiation phase such that a distributed radiation source is obtained that differs for each harmonic. The theoretical formalism for this approach has been published elsewhere[4] and only the results of the formalism, and its implementation in numerical simulation codes will be discussed here.

In Section II a brief overview of the single-electron source model is given. The relation of this model to previous analytic models[5,6] and to the discrete model used in the simulation code FELEX[7] is described. In Section III numerical simulations of the harmonic emission of the Los Alamos[8] and Stanford Mark III[9,10] FEL oscillator experiments are discussed. The conclusions are given in Section IV.

II. The Harmonic Model

The paraxial approximation is ideally suited for the analysis of the tightly collimated coherent emission from FELs. Within the confines of this approximation, the wave equation

for the linearly polarized electric field from a plane polarized wiggler has the form[4]

$$\left[2ifk_s \frac{d}{dz} + \frac{\partial^2}{\partial \bar{r}_\perp^2}\right] E_\perp^f = -\frac{i8\pi f k_s}{\lambda_s c} \int_z^{z+\lambda_s} d\zeta J_\perp(x, y, \zeta) e^{-if[k_s \zeta - \omega_s t]} \quad (1)$$

where $k_s = 2\pi/\lambda_s = \omega_s/c$ is the optical wavenumber, c is the speed of light, f is the harmonic number and $J_\perp$ is the transverse current given by

$$J_\perp(x, y, z(t)) = -ec \sum_{i=1}^{N_{pond}} \frac{a_{wi}}{\gamma_i} \delta(z - z_i(t)) \delta(x - x_{0i} - \beta_{x0i}z) \delta(y - y_{0i} - \chi_i \sin k_w z - \beta_{y0i}z) \quad (2)$$

The sum over i represents the contributions from all the electrons in a ponderomotive wavelength ($\lambda_{pond}^{-1} = \lambda_s^{-1} + \lambda_w^{-1}$). Here we have assumed a plane polarized wiggler magnetic field with period λ_w and amplitude $\vec{B} = \hat{x} B_w \sin k_w z$. The electrons have charge, $q = -e$, and energy, γmc^2 , such that they wiggle in the y -direction with their transverse positions given by $y = y_{0i} + \chi_i \sin k_w z + \beta_{y0i}z$ where $\chi_i \simeq \frac{a_w(x_{0i}, y_{0i})}{\gamma_i k_w}$ is the oscillation amplitude of the i^{th} electron and a_w is the wiggler vector potential given by

$$a_w = \frac{|e|B}{\gamma mc^2 k_w} \quad (3)$$

The quantities (x_{0i}, y_{0i}) and $(\beta_{x0i}, \beta_{y0i})$ are the guiding center position and transverse drift velocities of the i^{th} electron. To obtain a source function dependent on the electron's transverse y position alone, we average over a wiggler period yielding[4]

$$S_i^n(y) = \frac{i8\pi f k_s k_w}{\lambda_s} e^{-if\psi_i} \cos\{f[\xi \sin 2\theta_r(y) + \theta_r(y)]\} e^{if\sigma \sin \theta_r(y)} \quad (4)$$

for the odd harmonics and

$$S_i^n(y) = -\frac{8\pi f k_s k_w}{\lambda_s} e^{-if\psi_i} \sin\{f[\xi \sin 2\theta_r(y) + \theta_r(y)]\} e^{if\sigma \sin \theta_r(y)} \quad (5)$$

for the even harmonics where ξ is the interaction strength parameter given by

$$\xi = a_w^2/(4 + 2a_w^2) \quad (6)$$

and σ is the angular coupling factor defined

$$\sigma = 8\xi \left(\frac{\beta_{y0i}}{a_w/\gamma} \right) \quad (7)$$

Note that the bracketed term in Eq.(7) is a ratio of the transverse drift velocity of an electron to its peak wobble velocity. This ratio must be small for these results to be valid. We can then include the transverse drift contribution to the optical phase modulation, and ignore the radiation source due to the drift current beating against the modulated optical phase. Justification for this assumption is given in reference [4]. The transverse source functions in Eqs.(4) and (5) are a function of $\theta_r(y)$ given by

$$\theta_r(y) = \sin^{-1} \left(\frac{y - y_{0i}}{\chi_i} \right) \quad y_{0i} - \chi_i \leq y \leq y_{0i} + \chi_i \quad (8)$$

where we explicitly express the limits of $\theta_r(y)$, confining them to the transverse range of each electron. The source functions are complex quantities except when the electron is perfectly aligned with the wiggler axis ($\sigma \propto \beta_y \Rightarrow 0$). Plots of the source function for the second harmonic are given in Figure 1 in the limit $\sigma = 0$, i.e., zero transverse drift velocity. The three curves represent the source for $\xi = 10^{-5}$ (solid curve), $\xi = 1/6$ (dotted curve), and $\xi = 1/2$ (dot-dash curve). These values of ξ correspond to a_w values of .01, 1.0 and ∞ , respectively. The electron guiding center is at the point where all three curves cross the horizontal axis and the width of the curve reflects the transverse wobble range of the electron. Note that the peak of the curves moves closer to the guiding center position as the wiggler vector potential is increased. Plots of the source functions including angular effects with $\sigma = \xi = 1/6$ are given in Figures 2(a-c) for the fundamental, second and third harmonic. Since the source functions are now complex, we have plotted their real and imaginary parts with dot-dash and dotted lines, respectively. For reference, the solid line plots the source function for $\sigma = 0$. Note that the real part of the source function is only slightly modified from its $\sigma = 0$ value despite the large value of σ assumed for these calculations.

One-dimensional numerical models assume one discrete source for each electron located at the electron's guiding center position. The amplitude of this source is equal to the transverse average of the distributed source functions of Eqs.(4) and (5). This average is given by

$$\bar{S}_i = \int_{y_{0i} - \chi_i}^{y_{0i} + \chi_i} dy S_i(y) \quad (9)$$

Performing this average one finds

$$\bar{S}_i = \frac{i4\pi cfk_s a_{w1}}{\lambda_s} \chi_f^{(1)}(\xi, \sigma) \frac{e^{-if\psi}}{\gamma} \quad (10)$$

where

$$\mathcal{K}_f^1(\xi, \sigma) = (-1)^{f-1} \sum_{n=-\infty}^{\infty} J_n(f\xi) [J_{2n+f-1}(f\sigma) + J_{2n+f+1}(f\sigma)] \quad (11)$$

For an electron traveling on the wiggler axis, $\sigma = 0$, and the coupling coefficient given in Eq.(11) becomes

$$\mathcal{K}_f^{(1)}(\xi) = (-1)^{\frac{f-1}{2}} \left\{ J_{\frac{f-1}{2}}(f\xi) - J_{\frac{f+1}{2}}(f\xi) \right\} \quad f \text{ odd} \quad (12)$$

where the average sources for the even harmonics have vanished. Therefore, in the one-dimensional limit, the single-electron even-harmonic sources can only have non-zero averages caused by transverse drift velocities (angular effects). The coupling coefficients given in Eqs.(11) and (12) were originally derived by Colson[5,6] using a different technique.

We now wish to more accurately model the radiation pattern of an ensemble of electrons by retaining the distributed transverse source dependence of the individual electrons. This can be done by converting the smoothly varying distributed source functions of Eqs.(4) and (5) into discrete (δ -function) sources. For simulation purposes, these discrete sources can then be interpolated onto a numerical grid. The lowest order discrete source for an electron is just a δ -function located at the guiding center position of the electron. This term is referred to as the monopole source term. We saw previously that the amplitude of this source should be given by the transverse average of the analytic source functions. As seen from Eq.(12) the lowest order amplitude for the even harmonics in the limit $\sigma \Rightarrow 0$ is zero. To obtain a nonvanishing amplitude for the even harmonics in this limit we must go to higher order. The next order discrete source for an electron would be a pair of δ -functions in an odd configuration as shown in Figure 3. This term is referred to as the dipole source term. The amplitudes for this source and the third order discrete source (tripole) have been derived elsewhere[4]. The wave equation including these three discrete source terms can now be written

$$\left[2ifk_s \frac{d}{dz} + \frac{\partial^2}{\partial r_\perp^2} \right] E_\perp' = \sum_j^{1,2,3} \mathcal{S}_j(y) \quad (13)$$

where the three source terms on the right-hand side are given by:

- Mono-pole term

$$\mathcal{S}_1(y) = \left\langle \frac{i4\pi I f k_s a_{w0}}{c} \mathcal{K}_f^{(1)}(\xi, \sigma) \delta(y - y_{w0}) \frac{e^{-if\psi}}{\gamma} \right\rangle, \quad (14)$$

- Di-pole term

$$S_2(y) = \left\langle \frac{4\pi I f k_s a_{wi}}{c} \mathcal{K}_f^{(2)}(\xi, \sigma) \frac{1}{2} [\delta(y - y_{0i} - \chi/2) - \delta(y - y_{0i} + \chi/2)] \frac{e^{-if\psi}}{\gamma} \right\rangle_e \quad (15)$$

- Tri-pole term

$$S_3(y) = - \left\langle \frac{i4\pi I f k_s a_{wi}}{c} [\mathcal{K}_f^{(1)}(\xi, \sigma) - \mathcal{K}_f^{(3)}(\xi, \sigma)] \times \left\{ \delta(y - y_{0i}) - \frac{1}{2} [\delta(y - y_{0i} - \chi/2) + \delta(y - y_{0i} + \chi/2)] \right\} \frac{e^{-if\psi}}{\gamma} \right\rangle_e \quad (16)$$

where

$$\mathcal{K}_f^{(m)}(\xi, \sigma) = (-1)^f \sum_{n=-\infty}^{\infty} J_n(f\xi) [(-1)^m J_{2n+f-m}(f\sigma) - J_{2n+f+m}(f\sigma)] \quad (17)$$

Including higher order source terms in the sum in Eq.(13) will not significantly improve the accuracy of the calculation since these terms are effectively averaged out by the electron ensemble, assuming the transverse density profile is smoothly varying.

III. Numerical Simulations

The discrete distributed source model summarized in Eqs (13)-(17) has been implemented in the 3-D code FELEX[7]. The single phase front model assumes that each electron radiates monochromatically at each harmonic frequency. This is a good approximation when modeling coherent-spontaneous emission where the electrons are bunched by a strong fundamental wave. Special care was taken to insure that the proper coupling of each electron to each harmonic was achieved. As can be seen from Eq.(17) the coupling coefficient for each electron is dependent on the angle the electron makes with the wiggler axis through σ . This angle varies as the electron travels through the wiggler due to electron beam misalignment, betatron motion, wiggler field errors and electron beam focusing. In order to calculate the proper coupling coefficient, the instantaneous guiding center angle is calculated for each electron as it travels through the wiggler. As the angle for each electron changes, the coupling coefficient for each electron for each harmonic is modified by choosing the appropriate tabulated value.

Using the discrete single-electron multipole sources, simulations were conducted for the Stanford Mark III FEL and the Los Alamos FEL oscillator. Typical parameters assumed in the simulations were:

- Mark III FEL

$\gamma = 86.11$, $\Delta\gamma/\gamma = .3\%$, $\epsilon_x = .087\pi\text{mm-mrad}$, $\epsilon_y = .032\pi\text{mm-mrad}$, $I = 30$. Amps, $\lambda_w = 2.3\text{cm}$, $B_w = 6570\text{Gauss}$, $L_w = 108$. cm, $\lambda_p = 3.16\mu\text{m}$, $P_{\text{circ}} = 40\text{MW}$, $z_r = 73.6\text{cm}$, $G = 33\%$, $\eta = .98\%$

- Los Alamos FEL

$\gamma = 46.$, $\Delta\gamma/\gamma = 1.2\%$, $I = 400$. Amps, $\epsilon = 2.1\pi\text{mm-mrad}$, $\lambda_w = 2.73\text{cm}$, $B_w = 3927\text{Gauss}$, $L_w = 100$. cm, $\lambda_p = 10.36\mu\text{m}$, $P_{\text{circ}} = .5\text{GW}$, $z_r = 49.5\text{cm}$, $G = 16\%$, $\eta = .84\%$

Comparison of the power produced at the second through seventh harmonic for the Stanford Mark III FEL were initially conducted. The simulation results were in good agreement with the experimental measurements for the first four harmonics as reported by Bamford and Deacon[10]. Additional simulations were conducted to explore the effects of wiggler field errors on the level of harmonic power. For the Stanford Mark III FEL a rms field error level of 1.0% gave little or no degradation in harmonic power. This result persisted as long as the field error level was small enough to keep the fundamental gain in the wiggler within 90% of its zero field-error level.

Various simulations were conducted to study how the shape of the radiation pattern at the various harmonics was affected by different operating conditions. In Figure 4a-c, the *magnitude* of the second harmonic transverse electric field profiles at the exit of the wiggler for the Stanford Mark III oscillator are given for three different electron beam configurations. When the electron beam is perfectly aligned, the double peaked profile given in Figure 4a is obtained. This profile has odd symmetry in the electron's wiggler plane (where the negative lobe has been inverted for plotting purposes). The profiles in Figures 4b and 4c show a progressive decrease in one of the electric field lobes caused by an increasing offset of the electron beam of 0.25 and 0.75mm, respectively, at the entrance to the wiggler in the electron wiggler plane. This induces an electron beam slew across the wiggler axis. More complex beam slewing can also be induced by wiggler field errors. Since the wandering (slewing) of the electron beam due to field errors is more complicated than a linear slew, the resulting harmonic radiation patterns will undoubtedly be complex. Spatial profiles of the harmonic intensity have been measured on the Stanford Mark III oscillator[11]. The experimental plots do not show the odd symmetry predicted in the simulation for the second (even) harmonic. Causes for this discrepancy could be due to

diagnostic averaging of the intensity over several macropulses (hundreds of micropulses) such that electron beam jitter averages away the narrow null at the center of the pattern. Additionally, the relatively large aperture used for the experimental measurement may have made it impossible to resolve the narrow null region.

Analogous simulations have been conducted for the Los Alamos FEL oscillator experiment with qualitatively similar results due to electron beam misalignment. Experimental measurements of the second harmonic transverse intensity profiles were made[8] and are shown in Figure 5a-c. These profiles vary from fairly symmetric looking doubled peaked shapes as in Figure 5a, to single peaked, offset profiles as seen in Figure 5c which occur when the electron beam slews through the wiggler. These profiles are in excellent qualitative agreement with the simulation results. Closer agreement cannot be achieved unless the exact dynamics of the electron beam in the wiggler is known.

IV. Conclusions

A discrete monochromatic single-electron distributed source model has been implemented in the three-dimensional code FELEX to more accurately model the harmonic radiation processes in free-electron lasers. The model includes modifications to the radiation caused by misalignment between the wiggler axis and an individual electron's guiding center trajectory. Simulations show that the odd-harmonics are radiated as gaussian-like modes with even transverse symmetry, while the even-harmonics are radiated in modes with odd transverse symmetry in the wiggle plane. The odd symmetry of the even-harmonics is easily broken by misalignment of the electron beam and wiggler axes. For misalignments on the order of the electron beam radius, the second harmonic transverse profiles becomes singled-peaked. These simulations have been qualitatively verified by experimental observations from the Los Alamos FEL oscillator. Closer comparisons of experiment and simulation will require more accurate measurement of the electron beam location in the wiggler and resolution of the transverse harmonic intensity profiles on a micropulse time scale.

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Figure 1. Plots of the normalized transverse source dependence in the limit $\sigma = 0$ for the second harmonic. The three curves represent the source for $\xi = 10^{-5}$ (solid curve), $\xi = 1/6$ (dotted curve), and $\xi = 1/2$ (dot-dash curve). These values of ξ correspond to a_w values of .01, 1, and ∞ , respectively.

Figure 2(a-c). Plots of the source functions including angular effects with $\sigma = \xi = 1/6$ for the fundamental, second and third harmonic. The real and imaginary parts have been plotted with dot-dash and dotted lines, respectively. For reference, the solid line plots the source function for $\sigma = 0$. Note that the large imaginary part introduces a source with opposite symmetry from the $\sigma = 0$ reference case.

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with the Wiggler Vector Potential

Plots of
distributed source
function
 $\sin\{2[\xi \sin 2\theta + \theta]\}$
where
 $\xi \approx \frac{a_w^2}{4(1+a_w^2/2)}$
and
 $\theta = \sin^{-1}(y)$

Collapsed Source Function for the Fundamental **Including Large Angle Coupling**

Figure 2a

Collapsed Source Function for the Second Harmonic

Including Large Angle Coupling

Collapsed Source Function for the Third Harmonic

Including Large Angle Coupling

Discrete Second Harmonic Distributed Single-Electron Source

~~(b)~~

Figure 3

(a)

(b)

(c)

Figure 4

Los Alamos Fel Oscillator

Experimental Transverse Second Harmonic Intensity Profiles

**Experimental Transverse Second Harmonic Intensity Profiles
with Fundamental “Peaking”**

Los Alamos Fel Oscillator

**Experimental Transverse Second Harmonic Intensity Profiles
with Tilted Electron Beam**

Figure 5c